



1 Effects of current on wind waves in strong winds

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15 **Keywords:** wind waves, current, Doppler shift

16

17 **Abstract**

18 It is important to investigate the effects of current on wind waves, called the Doppler
19 shift, both at normal and extreme high wind speeds. Three different types of wind-wave
20 tanks along with a fan and pump are used to demonstrate wind waves and currents in
21 laboratories at Kyoto University, Japan, Kindai University, Japan, and the Institute of
22 Applied Physics, Russian Academy of Sciences, Russia. Profiles of the wind and current
23 velocities and the water-level fluctuation are measured. The wave frequency, wavelength,
24 and phase velocity of the significant waves are calculated, and the water velocities at the
25 water surface and in the bulk of the water are also estimated by the current distribution.
26 The results show that 27 different types of currents can be generated at wind speeds
27 ranging from 7 to 67 m s⁻¹. At normal wind speeds under 30 m s⁻¹, wave frequency,
28 wavelength, and phase velocity depend on wind speed and fetch. The effect of the
29 Doppler shift is confirmed at normal wind speeds, i.e., the significant waves are
30 accelerated by the surface current. The phase velocity can be represented as the sum of
31 the surface current and artificial phase velocity, which is estimated by the dispersion
32 relation of the deep-water waves. At extreme high wind speeds, over 30 m s⁻¹, a similar
33 Doppler shift is observed as under the conditions of normal wind speeds. This suggests
34 that the Doppler shift is an adequate model for representing the acceleration of wind



35 waves by current, not only for the wind waves at normal wind speeds but also for those
36 with intensive breaking at extreme high wind speeds. A weakly nonlinear model of
37 surface waves at a shear flow is developed. It is shown that it describes well the
38 dispersion properties of not only small-amplitude waves but also strongly nonlinear and
39 even breaking waves, typical for extreme wind conditions (over 30 m s^{-1}).

40

41 **1. Introduction**

42 The oceans flow constantly, depending on the earth's rotation, tides, ground shape,
43 and wind shear. High-speed continuous ocean flows are called currents. Although the
44 mean surface velocity of the ocean is approximately 0.1 m s^{-1} , the maximum surface
45 velocity for the currents is 1 m/s (e.g., Kawabe, 1988; Kelly et al., 2001). The interaction
46 between the current and wind waves generated by the wind shear have been investigated
47 in several studies. The acceleration effects of the current on wind waves (well known as
48 the Doppler shift), the effects of the current on the momentum and heat transfer across a
49 sea surface, and the modeling of waves and currents in the Gulf Stream have been the
50 subject of experimental and numerical investigations (e.g., Dawe and Thompson, 2006;
51 Kara et al., 2007; Fan et al., 2009; Shi and Bourassa, 2019). However, these studies were
52 performed at normal wind speeds only, and few studies have been conducted at extreme
53 high wind speeds, for which the threshold velocity is $30 - 35 \text{ m s}^{-1}$, representing the
54 regime shift of the air-sea momentum, heat, and mass transport (Powell et al., 2003;
55 Donelan et al., 2004; Takagaki et al., 2012, 2016; Troitskaya et al., 2012; Iwano et al.,
56 2013; Krall and Jähne, 2014; Komori et al., 2018; Krall et al., 2019). At such extreme
57 high wind speeds, owing to the intensive breaking by the strong wind shear, the local
58 ocean flows might be strong. Furthermore, under a hurricane, the directions of the wind
59 and ocean flows rapidly change; thus, the wind waves under a hurricane might be
60 strongly affected by complicated local ocean flows. However, the effects of the current on
61 wind waves have not yet been clarified.

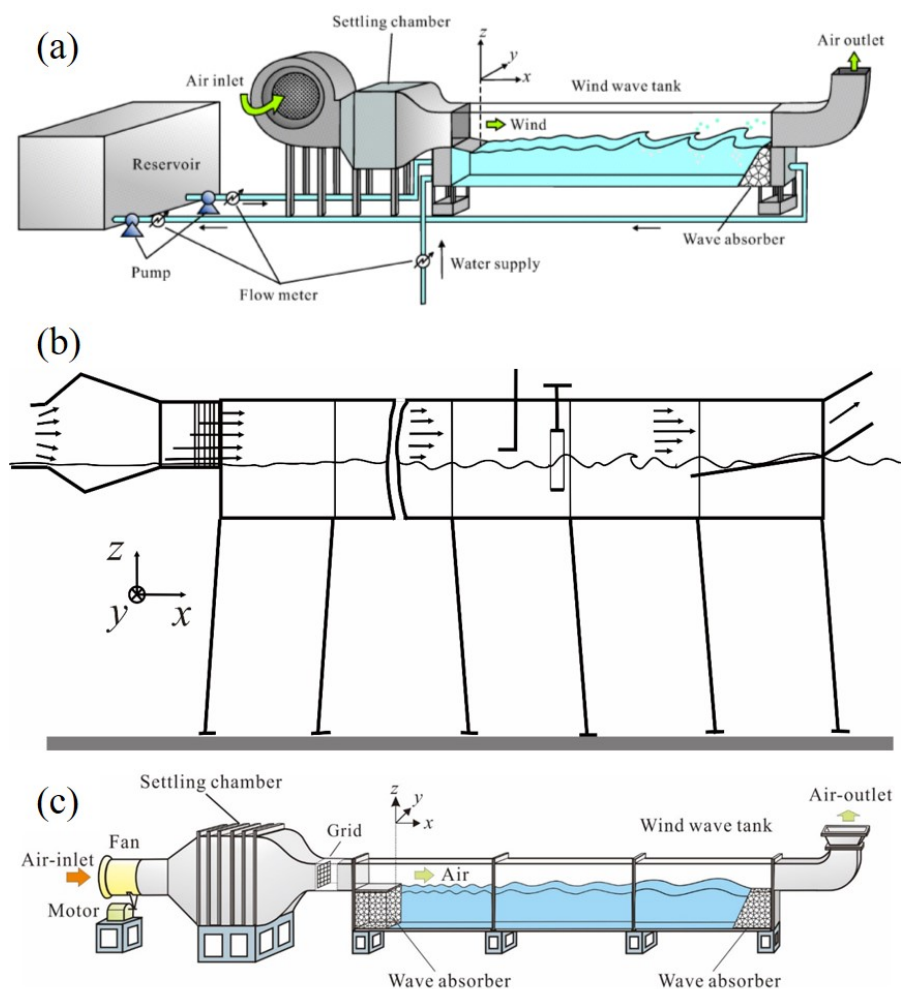
62 Therefore, the purpose of this study is to investigate the effects of the current on wind
63 waves in strong winds through the application of three different types of wind-wave tanks,
64 along with a pump.

