Response to Dr. David Webb by Guohong Fang and Di Wu

Overview

This is a classic semi-analytical study of a partially enclosed tidal system. The mathematics is fairly straightforward but the authors use the results to obtain a better physical understanding for the position of the amphidromes in the strait between Korea and Japan. The paper is well laid out and easy to read and understand. I think that in principal it should be published.

Reply: We sincerely thank Dr.Webb for his careful reading of our manuscript and constructive comments and suggestions, which are of great help in improving our study. We have addressed all these comments; our responses are given below.

Main suggestions

As I said the mathematics is fairly straightforward (maybe that is why JPO rejected the m/s), so I do not think all the details are needed in the final paper. In particular I think that the content of the appendices may be better placed in a separate document as supplementary material (a possibility with Ocean Science).

Reply: The appendix has been deleted and will be submitted separately in the form of supplementary material.

I am also concerned that this branch of oceanic literature always ignores similar studies that have occurred in related fields of physics - in particular microwave wave guides. There used to be a complaint about the different branches of physics reinventing the wheel and to a certain extent this is true here as the Coriolis term does not necessarily introduce major changes.
For that reason I suggest that the authors, who appear to be applied mathematicians, talk to someone with a physics or microwave background about reflections from discontinuities in impedance (refractive index in the case of light). This should give a bit more insight which they could usefully add to their conclusions.

Reply: The behaviour of water wave reflection in a nonrotating channel is indeed similar to the microwave reflection, or the light refraction. However, when the wave propagates in a rotating channel and the period of the wave is comparable to that of Earth's rotation the Coriolis force will have significant influence on the wave propagation and reflection. As an example, we revisit the problem of the reflection of the Kelvin wave in a semi-infinite channel first studied by Taylor (1922). Taylor' result shows that when the incident Kelvin wave is reflected at the southern shore of the North Sea, a time lag of 1.4 hr occurs due to the Coriolis force. The details are given in the appendix to this response. The main conclusion is that the Coriolis parameter has significant influence on the tidal wave reflection in a semi-infinite channel. This conclusion should also hold for the case studied in the present paper.

As another possibility for future work I would also suggest treating all variables as complex and investigating how the solutions at key points change with complex angular velocity - to understand how the resonant properties of the system affect the solution.

Reply: This is a very useful suggestion for our future work. We will try to apply the complex angular velocity to the Taylor method.

Detailed comments

1. Title

I suggest "Study of the ..."

Reply: Revised as suggested.

2. Page 1, line 9

Similarly "studies of the tides ..."

Reply: Revised as suggested.

3. Page 1, line 23

" ... the Yellow Sea ..."

Reply: Revised as suggested.

4. Page 1, line 26

Delete "vast".

Reply: Revised as suggested.

5. Page 1, line 27

Knives are sharp, continental slopes are steep.

Reply: The word "sharp" has been replaced with "steep" in page 1, line 27 and page 16, line 4.

6. Page 2, line 18

I disagree with "analytical", this is a semi-analytical method, using the numerical solution of a large set of equations.

Reply: The word "analytical" has been replaced with "semi-analytical".

7. Page 4, line 21

This is angular velocity (radians per second) Anything with frequency refers to full cycles of something.

Reply: The term "angular frequency" has been replaced with "angular velocity" in page 1, line 18, page 4, line 21 and page 5, line 13. (Please note that angular frequency is a synonym of angular velocity, see Weik M.H. (2000) angular frequency. In: Computer Science and Communications Dictionary. Springer, Boston, MA. https://doi.org/10.1007/1-4020-0613-6_670).

8. Page 5, line 8
Change to “with momentum ...”

Reply: The article “the” has been deleted.

9. Page 16, lines 10 onwards.
This is all very standard in other areas of physics as well, so I do not think the work of Dean and Dalrymple needs to be spelt out in such detail. I suggest that you just give the results you need.

Reply: According to this comment, we have revised this paragraph as follows: “If the second area is semi-infinitely long, allowing for the wave to radiate out from the second area freely, then a part of the wave is reflected at the connecting point and another part is transmitted into the second area. The amplitude of the transmitted wave is (see e. g. Dean and Dalrymple (1984))”.

10. Page 17, line 1
You do not make clear which case you are writing about - yours or that of Dean and Dalrymple.

Reply: This equation is the same as that given by Dean and Dalrymple (1984), but in a more understandable form. In order to clarify this the words “(see also Dean and Dalrymple (1984))” have been added above this equation.

11. Page 18, line 9 and following
“can be attributed to ...”. This is a bit of a cop out, the classic response of a committee shirking responsibility. It would read better if you were disappointed about the discrepancy but that it may be due to ... .

