

Response to the Simple Thought Experiment

“ I ask the author to consider the following situation where the planet is an aqua planet, and the ocean is not in motion. This requires that in situ density is constant at each point on the real geoid surface (not the ellipsoidal approximation to it). The author's GFD is however non-zero and large in this situation; that is, his equation (22). But this turns out only to be that he has not chosen his vertical distance to be measured from the real geopotential. Rather he has chosen the zero of his height to be in an ellipsoidal surface. So his equations show substantial motion, but we know that there should be no motion.”

I disagree.

Equation (22) in the manuscript is written by

$$\text{GDF} = \int_{-H}^0 \left(\frac{\partial \rho}{\partial x} \frac{\partial T}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial T}{\partial x} \right) dz = \int_{-H}^0 \left[\mathbf{k} \cdot (\nabla \rho \times \nabla T) \right] dz$$

Consider that the in-situ density is constant at each point on the true geopotential surface, i.e., the isopycnal surface coincides with the true geopotential surface. This requires that the two vectors $\nabla \rho$ and ∇T are parallel, i.e.,

$$\nabla \rho \times \nabla T = 0$$

which leads to

$$\text{GDF} = 0$$

which shows that the GDF does not drive any motion. It is the opposite outcome as you thought. This simple thought experiment demonstrates the merit of the manuscript.

Response on the Geopotential and Geopotential Surface

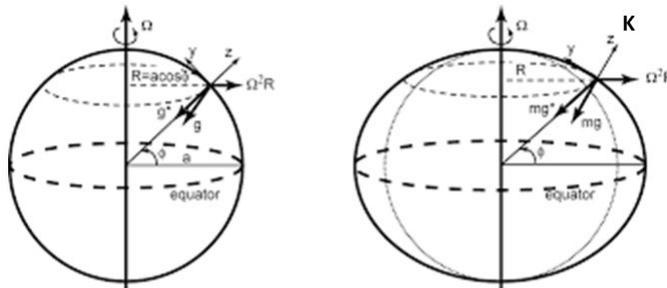
“The development of the equations with respect to the geoid is done in textbooks, for example in the early pages of the text "Fundamentals of Ocean Climate Models" by S. M. Griffies, published in 2004. These ocean models do not put the ocean in motion if the in situ density is constant on geopotential surfaces.”

The geopotential and geopotential surface used in oceanography and meteorology including in the text "Fundamentals of Ocean Climate Models" by S. M. Griffies are the normal geopotential and normal geopotential surface, but not the TRUE GEOPOTENTIAL and TRUE GEOPOTENTIAL SURFACE.

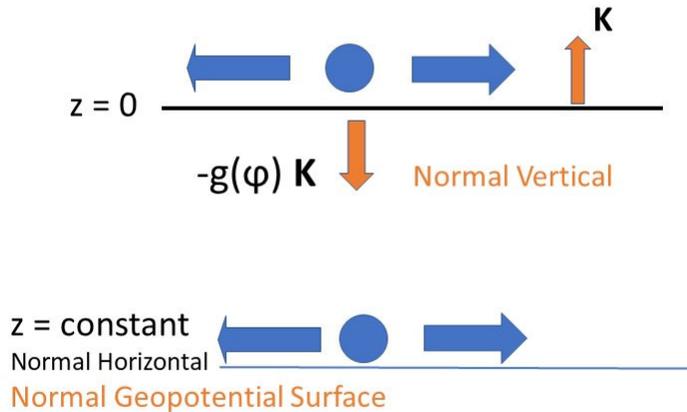
The two attached figures illustrate the difference between the normal gravity which is called the effective gravity and used in oceanography and meteorology, and the true gravity which is the most important variable in geodesy.

Figure A shows the main features of the effective gravity $[-g(\varphi)\mathbf{K}]$: (1) it is determined from the solid Earth with rotation and uniform mass density; (2) the unit vector \mathbf{K} is perpendicular to the z surface ($z = \text{constant}$) and points the normal vertical; (3) the z surface is the normal horizontal and coincides with the normal geopotential surface; (4) any movement on the z surface (i.e., normal geopotential surface) is not against the normal gravity.