65

66 **2. Experiment**

67 **2.1. Equipment and measurement methods**

68 Wind-wave tanks at Kyoto University, Japan and the Institute of Applied Physics,
69 Russian Academy of Sciences (IAP RAS) were used in the experiments (Figs. 1a, 1b).
70 For the tank at Kyoto University, the glass test section was 15 m long, 0.8 m wide, and 1.6



71
72 **Figure 1.** Schematics of wind-wave tanks. (a) High-speed wind-wave tank of Kyoto University. (b)
73 Typhoon simulator of IAP RAS. (c) Wind-wave tank of Kindai University.

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76 m high. The water depth D was set at 0.8 m. For the tank at IAP RAS, the test section in
77 the air side was 15 m long, 0.4 m wide, and 0.4 m high. The water depth D was set at 1.5
78 m. The wind was set to blow over the filtered tap water in these tanks, generating wind
79 waves. The wind speeds ranged from 4.7 to 43 m s^{-1} and from 8.5 to 21 m s^{-1} in the tanks
80 at Kyoto and IAP RAS, respectively. Measurements of the wind speeds, water-level
81 fluctuation, and current were carried out 6.5 m downstream from the edge ($x = 0$ m) in



82 both the Kyoto and IAP RAS tanks. Here, the x , y , and z coordinates are referred to as the
83 streamwise, spanwise, and vertical directions, respectively, with the origin located at the
84 center of the edge of the entrance plate. Additionally, the fetch (x) is defined as the
85 distance between the origin and measurement point ($x = 6.5$ m).

86 In Kyoto, a laser Doppler anemometer (Dantec Dynamics LDA) and phase Doppler
87 anemometer (Dantec Dynamics PDA) were used to measure the wind velocity fluctuation.
88 A high-power multi-line argon-ion (Ar^+) laser (Lexel model 95-7; laser wavelengths of
89 488.0 and 514.5 nm) with a power of 3 W was used. The Ar^+ laser beam was shot through
90 the sidewall (glass) of the tank. Scattered particles with a diameter of approximately 1 μm
91 were produced by a fog generator (Dantec Dynamics F2010 Plus) and were fed into the
92 air flow over the waves (see Takagaki et al. (2012) and Komori et al. (2018) for details).
93 Water level fluctuations were measured using resistance-type wave gauges (Kenek
94 CHT4-HR60BNC). The resistance wire was placed into the water, and the electrical
95 resistance at the instantaneous water level was recorded at 500 Hz for 600 s using a digital
96 recorder (Sony EX-UT10). The energy of the wind waves (E) was estimated by
97 integrating the spectrum of the water-level fluctuations over the frequency (f). The values
98 of the wavelength (L_s) and phase velocity (C_s) were estimated using the cospectra
99 method (e.g., Takagaki et al., 2017). The current was measured using the same LDA
100 system.

101 In IAP RAS, a hot-wire anemometer (E+E Elektronic EE75) was used to measure the
102 representative mean wind velocity at $x = 0.5$ m and $z = 0.2$ m. The three wind velocities
103 (U_{10} , u^* , U_∞) at $x = 6.5$ m were taken from Troitskaya et al. (2012). The water-level
104 fluctuations were measured using three handmade capacitive-type wave gauges. Three
105 wires formed a triangle with 25 mm on a side. The wires were placed in the water, and the
106 output voltages at the instantaneous water level were recorded at 200 Hz for 5400 s using
107 a digital recorder through an AD converter (L-Card E14-140). The current was measured
108 through acoustic Doppler velocimetry (Nortec AS) at $x = 6.5$ m and $z = -10, -30, -50,$
109 $-100, -150, -220,$ and -380 mm (see Troitskaya et al. (2012) for details).

110

111 2.2. Artificial current experiments at Kindai University

112 Additional experiments were performed using a wind-wave tank at Kindai
113 University with a glass test section 6.5 m long, 0.3 m wide, and 0.8 m high (Fig. 1c). The
114 water depth D was set at 0.49 m. A Pitot tube (Okano Works, LK-0) and differential
115 manometers (Delta Ohm HD402T) were used to measure the mean wind velocity. The
116 water level fluctuations were measured using resistance-type wave gauges (Kenek
117 CHT4-HR60BNC). To measure L_s and C_p , another wave gauge was fixed downstream at



118 $\Delta x = 0.02$ m, where Δx is the interval between the two wave gauges. The current was then
119 measured through electromagnetic velocimetry (Kenek LP3100) with a probe (Kenek
120 LPT-200-09PS) at $x = 4.0$ m. The probe sensing station was 22 mm long with a diameter
121 of 9 mm. The measurements were performed at $z = -15$ to -315 mm at 30 mm intervals.
122 The sampling frequency was 8 Hz, and the sampling time was 180 s.

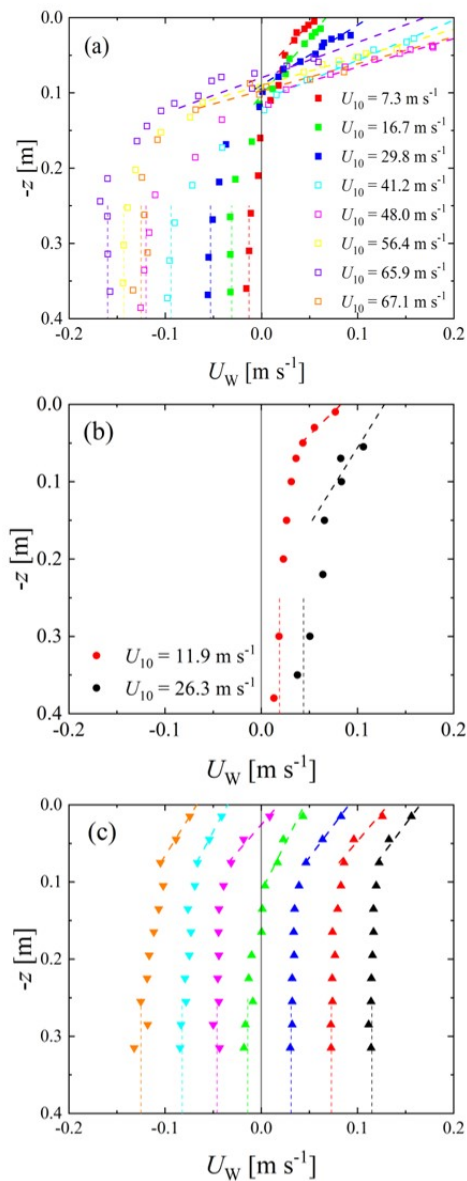
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124 **3. Results and discussion**

