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Interactive comment on "Spiciness theory revisited, with new views on neutral density, orthogonality and passiveness" *by* Rémi Tailleux

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Response to Referee 3

Tailleux presents new ideas around spiciness in the ocean. I think this is a worthwhile paper with some interesting points being made. A number of the key conclusions don't seem well supported though. Some points are presented as self-evident, yet their justification seems far from obvious. Furthermore, some analysis lacks rigour. I feel these are largely matters of presentation and I expect I will be able to recommend publication after major revision.

Response and proposed changes I thank Dr. Zika for his careful review and useful suggestions. I think that my analysis is rigorous enough but I agree in the light of Dr

Zika's comments that some of my arguments are not precise or tight enough. The main changes that I propose to implement following his suggestions, in addition to accounting for more specific suggestions, are:

1. Improve the discussion of orthogonality
2. Improve Fig. 11 by rescaling the variables by their standard deviation
3. Improve what I mean by 'optimality'

Specific issues:

1. **Orthogonality** Tailleux argues that the most appropriate spice variable should be orthogonal in geographical coordinates. I actually think this is a very important point but words like orthogonal and optimal are used frequently without their implementation actually being globally orthogonal, nor evidently 'optimal' in any way. Firstly, the importance of orthogonality is introduced with "As is well known, the most efficient way to represent a vector is achieved by decomposing it in an orthogonal basis" This statement (and similar statements about orthogonality) should be made more precisely. For example, does the word 'efficient' have a precise meaning here? If we are to apply rigour to the idea of developing an orthogonal basis, surely there is a fundamental issue that the gradient of any spice variable can vanish on an isopycnal (and clearly the along-section isopycnal gradient of all the spice variables shown in figure 11 vanish at various locations). The problem Tailleux is dealing with is in three-dimensional space yet neutral density and spice offer only two basis vectors. This should be clarified with regard to the motivation to have an orthogonal basis since the basis developed is clearly incomplete. I suggest a severe tone down of the language of 'orthogonal coordinates' unless these issues are to be discussed carefully. Perhaps more

crucially, it is unclear where and to what degree the modified spice variable eta' is actually orthogonal. How de we know if the reference profile $\xi_r(\sigma_1)$ is 'suitably constructed'? Fig.11 uses a polynomial fit of $\xi_r(\sigma_1)$ for a specific section for ξ_r. Doesn't this imply there is no perfect orthogonality anywhere? Why not choose η_r to be η at a specific latitude and longitude so at least local orthogonality is ensured? Or one could use the global isopycnal average of η Why not these other choices? More generally, there is no attempt to quantify how 'optimal' different methods for making eta' orthogonal are despite it the word optimal being used frequently throughout the paper.

Dr. Zika makes a number of legitimate points that I agree will need to be clarified in the revised version of the paper.

- The underlying physical problem that one tries to address here is how best to construct a new set of coordinates (γ, ξ) to isolate the active from the passive parts of (S, θ). Upon such a change of coordinates, functions of $f(S, \theta)$ are transformed into functions $\tilde{f}(\gamma, \xi)$. The gradient of such functions can be written in the following equivalent forms:

$$\nabla f = f_S \nabla S + f_\theta \nabla \theta = \tilde{f}_\gamma \nabla \gamma + \tilde{f}_\xi \nabla \xi = (\tilde{f}_\gamma + \tilde{f}_\xi \xi_r'(\gamma)) \nabla \gamma + \tilde{f}_\xi \nabla (\xi - \xi_r(\gamma)) \quad (1)$$

  Physically, when f is taken to be in-situ density $\rho = \tilde{\rho}(\gamma, \xi, p)$, one wants to:
  - Minimise the dependence of $\tilde{\rho}$ on ξ, that is make the partial derivative $\partial\tilde{\rho}/\partial\xi$ as small as possible. This is equivalent to make γ as neutral as feasible.
  - One also would like to minimise the contribution $\tilde{\rho}_\xi \nabla(\xi - \xi_r(\gamma))$ so that the term proportional to γ maximally projects on the neutral vector. To that end, it is easy to establish that one needs to maximise the orthogonality of $\nabla\xi'$ and $\nabla\gamma$. Locally, this is possible if one define $\xi_r(\gamma)$ so that

$$\xi_r'(\gamma) = \frac{\nabla\xi \cdot \nabla\gamma}{|\nabla\gamma|^2} \quad (2)$$