Reply: The words “can be attributed to” have been replaced with “may be duo to”.

12. Page 19, Line 21.
I would suggest you delete this line. It is doing nothing useful.

Reply: This line has been deleted.

Appendix: A Short Note on the Reflection of the Kelvin Wave in a Semi-infinite Channel: Taylor’s Example Revited

1. Introduction

Taylor (1922) studied the tidal system in a semi-infinite channel, with especial attention paid to the refection of the Kelvin wave at the closed end of the channel. The channel he studied is semi-infinite with a width of W and a uniforn depth of h as shown in Fig. 1. He showed that when an incident Kelvin wave propagates into the rotating channel, the wave would be reflected at the closed end to form a reflected Kelvin wave and an amphidromic system. Meanwhile, a series of Poincare modes would be induced in the vicinity of the closed end.

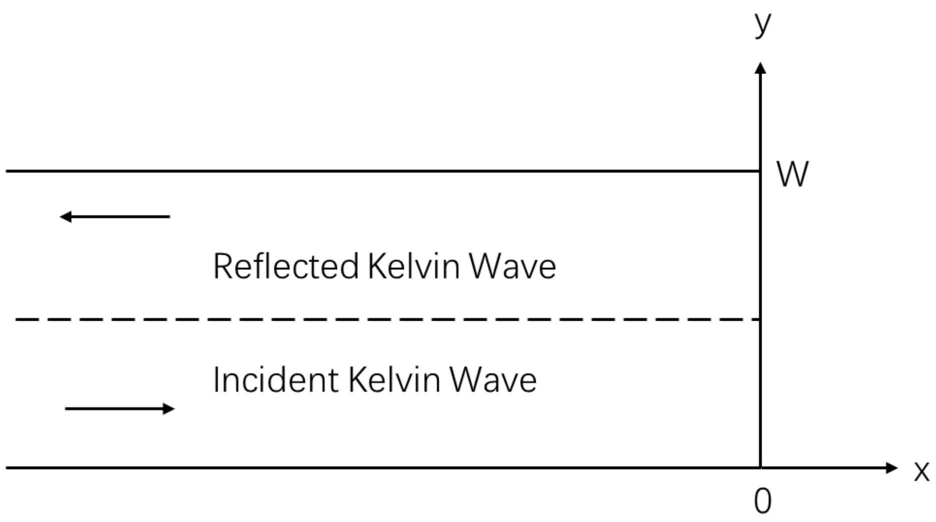

Fig.A1 Sketch of the semi-infinite channel.

2. Taylor's Example

Let

$$\begin{cases} \alpha = \frac{f}{c}, \\ k = \frac{\sqrt{\sigma^2+f^2}}{c}, \end{cases} \tag{R1}$$

where f is Coriolis parameter, σ is the angular velocity of the wave, c is defined as

$$c = \frac{\pi}{W}\sqrt{gh} \tag{R2}$$

with W and h representing the width and depth of the channel respectively. Taylor (1922) specifically computed the relationship between the incident and reflected Kelvin waves with period equal to 12 hr (equivalent to an angular velocity of $1.4544 \times 10^{-4}\text{s}^{-1}$) for the case

$$\begin{cases} \alpha = 0.7, \\ k = 0.5, \end{cases} \tag{R3}$$

which corresponds to the dimensions of the North Sea. This case was referred to as Taylor's example by Brown (1973). The estimated phase-lag increase θ of the reflected Kelvin wave versus the incident Kelvin wave at the closed end of the channel was equal to 42.10° (Taylor, 1920, p. 166). This result indicates that when the incident Kelvin wave is reflected at the southern shore of the North Sea, a time lag of 1.4 hr occurs due to the Coriolis force. The value of θ was estimated again by Brown (1973), yielding θ =42.18° (see also Thiebaux, 1988, p.369).

3. Influence of the Coriolis parameter on the reflection of the incident Kelvin wave

To illustrate the Influence of the Coriolis parameter on the reflection of the incident Kelvin wave, we artificially change the values of the Corisolis parameter, and apply the method discribed in our paper to the semi-infinite channel shown in Fig.1. The channel is taken 463.3 km wide (corresponding to 250 nautical miles as given by Taylor (1922)) and 63.4 m deep, then we truncate the Poincare modes up to 100 terms and calculate the values of θ for various values of f. The result is shown with the red curve in Fig. 2. This figure indicates that the value of θ is zero when $f = 0$, and can be up to nearly 50° when $f = 1.4 \times 10^{-4}\text{s}^{-1}$.