125 **3.1. Waves and current**

126 Figure 2 shows the vertical distributions of the streamwise water velocity. The
127 water velocities in the three different wind-wave tanks at Kyoto University, Kindai
128 University, and IAP RAS are separately shown in each subfigure. In Fig. 2a, the bulk
129 velocity of water U_{BULK} shows negative values ($U_{\text{BULK}} = -0.16$ to -0.01 m s⁻¹) at Kyoto
130 University, which is generated as the counterflow against the Stokes drift at the wavy
131 water surface. In Fig. 2b, the bulk velocity of water demonstrates positive values (U_{BULK}
132 = 0.019 to 0.044 m/s) at IAP RAS. This is because the wind-wave tank at IAP RAS is an
133 open tank; thus, the Stokes drift on the wavy water surface does not provide the
134 counterflow for the bulk water, unlike in the closed tank at Kyoto University. From Fig.
135 2c, it is clear that the bulk velocities of the water vary in each case at Kindai University
136 with the use of the pump. Furthermore, the water bulk velocities change from negative to
137 positive ($U_{\text{BULK}} = -0.13$ to -0.17 m s⁻¹). The bulk velocities of water were defined as the
138 mean velocity with $z = -0.4$ to -0.25 m (see dotted lines in Fig. 2), and the velocities are
139 listed in Table 1. Experiments were performed under 27 different conditions, with the
140 bulk velocity of water provided in the three different wind-wave tanks. The surface
141 velocities of water, U_{SURF} , also varied in the three tanks with respect to wind speed (see
142 Fig. 2). The U_{SURF} values were estimated by the linear extrapolation lines (dashed lines)
143 as the water velocity at the surface ($z = 0$ m) shown in Fig. 2, and the velocities are listed
144 in Table 1.

145 Figure 3 shows the wind-velocity dependency of the wave frequency f_m ,
146 wavelength L_s , phase velocity C_s , surface velocity of water U_{SURF} , and bulk velocity of
147 water U_{BULK} . From Figs. 3a–3c, it is clear that both the Kyoto and IAP RAS data
148 demonstrate that the wind waves develop with wind shear. Although f_m in both cases
149 correspond to each other, L_s and C_s in IAP RAS are different from those in Kyoto. The
150 disagreement might be caused by the difference in the wind-wave development or
151 Doppler effect; this is discussed below. From Figs. 3d and 3e, U_{SURF} and U_{BULK} increase
152 with an increase in U_{10} in IAP RAS. However, in Kyoto, U_{SURF} increases, but U_{BULK}
153 decreases with an increase in U_{10} . Moreover, U_{SURF} in IAP RAS corresponds to U_{SURF} in



154

155 **Figure 2.** Vertical distributions of water-flow velocity; (a) Kyoto University, (b) IAP RAS, and (c)

156 Kindai University. In (c), plots indicate cases 21–27 starting from right. Dotted and dashed lines

157 indicate the lines used to estimate U_{BULK} and U_{SURF} , respectively. Open symbols show the

158 high-wind-speed cases.

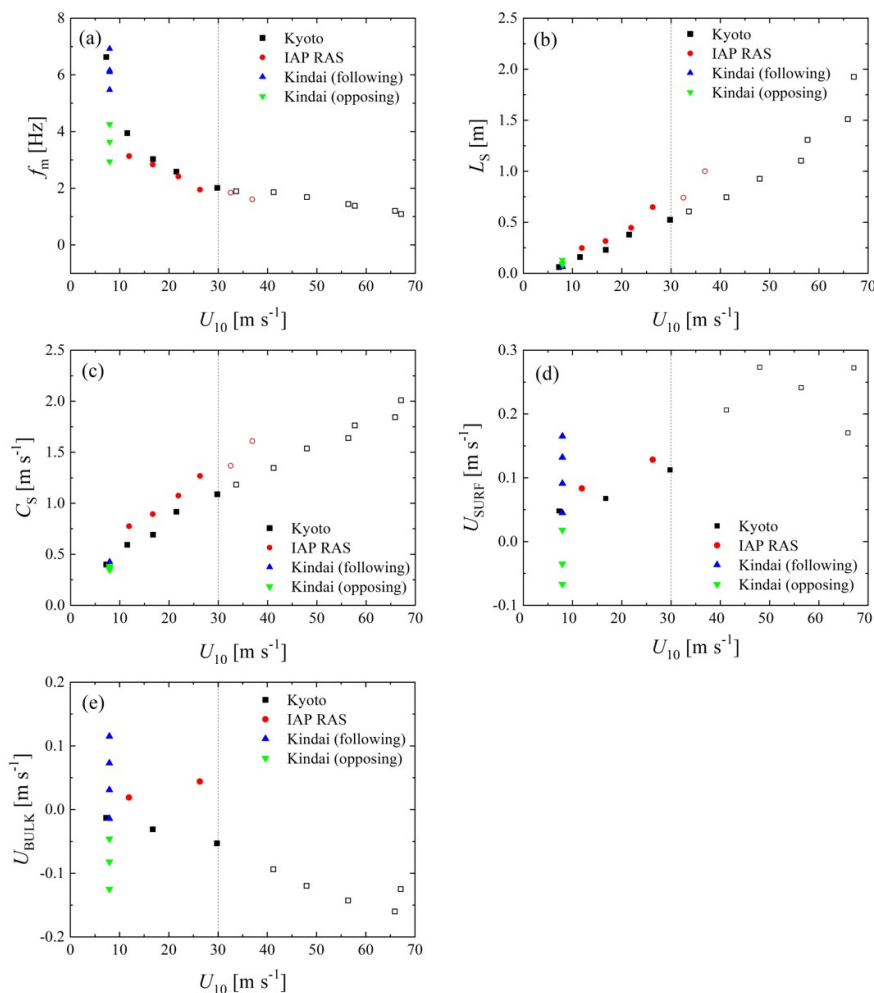


159 **TABLE 1.** Wind and wind-wave properties. F : fetch; N_{PUMP} : pump inverter frequency; U_{∞} :
 160 freestream wind speed; u^* : friction velocity of air; U_{10} : wind speed at 10 m above the sea surface;
 161 U_{SURF} : surface flow velocity of water; U_{BULK} : bulk flow velocity of water; C_D : drag coefficient; H_s :
 162 significant wave height; T_s : significant wave period; E : wave energy; f_m : significant frequency; C_s :
 163 phase velocity; L_s : significant wave length; $C_{s\text{-theor-l}}$: phase velocity predicted by theoretical linear
 164 model; $C_{s\text{-theor-nl}}$: phase velocity predicted by theoretical nonlinear model. The values of u^* , U_{10} , and
 165 C_D in Kindai were estimated using the empirical curves by Iwano et al. (2013) from U_{∞} . Superscripts †
 166 and †† indicate the artificial following and opposing flows, respectively.