- The above equation provides a new way of computing $\xi_r(\gamma)$ that I did not fully realise when I originally wrote the paper, but which is different from specifying it in terms of polynomial interpolation or from some other form of isopycnal mean. I propose to revise the paper to be based on the above construction, which is more logical.
- Other than that, Dr. Zika is right that exact orthogonality cannot be imposed in the most general case. In that case, all what one can do is to maximise orthogonality, rather than strictly enforce it.
- Dr Zika is also right that one may want to use a more regional construction of $\xi_r(\gamma)$. However, if one wants to be able to compare the spiciness of various parts of the global ocean, $\xi_r(\gamma)$ has to be constructed globally.
- Note that for functions $f(S, \theta)$, ∇f is generated by only two vectors ∇S and $\nabla\theta$, so that its dimensionality is two rather than 3. The fact that ∇f lives in 3D space is irrelevant. With $\nabla\gamma$ and $\nabla\xi'$, one wants to create a basis to represent ∇f, not a basis for all possible three-dimensional vectors.

2. **Fig 11.** I think the variables shown in Fig. 11 are even closer than they appear. Both potential spiciness and spicity are in units of kg/mËĘ3 while Θ is in $^\circ C$ and S is in g/kg. As a consequence $\Theta - \Theta_r$ is saturated and $S - S_r$ is poorly resolved by the colour scale. There seems to not be a fundamental reason to care about the units of any of these coordinates since their utility is primarily in tracing water masses. So, I strongly encourage the author to rescale the colour axes (e.g. by dividing each by 1 standard deviation) so the variations in each variable are highlighted rather than their absolute values. This will likely show that all four variables look very similar in terms of their relative variations.
This is a very good idea that I will implement in the revision.

3. I am not sure if I saw it mentioned but it would be nice to see it pointed out that if the equation of state is indeed linear then all four of the diagnostics shown in

Fig.11 should be proportional (at least I am sure this is the case for Θ and S).

4. General references to previous work There are a lot of instances where what is written in previous work is generalised. These need to be either removed or replaced with concrete examples. For example on line 120 it says "So far, studies that have pursued orthogonality: : :have taken for granted: : :". Unless complete knowledge of all such studies can be claimed, it would be more appropriate to just point out that this has happened in some studies and provide references.
Thank for pointing this out. I'll endeavour to be more factual and specific in the revision.

Other comments and suggestions:
Thank for these. These will be accounted for when revising the paper

1. There were a large number of typos and a few terms left un-defined.
2. Generally, it makes more sense to me that 'I' is used instead of 'we' since this is a sole author paper.
3. A lot of the mathematics was difficult to follow often because basic variables and notation were not defined.
4. Line 14: What is a 'binary fluid'
5. Line 25: What is "de-compensate"
At the surface, a density-compensated temperature anomaly will be modified by air-sea interactions, without necessarily modifying the associated density-compensated salinity anomaly. As a result, the modified temperature anomaly can no longer be density compensated, hence the term 'de-compensate'.

6. Line 28: "isopycnal"
7. Line 45: "As *shown* in this paper" Line 72: I think I understand that f can be either gamma or eta. But as written it looks like f maps from Theta and S into gamma and eta space (e.g. the author writes $f = (\gamma, \eta)$). This whole paragraph could be expanded for clarity as it is important.
8. Line 80: What is γ_S? The partial derivative of gamma with respect to S?
9. Line 102: "As shown by" or "As Tailleux (2016a) showed" Eq 2: Define ρ_p and ρ_η
10. Line 120: "in a join*t* system". Also – its not clear what a 'joint system of physical units' is. Eq (5): Why no brackets around what is being logged here?
11. Line 139: Why ρ_{00} and not just ρ_0?
12. Line 177: Define 'quasi-material' Personal note: In our recent paper, Zika, J. D., J-B. Sallée, A. C. Naveira-Garabato, A. J. Watson, A. Meijers, M-J. Messias, B. King, 2020: Tracking the spread of a passive tracer through Southern Ocean water masses. Ocean Science.,16, 323–336, 2020, we attempted to construct a coordinate which was locally orthogonal to the along isopycnal direction and also materially conserved. The coordinate was essentially $S - S_r$. We chose $S - S_r$ because it was simpler to define than spice. Fig. 11 of this paper suggests this was a reasonable choice. Our salinity anomaly variable was used to help understand the ispoycnal spreading of a passive tracer. There are likely other examples of work that benefited from, or would have benefitted from, such 'spicy' coordinates. I feel this paper would be better motivated if more references were made to such studies.
I now remember that Dr. Zika mentioned this study to me at Ocean Sciences, and I am sorry that I forgot to cite it. This will be corrected in the revision.

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