Authors' responses to Reviewer 1 comments on the MS 'Model-to-model data assimilation method for fine resolution ocean modelling' by Shapiro and Ondina

Comment. *The manuscript presents a novel method for DA in a high-resolution ocean model*

Response. Thank you

Comment. *My main concern with this manuscript is how well suited the three cases are to demonstrate the potential of the method for the proposed application, namely high-resolution ocean models. All three examples are two-dimensional fields of one state variable that can be described by smooth functions.*

Response. The two-dimensional fields used in the MS represent data from a single computational (geopotential or sigma) level taken from a full 3D mesh. In order to carry out DA in full 3D the process has to be repeated for all levels. Such approach is widely used in practical applications of DA in ocean and atmospheric modelling. For example, the papers referenced in our original MS (Adhikary el al. 2008) and (Bell et al. 2000) present schemes where forecast error correlations only include horizontal dependencies and DA is performed level-by-level in the same way that is used in our method. Clarification has been given in the text.

Comment. *I find the lack of small-scale variations that would mimic sub-mesoscale features in the true fields to be an unrealistic assumption in the presented experiments, as such variations will always be present in any realistic case.*

Response. Whether an ocean feature is mesoscale or sub-mesoscale depends on the Rossby radius (commonly the first baroclinic radius is used)- see (Robinson, 1983). If the Rossby radius is about R=50 km (as in the mid Atlantic) or even 10 km (as in some coastal areas, e.g. the Persian Gulf) then the eddies of 6 km in radius, treated in the Example b) of the MS are definitely sub-mesoscale. The eddies of 46 km radius again considered in Example b) could be classed as mesoscale at R=50 km. The minimum size of sub-mesoscale features resolved by a model is determined by the resolution of the child (fine) model not by the SDDA methodology. The SDDA method and the examples in the MS are not specific to a certain value of Rossby radius, and therefore can be applied to both meso and sub-meso features. In order to clarify this issue, we have added the relevant explanation.

Comment. *The examples in the manuscript do not address the suitability of the proposed method for regions where increased resolution is applied to better resolve complex topography and coastlines with mismatch in land mask between parent and child models, nor is this issue addressed in the discussion.*

Response. A potential mismatch between fine and coarse grids due to finer features of the coastline and bathymetry can only relate to the first step of the SDDA method, namely the downscaling. This issue is not specific to SDDA and may appear when gridded observational data of different resolution (e.g. satellite imagery) is used in common data assimilation procedures. The downscaling of the coarse model data onto the fine model grid is carried out using the SDD algorithm. As shown in (Shapiro et al, 2021) in the example of the coastal areas of the Red Sea, this mismatch is natively resolved during the SDD downscaling. Better resolution of the coastline/bathymetry could result in higher vorticity/enstrophy values in the downscaled field compared to the parent coarse model as shown in Fig 11-15 of the above paper. After downscaling the data from the parent and child model are available exactly on the same (fine) grid, and the grid mismatch issue does not appear. Additional reference and clarification are given in the revised MS.

Comment. *Another question that remains unanswered is whether the description of the ocean state is dynamically consistent after an assimilation procedure that is applied point-wise for each state variable. If this is not the case, it will most likely result in numerical instabilities when initializing the forecast from such an analysis and thus additional post-processing will be required for the method to be applicable.*

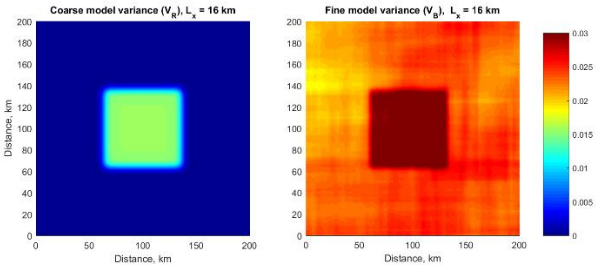
Response. This issue is common to a variety of data assimilation methods, it is not specific to assimilating data from another model instead of observations. In a wider context, hydrostatic instability could appear when external data are incorporated into the model. An example is the initialisation of numerical model from climatological fields of temperature and salinity when separate interpolation of the state variables to the model grid may result in inversion in density. The issue is well known and is dealt with in a number of ways. For example, in the NEMO model, density inversions are treated with highly enhanced vertical diffusion, so that the inversions are removed in a few time steps. Clarification is added to the text.

Comment. *Line 63-64: The synthetic cases are not introduced or otherwise mentioned in the main body of the manuscript before this point. I would suggest either rephrasing or including a sentence or two in the above paragraph.*

Response. The text amended as advised.

Comment. *$\boldsymbol{V}_B$ and $\boldsymbol{V}_R$ determine how much weight is given to the fluctuations of the background vs the fluctuations of the downscaled product. I cannot see that there's any mention of how the values of these key parameters are set, nor a discussion on how these choices affect the results. The aim should be to weight the two solutions in a way where the child model is prevented from drifting from the assimilated parent model, while at the same time retaining high-resolution dynamical features that arise from e.g. improved topography.*