Case	Facility	F	N_{pump}	U_{∞}	u^*	U_{10}	U_{SURF}	U_{BULK}	C_D	H_s	T_s	$E^{0.5}$	f_m	C_s	L_s	$C_{s\text{-theor-l}}$	$C_{s\text{-theor-nl}}$
		[m]	[Hz]	[m s ⁻¹]	[m s ⁻¹]	[m s ⁻¹]	[m s ⁻¹]	[m s ⁻¹]	[×10 ⁻³]	[m]	[m]	[m]	[Hz]	[m s ⁻¹]	[m]	[m s ⁻¹]	[m s ⁻¹]
1	Kyoto	6.5	-	4.7	0.24	7.3	0.056	-0.01	1.1	0.0035	0.15	0.00092	6.63	0.40	0.06	0.369	0.374
2	Kyoto	6.5	-	7.2	0.43	11.5	-	-	1.4	0.0131	0.25	0.00353	3.95	0.59	0.16	-	-
3	Kyoto	6.5	-	10.3	0.67	16.7	0.067	-0.031	1.6	0.0231	0.32	0.00624	3.03	0.69	0.23	0.658	0.690
4	Kyoto	6.5	-	12.6	0.89	21.5	-	-	1.7	0.0357	0.39	0.00968	2.59	0.92	0.38	-	-
5	Kyoto	6.5	-	16.3	1.49	29.8	0.112	-0.053	2.5	0.0584	0.50	0.01570	2.01	1.09	0.52	0.972	1.044
6	Kyoto	6.5	-	18.8	1.70	33.6	-	-	2.5	0.0626	0.52	0.01691	1.89	1.18	0.60	-	-
7	Kyoto	6.5	-	22.2	2.08	41.2	0.206	-0.094	2.6	0.0631	0.53	0.01735	1.86	1.35	0.74	1.188	1.258
8	Kyoto	6.5	-	24.8	-	-	-	-	-	0.0668	0.55	0.01866	1.76	1.41	0.79	-	-
9	Kyoto	6.5	-	28.5	2.36	48.0	0.273	-0.120	2.4	0.0727	0.58	0.02058	1.68	1.54	0.93	1.325	1.424
10	Kyoto	6.5	-	31.1	-	-	-	-	-	0.0807	0.62	0.02309	1.58	1.60	1.07	-	-
11	Kyoto	6.5	-	34.8	2.69	56.4	0.241	-0.143	2.3	0.0944	0.68	0.02715	1.44	1.64	1.10	1.379	1.550
12	Kyoto	6.5	-	37.1	2.89	57.7	-	-	2.5	0.1043	0.73	0.03027	1.37	1.76	1.31	-	-
13	Kyoto	6.5	-	39.6	3.38	65.9	0.170	-0.160	2.6	0.1214	0.80	0.03553	1.20	1.84	1.51	1.531	1.694
14	Kyoto	6.5	-	43.3	3.31	67.1	0.272	-0.125	2.4	0.1609	0.93	0.04766	1.08	2.01	1.92	1.743	2.149
15	IAP RAS	6.5	-	8.5	0.40	11.9	0.083	0.019	1.1	0.0214	0.31	0.0056	3.14	0.78	0.25	0.690	0.715
16	IAP RAS	6.5	-	11.0	0.60	16.7	-	-	1.3	0.0305	0.36	0.0081	2.84	0.89	0.32	-	-
17	IAP RAS	6.5	-	13.5	0.90	21.9	-	-	1.7	0.0455	0.43	0.0121	2.41	1.07	0.45	-	-
18	IAP RAS	6.5	-	16.3	1.15	26.3	0.128	0.044	1.9	0.0790	0.50	0.0161	1.95	1.27	0.65	1.111	1.190
19	IAP RAS	6.5	-	18.9	1.50	32.5	-	-	2.1	0.0690	0.54	0.0246	1.85	1.37	0.74	-	-
20	IAP RAS	6.5	-	21.2	1.70	36.9	-	-	2.1	0.0847	0.60	0.0305	1.61	1.61	1.00	-	-
21	Kindai	4.0	15 [†]	5.8	0.28	7.9	0.165	0.115	1.2	0.0044	0.14	0.0012	6.92	0.43	0.06	0.484	0.492
22	Kindai	4.0	10 [†]	5.8	0.28	7.9	0.132	0.073	1.2	0.0050	0.16	0.0014	6.10	0.43	0.07	0.501	0.510
23	Kindai	4.0	5 [†]	5.8	0.28	7.9	0.091	0.031	1.2	0.0049	0.16	0.0014	6.16	0.38	0.06	0.410	0.420
24	Kindai	4.0	0	5.8	0.28	7.9	0.045	-0.014	1.2	0.0054	0.19	0.0014	5.47	0.38	0.07	0.382	0.393
25	Kindai	4.0	5 ^{††}	5.8	0.28	7.9	0.018	-0.046	1.2	0.0076	0.23	0.0021	4.25	0.36	0.08	0.384	0.400
26	Kindai	4.0	10 ^{††}	5.8	0.28	7.9	-0.035	-0.082	1.2	0.0098	0.27	0.0027	3.64	0.35	0.10	0.355	0.375
27	Kindai	4.0	15 ^{††}	5.8	0.28	7.9	-0.067	-0.125	1.2	0.0125	0.34	0.0035	2.94	0.38	0.13	0.381	0.402

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170 Kyoto. This is because the Stokes drift generated by the wind waves, rather than the
 171 current, is significant. For the Kindai data, although f_m , U_{SURF} , and U_{BULK} vary, L_s and C_s
 172 are concentrated at single points at $L_s = 0.1$ m and $C_s = 0.4$ m s⁻¹, respectively. This shows
 173 that the intensity and direction of the current do not significantly affect L_s and C_s but do
 174 affect f_m and U_{SURF} . Thus, this implies that the present artificial current changes the water
 175 flow dramatically but does not affect the development of the wind waves.



