Appendix: A Short Note on the Reflection of the Kelvin Wave in a Semi-infinite Channel: Taylor's Example Revited

1. Introduction

Taylor (1922) studied the tidal system in a semi-infinite channel, with especial attention paid to the refection of the Kelvin wave at the closed end of the channel. The channel he studied is semi-infinite with a width of W and a uniforn depth of h as shown in Fig. 1. He showed that when an incident Kelvin wave propagates into the rotating channel, the wave would be reflected at the closed end to form a reflected Kelvin wave and an amphidromic system. Meanwhile, a series of Poincare modes would be induced in the vicinity of the closed end.

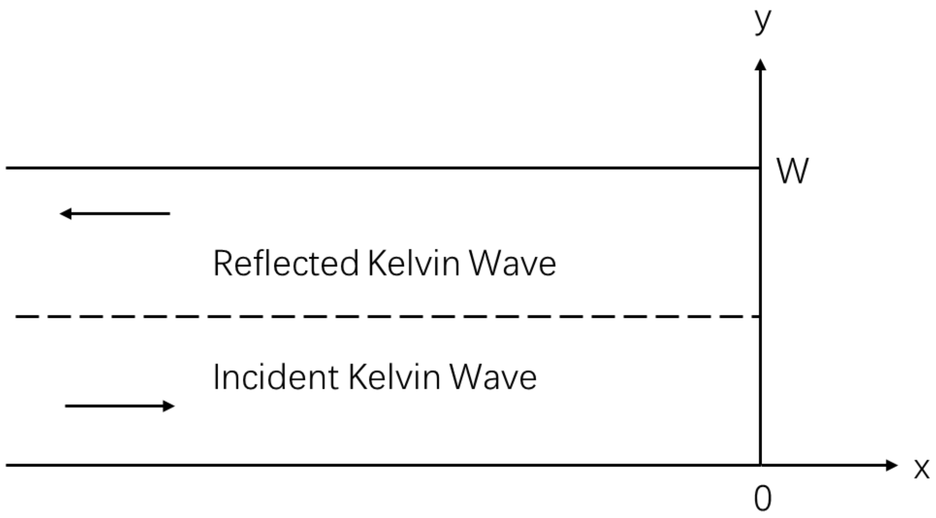

Fig.1 Sketch of the semi-infinite channel.

2. Taylor's Example

Let

$$\begin{cases} \alpha = \frac{f}{c}, \\ k = \frac{\sqrt{\sigma^2+f^2}}{c}, \end{cases} \tag{1}$$

where f is Coriolis parameter, σ is the angular velocity of the wave, c is defined as

$$c = \frac{\pi}{W}\sqrt{gh} \tag{2}$$

with W and h representing the width and depth of the channel respectively. Taylor (1922) specifically computed the relationship between the incident and reflected Kelvin waves with period equal to 12 hr (equivalent to an angular velocity of $1.4544 \times 10^{-4}\mathrm{s}^{-1}$) for the case

$$\begin{cases} \alpha = 0.7, \\ k = 0.5, \end{cases} \tag{3}$$

which corresponds to the dimensions of the North Sea. This case was referred to as Taylor's example by Brown (1973). The estimated phase-lag increase θ of the reflected Kelvin wave versus the incident Kelvin wave at the closed end of the channel was equal to 42.10° (Taylor, 1920, p. 166). This result indicates that when the incident Kelvin wave is reflected at the southern shore of the North Sea, a time lag of 1.4 hr occurs due to the Coriolis force. The value of θ was estimated again by Brown (1973), yielding θ =42.18° (see also Thiebaux, 1988, p.369).

3. Influence of the Coriolis parameter on the reflection of the incident Kelvin wave

To illustrate the Influence of the Coriolis parameter on the reflection of the incident Kelvin wave, we artificially change the values of the Corisolis parameter, and apply the method discribed in our paper to the semi-infinite channel shown in Fig.1. The channel is taken 463.3 km wide (corresponding to 250 nautical miles as given by Taylor (1922)) and 63.4 m deep, then we truncate the Poincare modes up to 100 terms and calculate the values of θ for various values of f. The result is shown with the red curve in Fig. 2. This figure indicates that the value of θ is zero when $f = 0$, and can be up to nearly 50° when $f = 1.4 \times 10^{-4}\mathrm{s}^{-1}$.