(A) Normal (or called Effective) Gravity
(Uniform Mass Density inside the Solid Earth)
Normal Geopotential Surface = z Surface



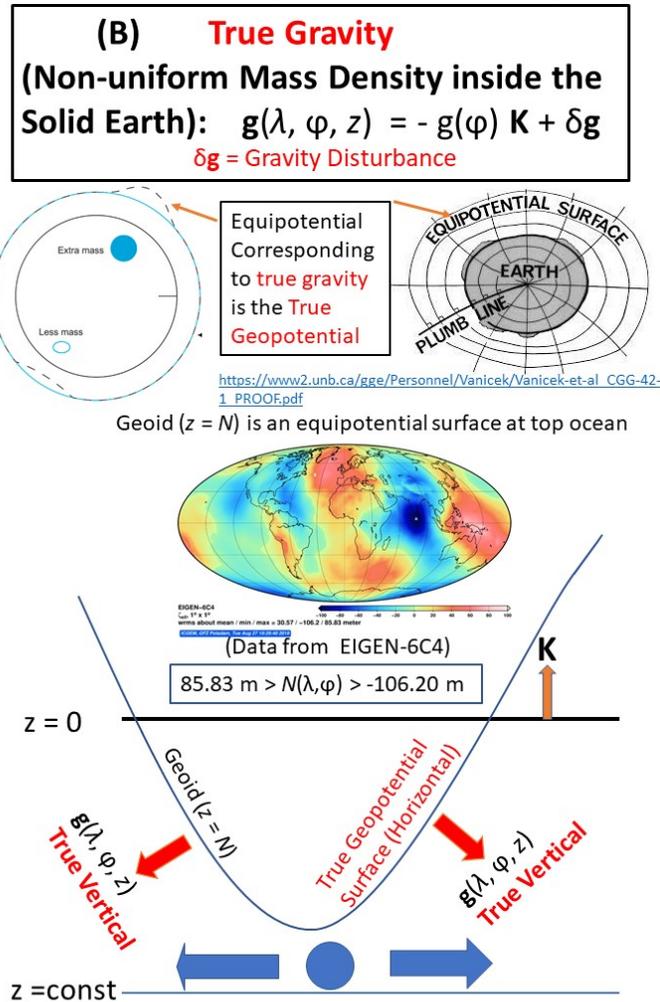
The two figures are from the website:
https://atoc.colorado.edu/~cassano/atoc5050/Lecture_Notes/hh_ch1.pdf



Any movement on $z = \text{constant}$ (i.e., z surfaces) is not against the **effective gravity**.
 The **normal geopotential surface** coincides with the z -surface.

Fig. A. Illustration of normal gravity, normal geopotential, normal vertical, and normal horizontal, which are used in atmospheric and oceanic dynamics.

Figure B shows the main features of the true gravity [$\mathbf{g}(\lambda, \varphi, z) = -g(\varphi)\mathbf{K} + \delta\mathbf{g}$]: (1) it is determined from the solid Earth with rotation and non-uniform mass density; (2) the true gravity has never been used in oceanography and meteorology; (3) the true gravity vector $\mathbf{g}(\lambda, \varphi, z)$ is perpendicular to the true geopotential surface such as the geoid surface, which represents the true horizontal; (4) any movement on the true geopotential surface is not against the true gravity; (5) any movement on the z -surface is against the true gravity. An additional force, the gravity disturbance (T), shows up in the z -surface momentum equations, such as in Equation (18) of the manuscript.



- (1) Any movement along the geoid surface (**true horizontal surface**), $z = N(\lambda, \varphi)$, (-106.20 m to 85.83 m, from EIGEN-6C4) is not against the **true gravity**.
- (2) Any movement on the z -surface is **against the true gravity**. An additional force, **Gravity Disturbance**, shows up in the z -surface momentum equations.

Fig. B. Illustration of true gravity, true geopotential, true vertical, and true horizontal, which should be used in atmospheric and oceanic dynamics