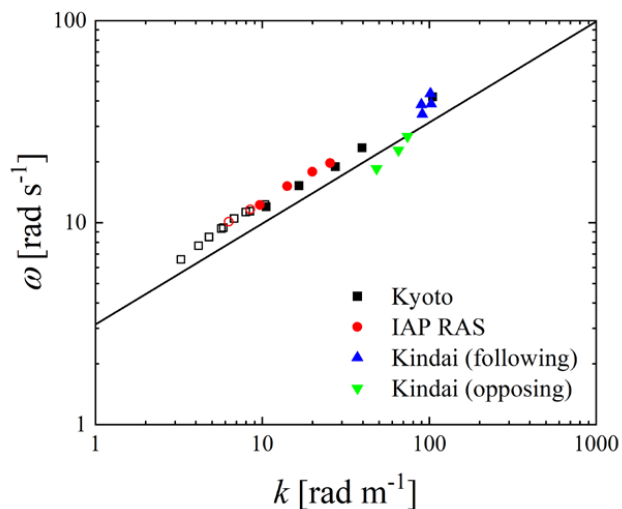
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177 **Figure 3.** Relationships between U_{10} and (a) significant frequency f_m , (b) significant wave length L_s ,
 178 (c) phase velocity C_s , (d) surface velocity of water U_{SURF} , and (e) bulk velocity of water U_{BULK} . Open
 179 symbols show the high-wind-speed cases.

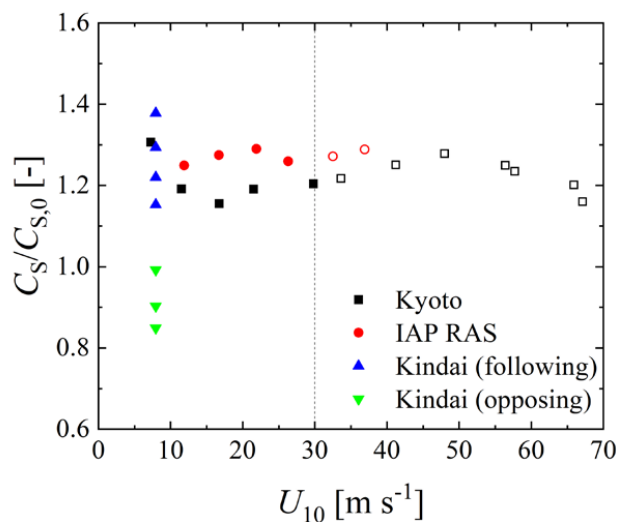
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182 Figure 4 shows the dispersion relation and demonstrates that the Kindai data
 183 points depend on the variation in the water velocity of the artificial current. The plots for
 184 the Kyoto University and IAP RAS cases at normal wind speeds (solid symbols) are



185
 186 **Figure 4.** Dispersion relation between angular frequency ω and wave number k . Open symbols show
 187 the high-wind-speed cases. Curve shows the dispersion relation of the deep-water waves ($\omega^2 = gk$).
 188

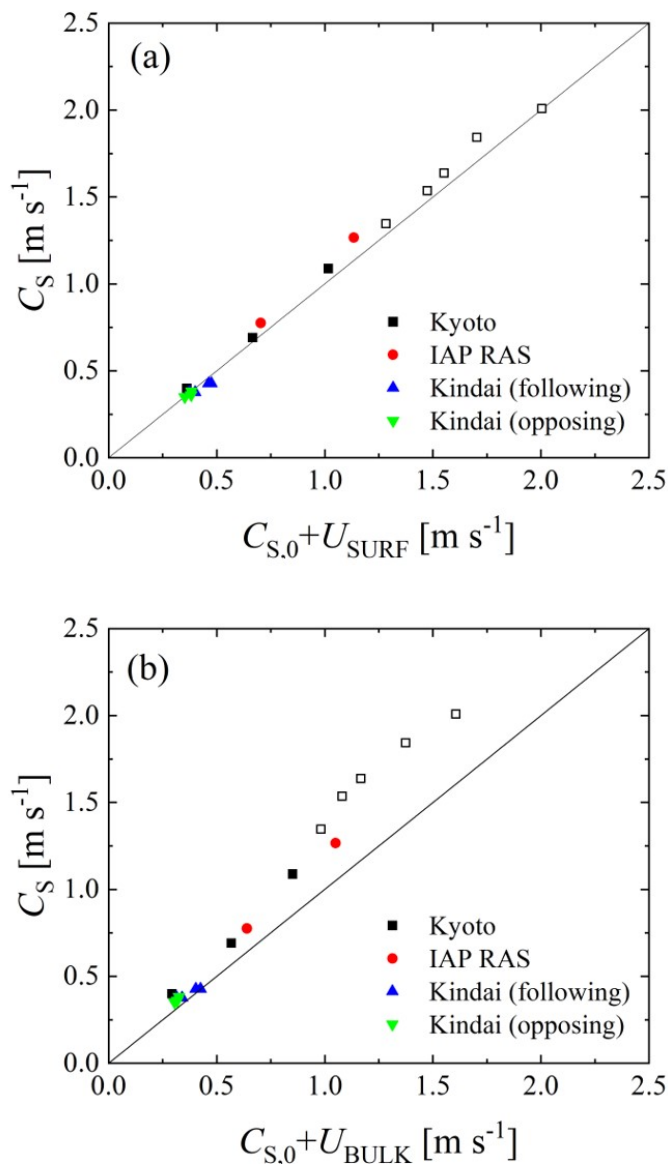


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 190 **Figure 5.** Relationship between the freestream wind speed and phase velocity C_S . The C_S is
 191 normalized by phase velocity $C_{S,0}$ without the Doppler effect, estimated by the dispersion relation of
 192 the deep-water waves ($C_{S,0} = (gL_s/2\pi)^{1/2}$). Open symbols show the high-wind-speed cases.



193 concentrated above the solid curve, showing the dispersion relation of the deep-water
194 waves ($\omega^2 = gk$). Meanwhile, the plots for extreme high wind speeds (open symbols) are
195 also concentrated above the solid curve. This implies that the wind waves, along with the
196 intensive breaking at extreme high wind speeds, are dependent on the Doppler shift. To
197 investigate the phase velocity trend, Fig. 5 shows the ratio of the measured phase velocity
198 C_S to the phase velocity $C_{S,0}$ estimated by the dispersion relation of the deep-water waves
199 ($C_{S,0} = (gL_S/2\pi)^{1/2}$) against the wind velocity. From the figure, the ratios at the normal
200 wind speeds assume a constant value (~ 1.21 in Kyoto or ~ 1.27 in IAP RAS). Moreover,
201 the ratios at the extreme high wind speeds take similar values of 1.23 and 1.28 for Kyoto
202 or IAP RAS, respectively. This implies that the phase velocities at extreme high wind
203 speeds are accelerated by the current just as those at normal wind speeds. However, the
204 Kindai values are scattered and increase in the following cases and decrease in the
205 opposing cases. It is clear that the artificial current accelerates (or decelerates) the phase
206 velocity.