For the case of Tayor's example which satisfies Eq. (3) we can obtain $f = 1.1835 \times 10^{-4}\mathrm{s}^{-1}$ through eliminating c in Eq. (1) and inserting Eq. (3). This particular value of f is indicated with a vertical dashed line in Fig.2, and the corresponding value of θ is 42.16°.

Fang and Wang (1966) proposed an approximate equation for θ as follows (note that the Eq. (60) of their paper is the expression for $\theta/2$):

$$\theta = \frac{8\nu^3}{\pi l(l^2+\nu^2)\sqrt{l^2+\nu^2-1}\,\mathrm{th}\frac{\pi\nu}{2l}}, \tag{4}$$

where

$$\nu = \frac{f}{\sigma}, \tag{5}$$

and

$$l = \frac{c}{\sigma}. \tag{6}$$

The values of θ derived from (4) as function of f are shown in blue curve in Fig. 2. In particular, the value of θ corresponding to $f = 1.1835 \times 10^{-4}\mathrm{s}^{-1}$ is equal to 41.68°.

Thiebaux (1988) also proposed an approximate method for calculating θ. His equation has the form

$$\theta = b_1\nu' + b_3\nu'^3 + \mathrm{O}(\nu'^5) \tag{7}$$

where

$$\nu' = \frac{\sigma}{f}. \tag{8}$$

Thiebaux (1988) did not provide any formula for calculating $\mathrm{O}(\nu'^5)$. The expressions of b_1 and b_3 are quite complicated, but their values can be calculated from his Eqs. (30) and (31) and his Table 1. The values of θ derived from (7) as function of f are shown in green curve in Fig. 2. In particular, the value of θ corresponding to $f = 1.1835 \times 10^{-4}\mathrm{s}^{-1}$ is equal to 37.19°.

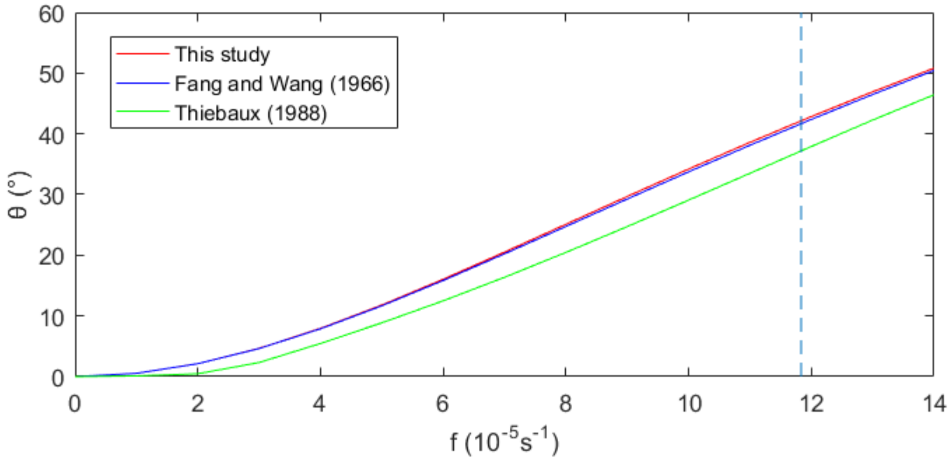

Fig. 2 The phase-lag increase (θ) of the reflected Kelvin wave versus the incident Kelvin wave at the closed end as function of the Coriolis parameter (f) in a semi-infinite channel, which has a width of 463.3 km and a unifom depth of 63.4 m.

4. Conclusion

The works of Taylor (1922), Fang and Wang (1966), Brown (1973), Thiebaux (1988) and the present study all show that the Coriolis parameter has significant influence on the tidal wave reflection in a semi-infinite channel.

References

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