For the case of Tayor's example which satisfies Eq. (3) we can obtain $f = 1.1835 \times 10^{-4}\mathrm{s}^{-1}$ through eliminating c in Eq. (1) and inserting Eq. (3). This particular value of f is indicated with a vertical dashed line in Fig.2, and the corresponding value of θ is 42.16°.

Fang and Wang (1966) proposed an approximate equation for θ as follows (note that the Eq. (60) of their paper is the expression for $\theta/2$):

$$\theta = \frac{8\nu^3}{\pi l(l^2+\nu^2)\sqrt{l^2+\nu^2-1}\,\mathrm{th}\frac{\pi\nu}{2l}}, \quad \text{(R4)}$$

where

$$\nu = \frac{f}{\sigma}, \quad \text{(R5)}$$

and

$$l = \frac{c}{\sigma}. \quad \text{(R6)}$$

The values of θ derived from (4) as function of f are shown in blue curve in Fig. 2. In particular, the value of θ corresponding to $f = 1.1835 \times 10^{-4}\mathrm{s}^{-1}$ is equal to 41.68°.

Thiebaux (1988) also proposed an approximate method for calculating θ. His equation has the form

$$\theta = b_1\nu' + b_3\nu'^3 + O(\nu'^5) \quad \text{(R7)}$$

where

$$\nu' = \frac{\sigma}{f}. \quad \text{(R8)}$$

Thiebaux (1988) did not provide any formula for calculating $O(\nu'^5)$. The expressions of b_1 and b_3 are quite complicated, but their values can be calculated from his Eqs. (30) and (31) and his Table 1. The values of θ derived from (7) as function of f are shown in green curve in Fig. 2. In particular, the value of θ corresponding to $f = 1.1835 \times 10^{-4}\mathrm{s}^{-1}$ is equal to 37.19°.

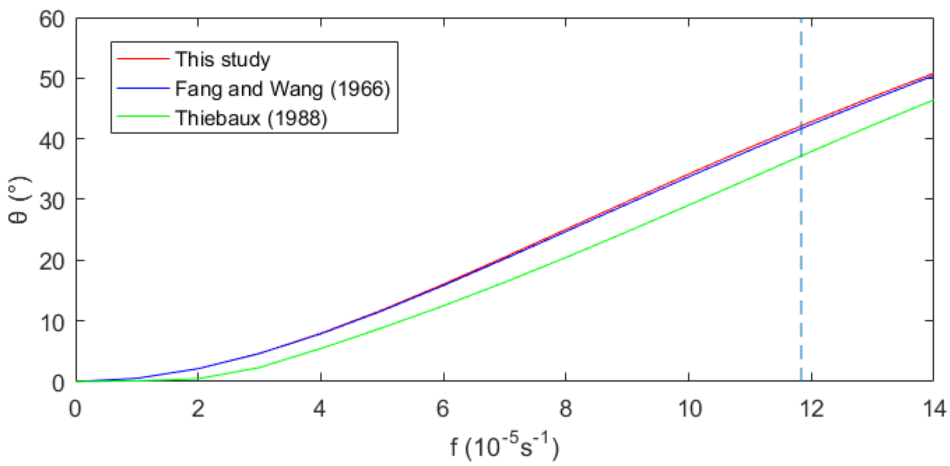

Fig. 2 The phase-lag increase (θ) of the reflected Kelvin wave versus the incident Kelvin wave at the closed end as function of the Coriolis parameter (f) in a semi-infinite channel, which has a width of 463.3 km and a unifom depth of 63.4 m.

4. Conclusion

The works of Taylor (1922), Fang and Wang (1966), Brown (1973), Thiebaux (1988) and the present study all show that the Coriolis parameter has significant influence on the tidal wave reflection in a semi-infinite channel.

References

Brown, P. J.: Kelvin-wave reflection in a semi-infinite canal. J. Mar. Res., 31, 1-10, 1973.

Fang, G., and Wang, J.: Tides and tidal streams in gulfs, Oceanol. Limnol. Sin., 8, 60–77, 1966. (in Chinese with English abstract).

Taylor, G. I.: Tidal oscillations in gulfs and rectangular basins. Proc. London Math. Soc., Ser. 2, 20, 148-181, https://doi.org/10.1112/plms/s2-20.1.148, 1922.

Thiebaux, M. L.: Low-frequency Kelvin wave reflection coefficient. J. Phys. Oceanogr., 18, 367-372, https://doi.org/10.1175/1520-0485(1988)018<0367:LFKWRC>2.0.CO;2, 1988.