207 To interpret the relationship among the measured phase velocity C_S , first phase
208 velocity $C_{S,0}$ estimated by the dispersion relation, and water velocity, two types of phase
209 velocity were evaluated: the sum of $C_{S,0}$ and surface velocity of water U_{SURF} and the sum
210 of $C_{S,0}$ and bulk velocity of water U_{BULK} . Figure 6 shows the relationship between C_S and
211 (a) $C_{S,0} + U_{SURF}$, and (b) $C_{S,0} + U_{BULK}$. In Fig. 6a, we can see that the Doppler shift is
212 confirmed at the normal wind speeds, i.e., the significant waves are accelerated by the
213 surface flow, and the real phase velocity can be represented as the sum of the velocity of
214 the surface flow and the virtual phase velocity, which is estimated by the dispersion
215 relation of the deep-water waves. At extreme high wind speeds over 30 m s^{-1} , a similar
216 Doppler shift is observed as under the conditions of normal wind speeds, as seen in Fig.
217 6a. Meanwhile, in Fig. 6b, although C_S corresponds to $C_{S,0} + U_{BULK}$ at low phase
218 velocities, C_S assumes values larger than $C_{S,0} + U_{BULK}$ at high phase velocities. This
219 suggests that the Doppler shift is an adequate model for representing the acceleration of
220 the wind waves by the current, not only for the wind waves at normal wind speeds but
221 also for those with intensive breaking at extreme high wind speeds. Moreover, the
222 Doppler shift of wind waves occurs due to a very thin surface flow, as the correlation
223 between C_S and $C_{S,0} + U_{SURF}$ is higher than the correlation between C_S and $C_{S,0} + U_{BULK}$.
224



225

226 **Figure 6.** Relationship between phase velocity C_S and (a) sum of $C_{S,0}$ and surface velocity of water
227 U_{SURF} , and (b) sum of $C_{S,0}$ and bulk velocity of water U_{BULK} . Open symbols show the high-wind-speed
228 cases.
229



230 **3.2. The theoretical model of waves at the shear flow**

231 The parameters of the observed Doppler shift can be explained more precisely
232 within the theoretical model of the capillary-gravity waves at the surface of the water
233 flows with the velocity profiles prescribed by the experimental data, which are plotted in
234 Fig. 2a–c. Because the dominant wind wave propagates along the wave and water flows,
235 we will consider the 2D-wave model in the 2D flow. This flow is described by the system
236 of 2D Euler equations:

237
$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0, \quad (1)$$

238
$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial z} = -g,$$

239 and the condition of non-compressibility:

240
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (2)$$

241 with the kinematical

242
$$\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} = w \Big|_{z=\eta(x,t)} \quad (3)$$

243 and dynamical boundary conditions

244
$$p \Big|_{z=\eta(x,t)} = 0 \quad (4)$$

245 at the water surface. Here, u and w are the horizontal and vertical velocity components, p
246 is the water pressure, x and z are the horizontal and upward vertical coordinates, g is the
247 gravity acceleration, and ρ is the water density. The boundary condition at the bottom of
248 the channel is $w \Big|_{z=-D} = 0$. It should be noted that the water depth in almost all the
249 experimental runs exceeded half of the wavelength of the dominant waves (see Table 1).
250 In this case, the deep-water approximation is applicable for describing the surface waves,
251 and the boundary condition of the wave field vanishing with the distance from the water
252 surface can also be used.

253 Because the fluid motion under consideration is 2D, the stream function can be
254 introduced as follows:

255
$$u = \frac{\partial \psi}{\partial z}; w = -\frac{\partial \psi}{\partial x}. \quad (5)$$

256 To derive the linear dispersion relation for the surface waves at the plane shear flow with
257 the horizontal velocity profile $U_w(z)$, we consider the solution to Eqs. (1, 2) in terms of the
258 stream function as the sum of the undisturbed state with steady shear flow and



259 small-amplitude disturbances. Then, the stream function ψ and pressure p are as follows:

260
$$\psi(x, z, t) = \int U_w(z_1) dz_1 + \varepsilon \psi_1(x, z, t); \quad (6)$$

261
$$p(x, z, t) = -\rho g z + \varepsilon p_1(x, z, t), \quad (7)$$

262 where $\varepsilon \ll 1$, and the water elevation value is also the order of ε , namely $\varepsilon \eta_1(x, t)$.

263 In the linear approximation in ε , the system of Eqs. (1, 2) and the boundary
 264 conditions of Eqs. (3, 4) take the form:

265
$$\left(\frac{\partial}{\partial t} + \frac{U_w(z) \partial}{\partial x} \right) \left(\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_1}{\partial z^2} \right) - \frac{\partial \psi_1}{\partial x} \frac{d^2 U_w(z)}{dz^2} = 0,$$

266
$$\frac{\partial \eta_1}{\partial t} + U_w(0) \frac{\partial \eta_1}{\partial x} = - \frac{\partial \psi_1}{\partial x} \Big|_{z=0}, \quad (8)$$

267
$$\frac{\partial p_1}{\partial x} \Big|_{z=0} - \rho g \frac{\partial \eta_1}{\partial x} = 0,$$

268
$$\psi_1 \Big|_{z=-D} = 0.$$

269 Excluding p_1 with use of the first equation of the system in Eq. (1) and eliminating η_1
 270 yields one boundary condition at the water surface for ψ_1 :

271
$$\left[\left(\frac{\partial}{\partial t} + \frac{U_w(0) \partial}{\partial x} \right)^2 \frac{\partial \psi_1}{\partial z} - \left(\frac{\partial}{\partial t} + U_w(0) \frac{\partial}{\partial x} \right) \frac{\partial \psi_1}{\partial x} \frac{dU_w}{dz} - g \frac{\partial^2 \psi_1}{\partial x^2} \right] \Big|_{z=0} = 0. \quad (9)$$

272 For the harmonic wave disturbance, where

273
$$\psi_1(x, z, t) = \Psi_1(t) \exp(-i(\omega t - kt)), \quad (10)$$

274 substituting into Eqs. (8, 9) yields the Rayleigh equation for the complex amplitude of the
 275 stream function disturbance:

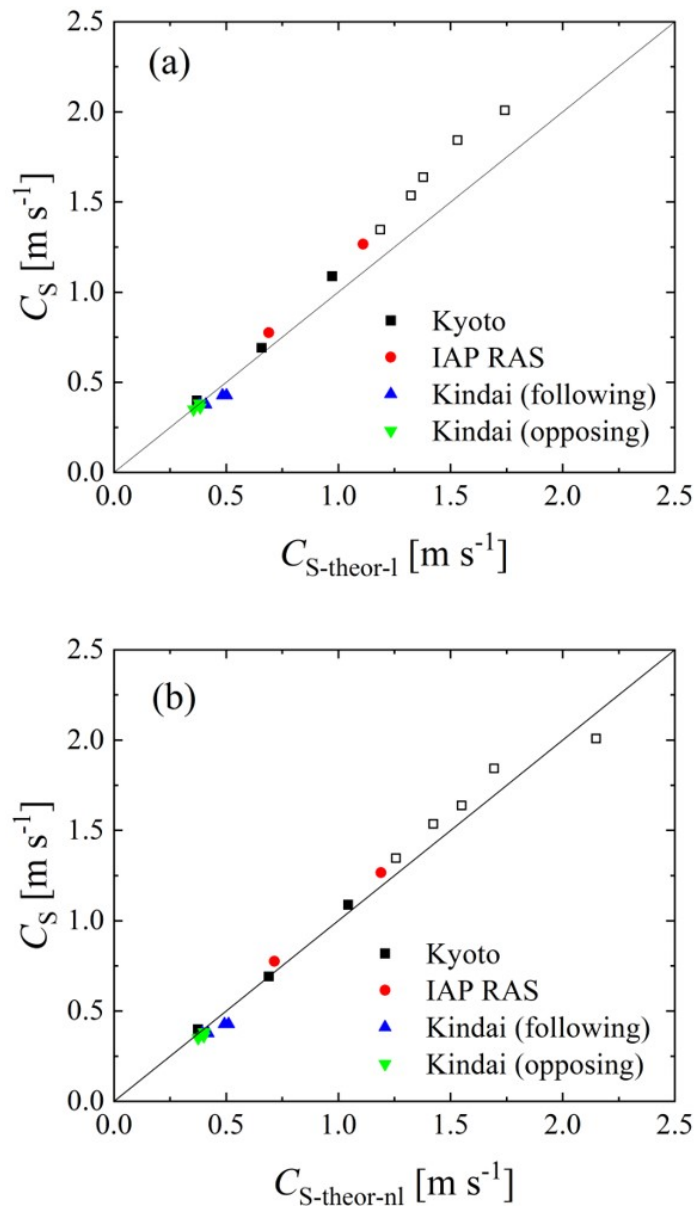
276
$$(\omega - U_w(z)k) \left(\frac{d^2 \Psi_1}{dz^2} - k^2 \Psi_1 \right) + \frac{d^2 U_w(z)}{dz^2} k^2 \Psi_1 = 0, \quad (11)$$

277 with the following boundary condition:

278
$$(\omega - U_w(0)k)^2 \frac{d\Psi_1(0)}{dz} + (\omega - U_w(0)k)k\Psi_1(0) \frac{dU_w(0)}{dz} - k^2 g\Psi_1(0) = 0, \quad (12)$$

279
$$\Psi_1 \Big|_{z \rightarrow -\infty} \rightarrow 0.$$

280 Numerically solving the boundary layer problem for Eq. (11) with the boundary
 281 conditions in Eq. (12) enables one to obtain the dispersion relation $\omega(k)$ for the surface
 282 waves at the inhomogeneous shear flow. Note that because the phase velocity of the
 283 waves significantly exceeded the flow velocity in all experiments (cf. Figs. 2 and 3), the
 284 Rayleigh equation did not have a singularity, and the calculated frequency and phase



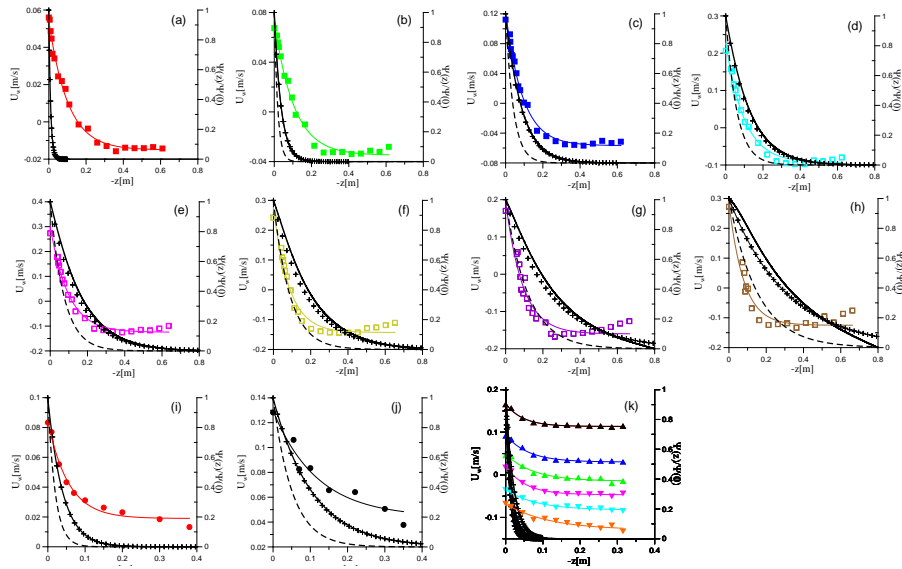
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287

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Figure 7. The measured phase velocity C_S versus theoretical prediction: (a) linear model, and (b) nonlinear model.



289

290 **Figure 8.** Vertical velocity profiles (points), their fitting (thin color line), the eigenfunction of the Eq.
 291 (8) with the boundary conditions Eq. (9) (black solid curve), the function e^{kz} (crosses), the function e^{2kz}
 292 (dashed line). The panels (a)-(j) corresponds to the experiments No. 1, 3, 5, 7, 9, 11, 13-15, 18
 293 respectively, the panel (k) corresponds to the experiments No. 21-27.

294

295

296 velocity of the wave were real values, i.e., the current was neutral stable.