Response. Diagonal matrices $\boldsymbol{V}_B$ *and* $\boldsymbol{V}_R$ are related to the error covariance matrices $\boldsymbol{B}$ and $\boldsymbol{R}$ of the child and parent models respectively as specified in lines 114-115 of the original MS. Therefore, the values of these key parameters are not set by a modeller, i.e., they are not tuning coefficients but calculated using an algorithm similar to other variational DA methods. This point is briefly discussed in the original MS (lines 436-438). As both data sets are from models (the parent model outputs are used instead of real observations) then any suitable method to calculate the matrix$\boldsymbol{B}$, can be applied to calculate matrix $\boldsymbol{R}$, and hence $\boldsymbol{V}_B$ *and* $\boldsymbol{V}_R$, for example the NMC (Parrish, D. F. and Derber, J. C., 1992: The National Meteorological Center's spectral statistical interpolation analysis system. Mon. Weather Rev., 120, 1747-1763) or 'Canadian' (Polavarapu et al, 2005, Data assimilation with the Canadian middle atmosphere model, Atmosphere-Ocean, 43:1, 77-100, DOI: 10.3137/ao.430105) methods. In the examples presented in the MS, the diagonal elements V_{Bii} and V_{Rii} (see Eq 10) required for the second step of SDDA are calculated, for consistency, in the same way as for the first step (downscaling) , namely of by spatial averaging and calculating dispersion of fluctuations over a small trial area around the node at the same time point, see lines 152-153 of the original MS which refers the reader to (Shapiro et al, 2021) for details. The trial area was a square of 68 x 68 km centred at each node (see lines 203-204 of the original MS). We agree that it would be easier for a reader if such details are presented in greater detail in the actual MS. The text of the revised MS is extended to incorporate this and a new figure (see below) showing an example map of VBii and V_{Rii} is added, along with the clarifying text .



Comment. *The proposed SDDA method consists of two steps, namely a stochastic downscaling of the parent model to the grid of the child model, and an assimilation step where the downscaled parent model values are treated as observations and combined with the first guess of the child model. I think an analysis of how these two steps contribute to the total improvement of the SDDA could provide valuable insights. Given the smooth nature of the chosen examples, I suspect the DA step might increase RMSE and bias compared to the intermediate solution given by the SD step.*

Response. An additional analysis is provided in the discussion section of the revised MS as advised.

Comment. *Again, the choices for **B** and **R** will strongly affect the results and it would thus be relevant to report whether or not they differ significantly between the two methods, as well as how changing their values affect the results.*

Response. The **B** and **R** matrices are not prescribed (chosen) but calculated according to the SDDA and H-L methods accordingly, so there is no option to change them arbitrarily in order to see how their values affect the results**.** The procedures for calculation of ***B*** and ***R*** matrices in the SDDA and H-L methods are different and therefore they produce different matrices. For SDDA, both matrices represent the covariances of the background error of the downscaled parent and child models respectively and are calculated in a similar and consistent way as both data sets are from models. Therefore, whilst both matrices have different error variances (diagonal elements), they have the same spatial correlations. Conversely, in the 'standard' method, the matrices are calculated using the H-L method that assumes that the observation errors have no spatial correlation. This assumption can be valid for some types of observations, but it is not valid for model-to-model DA.

The revised MS is amended to include the discussion of differences in **B** and **R** matrices for the two methods as advised.

Comment. *Evaluating the cost function values in addition to RMSE and bias could perhaps also help to shed light on the differences.*

Response. The cost functions for the 'standard' (see Eq (1)) and the SDDA (see Eq (2)) methods are different, so it is not clear what information can be gained from the direct comparison. Both methods minimize their own cost functions. Clarification is given in the revised MS.

Comment. *As spectral nudging (see e.g. Katavouta and Thompson, 2016) addresses the very same issue as the proposed method, namely ensuring that a high-resolution model does not drift away from the large-scales that are well constrained by observations assimilated in coarser ocean models, I think a discussion on how the SDDA method compares with spectral nudging both in terms of quality of the results and computational efficiency would be of interest to the target audience for this manuscript.*

Response. The study by Katavouta and Thompson (2016) used a spectral nudging method in order to restrict the drift of the fine scale model from the global model. They described a method that 'nudges' the large scale spectral components of a regional model to those of a global model. The main difference with the SDDA is that the nudging technique uses weighting coefficients that are tuning parameters and are prescribed in advance, while the SDDA variational method uses weights computed from the variance of the errors by minimising the cost function and therefore they cannot be changed at will. In contrast to the spectral nudging, the SDDA method corrects both large and small scale components of the child model as seen in amplitude spectra shown in Fig.14 of the original MS. The removal of bias in the Fourier space can be seen in Fig. 14 of the original MS. Some similarity can be found in the fact that the bias of the child model is replaced with the bias from the parent model which could be interpreted as aggressively nudging of a single long-wave component of the field. The relevant reference and discussion are added in the revised MS as advised.

Comment. *The authors might want to consider adopting scientific colormaps and avoid using red and green colored lines in the same plots. See e.g. Crameri et al. (2020).*

Response. The green colour is replaced with a different shade in line-art plots in the revised MS as advised. Different markers have been also added to improve readability of plots with many different lines. The jet colour map which contains both red and green colour is used for compatibility with the EU Copernicus Marine Service products, and other recent papers published in Ocean Science, where maps contain both green and red, e.g. doi.org/10.5194/os-17-1385-2021 ; doi.org/10.5194/os-17-833-2021; doi.org/10.5194/os-17-615-2021 etc.

Comment. *Equation 1: the observation vector* ***y*** *should be in bold font*

Response. Corrected as advised.

Comment. *Line 84: A period is missing after the reference*

Response. Corrected as advised.

Comment. *Line 96: a "b"-superscript seems to be missing from the left-hand side of the equation.*

Response. Corrected as advised.

Comment. *Line 241: "an analysis" or perhaps initial conditions?*
Response. The MS considers only one assimilation cycle, therefore the 'analysis' of the previous cycle acts as initial condition for the next cycle. Clarification is given.

Comment. *Although the section indeed presents results for cases A-D, the statement at line 242 of "four examples" reads as a typo in the current context.*

Response. The sentence is corrected to include all 4 examples as advised

Comments. *Line 424: observation**s**.*
*Line 468: mode**l**-to-model*

Responses. Both typos corrected as advised.