297 The wave phase velocities $C_{S\text{-theor-1}} = \omega(k)/k$ were calculated for the parameters of
 298 those experiments that contained complete information about the course and
 299 characteristics of the waves, namely 1, 3, 5, 7, 9, 11, 13–15, 18, and 21–27 from Table 1.
 300 The results are presented in Fig. 7a as the measured phase velocity C_s versus calculated
 301 phase velocity $C_{S\text{-theor-1}}$. One can see that the model corresponds to the data substantially
 302 better than does the model of linear potential waves at the homogeneous current U_{BULK}
 303 (compare Fig. 6b). Considering the structure of the wave disturbances of the stream
 304 function, $\Psi_1(z)$, which was found as the eigenfunction of the boundary problem of Eqs.
 305 (11, 12). The profiles of $\Psi_1(z)$ are presented in Fig. 8. One can see that in all cases the
 306 functions $\Psi_1(z)$ are close to e^{kz} at the background of the mean velocity profiles. Moreover,
 307 for experiments No. 1, 3, 5, 15, and 21–27 (see Fig. 8a, 8b, 8c, 8i, and 8k), the wave field
 308 is concentrated near the surface at a distance less than the scale of the change in the mean
 309 flow, where the flow velocity is approximately equal to U_{SURF} . This explains the good
 310 correlation in these cases of the observed phase velocity with the phase velocity of waves



311 at the homogeneous current U_{SURF} presented in Fig. 6a. At the same time, for experiments
 312 No. 7, 9, 5, 11, 13, 14, and 18 (see Figs. 8d–8h, and 8j), the scale of the variability of the
 313 flow is significantly smaller than the scale of the wave field. Under these conditions, a
 314 significant difference between the phase velocity of the waves and that given by the linear
 315 dispersion relation can be due to the influence of nonlinearity.

316 To estimate the nonlinear addition to the wave phase velocity, we used the results
 317 of the weakly nonlinear theory of surface waves for the current with a constant shear. Of
 318 course, the flow in the experiments of the present work does not have a constant shift, and
 319 this was considered when obtaining the linear dispersion relation. However, it should be
 320 taken into account that the contributions of the n -th harmonic to the nonlinear dispersion
 321 relation are determined by wave fields in the n -power, which have a scale that is n time
 322 smaller than the first harmonic. Additionally, the model of constant shear of the mean
 323 current velocity is already approximately applicable for the 2nd harmonic (see Fig. 8).

324 We use the nonlinear dispersion relation for waves in the current with a constant
 325 shift in the deep-water approximation, which was obtained by Simmen and Saffman
 326 (1985):

$$\begin{aligned}
 327 \quad & (\omega - U_w(0)k)^2 \frac{d\Psi_1(0)}{dz} + (\omega - U_w(0)k)k\Psi_1(0) \frac{dU_w(0)}{dz} - k^2 g\Psi_1(0) = \gamma(ka)^2, \\
 328 \quad & \gamma = \frac{(\omega_0 - U_w(0)k)^2}{2k} \left(1 - \frac{1}{2}\Omega^2 + \left(1 + 2\Omega + \frac{1}{2}\Omega^2 \right)^2 \right), \quad (13) \\
 329 \quad & \Omega = \frac{1}{(\omega_0 - U_w(0)k)} \frac{dU_w(0)}{dz},
 \end{aligned}$$

330 Here, ω_0 is the solution of the linear dispersion equation. Eq. (13) is rewritten in the
 331 notation of this work and formulated in a reference frame in which the surface of the
 332 water has the velocity $U_w(0)$. Note that the linear part of Eq. (13) coincides with Eq. (12).
 333 The results of solving Eq. (13) are presented in Fig. 7b similarly to Fig. 7a as the
 334 measured phase velocity C_s versus calculated phase velocity $C_{s\text{-theor-nl}} = \omega(k)/k$, where
 335 one can see their good agreement with each other. Thus, the wave frequency shift can be
 336 explained by two factors, including the Doppler shift at the mean flow and the nonlinear
 337 frequency shift, while, the latter can also be interpreted in its physical nature as the wave
 338 frequency shift in the presence of its orbital velocities.

339 Recent studies have indicated a regime shift in the momentum, heat, and mass
 340 transfer across an intensive broken wave surface along with the amount of dispersed
 341 droplets and entrained bubbles at extreme high wind speeds over 30 m s^{-1} (e.g., Powell et
 342 al., 2003; Donelan et al., 2004; Takagaki et al., 2012, 2016; Troitskaya et al., 2012; Iwano
 343 et al., 2013; Krall and Jähne, 2014; Komori et al., 2018; Krall et al., 2019). Thus, there is



344 the possibility of a similar regime shift in the Doppler shift of wind waves by the current
345 at extreme high wind speeds. However, the present study reveals that such a Doppler shift
346 is observed as under the conditions of normal wind speeds. In this case, the weakly
347 nonlinear approximation turns out to be applicable for describing the dispersion
348 properties of not only small-amplitude waves but also nonlinear and even breaking waves.
349 This implies that the intensive wave breaking at extreme high wind speeds occurs with
350 the saturation (or dumping) of the wave height rather than the wavelength. This evidence
351 might be helpful in investigating and modelling the wind-wave development at extreme
352 high wind speeds.

353

354 **4. Conclusion**

355 The effects of the current on wind waves were investigated through laboratory
356 experiments in three different wind-wave tanks along with a pump at Kyoto University,
357 Japan, Kindai University, Japan, and IAP RAS. In this experiment, 27 different types of
358 currents were generated at wind speeds ranging from 7 to 67 m s⁻¹. At normal wind speeds
359 under 30 m s⁻¹, the wave frequency, wavelength, phase velocity of waves, and surface
360 velocity of the water were found to depend on the wind speed. However, the bulk velocity
361 of the water showed a dependence on the tank type, i.e., open tank (IAP RAS) or closed
362 tank (Kyoto University). The effect of the Doppler shift was confirmed at normal wind
363 speeds, i.e., the significant waves were accelerated by the surface flow, and the phase
364 velocity was represented as the sum of the surface velocity of water and the phase velocity,
365 which is estimated by the dispersion relation of the deep-water waves. At extreme high
366 wind speeds over 30 m s⁻¹, a Doppler shift was observed similar to that under the
367 conditions of normal wind speeds. This suggests that the Doppler shift is an adequate
368 model for representing the acceleration of wind waves by the current, not only for the
369 wind waves at normal wind speeds but also for those with intensive breaking at extreme
370 high wind speeds. The data obtained by the artificial current experiments conducted at
371 Kindai University were used to explain how the artificial current accelerates (or
372 decelerates) the significant waves. A weakly nonlinear model of surface waves at a shear
373 flow was developed. It was shown that it describes well the dispersion properties of not
374 only small-amplitude waves but also strongly nonlinear and even breaking waves, typical
375 for extreme wind conditions, with speeds, U_{10} , exceeding 30 m s⁻¹.

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