Final author response: "Reassessment of long-period constituents for tidal predictions along the German North Sea coast and its tidally influenced rivers"

We would like to thank again the three reviewers and the journal editor for their constructive feedback. Detailed point-to-point responses to all referee comments can be found in the respective author comments from the discussion phase (repeated below for completeness).

The changes to the manuscript are explained in the individual answers in the author comments. One major change (and a deviation from the answers to the reviewers) is the inclusion of a new section 6 which provides a comparison of predictions made with the Harmonic Representation of Inequalities and with the Harmonic Method for the two tide gauge locations Cuxhaven and Hamburg. We hope that this analysis satisfies the repeatedly expressed wish for a comparison of the two prediction methods. Additionally, some minor editorial corrections have been applied (e.g. corrections for spelling or sentence structure).

Please find the marked-up manuscript version showing the changes made below the author comments.

Best regards,
Andreas Boesch and Sylvin Müller-Navarra

Authors' response to Referee Comment #1

Dear Reviewer #1.

Thank you very much for your attention to the manuscript and your review report. Please find below our replies to your comments. The different items from the review report are first cited, followed by our responses.

1) "But personally I found the paper of interest not for that, but rather just for the description of the HRoI method, about which I was completely unaware. Evidently developed by Horn in the 1950s, it is an unusual approach to tidal analysis. I'm especially appreciative of the fact it is developed for use in analyzing high and low water (rather than the more standard hourly data), and I can foresee more applications of the method once readers become familiar with it."

>> Yes, this paper is also an opportunity to (re-)introduce the Harmonic Representation of Inequalities (HRoI) to the scientific community. Although the method has been around for a long time, many tidal scientists are not aware of it (anymore). We agree that the method can be interesting for others, especially when studying tides in estuaries or when tide gauges run dry around low water.

2) "The one drawback of the method, according to how the authors describe it, is on page 8, lines 10-15, where it seems a 19-year time series is needed."

>> Best results are, of course, achieved when using a 19-year time series, because in this case all relevant tidal information is contained within the data. However, shorter time series can also be analysed. The rank R in table 4 indicates which partial tides need to be dropped in the case of shorter time series. In operational usage, we use the method directly if 10 years or more years are available. For time series between 10 and 19 years, 5 of the 39 partial tides are dropped (see figure 6). Analyses with even shorter time series are also possible but with decreasing accuracy; in these

cases the transfer of a good prediction from a nearby station often gives better results.

3) "For my own interest, I would liked to have seen more standard methods of prediction included in the tests of Tables 5-6, but I won't insist on this, because it would involve the authors using methods they may not have ready at hand. Others can perform this extended testing."

>> The comparison of the HRoI with other methods (e.g. the harmonic method) is not the subject of this paper and would be beyond its scope. We agree that this testing is interesting and important. We are starting to develop tools for extensive comparison of the HRoI and the harmonic method, and we will share these results with the scientific community when results are available in the future. We also invite others to use the HRoI for their applications and comparisons.

4) "The paper is well-written and the English is quite good, but there is a number of misspellings which I noticed. The authors should run an English-language spell-checker on the text to pick these up. But a spell-checker may not catch: page 5, line 17: frequency depended –> frequency-dependent page 4, line 78: what is "appodization" ?"

>> We will look carefully through the manuscript to catch the remaining spelling mistakes. The discussion paper version (one column) had already been improved in this regard, compared to the initially uploaded two-column version (which you probably read according to the cited pages and line numbers).
The word "appodization" should be spelled "apodization" and is a window function that is applied (multiplied) to the data in order to reduce side lobes in the periodogram. Otherwise, these side lobes could be identified as true signals by mistake. In the revised manuscript, we will add more information and references in the corresponding paragraph.

5) "Page 4, line 51: I understand why lunar transit times are computed, as they are fundamental to the method, but I do not understand why "lunar coordinates" are also needed. Or do the authors mean merely the mean longitudes needed to evaluate the Doodson arguments?"

>> The lunar transit times are computed using the algorithm published in chapter 15 of Meeus (1998). Inputs to this algorithm are the right ascension and declination of the transiting body, i.e. the moon. These coordinates are calculated using the lunar theory by Chapront-Touzé and Chapront (1991). We will make the corresponding sentence clearer in the revised manuscript.

6) "Page 4, lines 57-60: Regarding removal of "extreme events" – were these data also removed when the tests of Tables 5-6 were computed? Or do Tables 5-6 include ALL data from 2016 ?"

>> The observed water levels of the year 2016 used for comparison are also filtered as described in Sect. 4.1. This way, the tidal predictions are compared to observations that represent the tidal behaviour better than the full data sets including extreme events. We will add this information explicitly in Sect. 5.1.

7) Do any of the German stations experience a double high tide? This occurs in some locations in the English Channel. If that occurs, how does the time indexing change?

>> There are no German stations with a double high tide. In its current form, the Harmonic Representation of Inequalities is tailored to strictly semi-diurnal tides because it was developed for the conditions in the German Bight. The possible

adaption to other tidal forms is a very interesting question. If the double high tide appears in every cycle (and if this is known to the analyst), it should be no problem to introduce four more equations of the type of Eq. (1); one model equation for each of the heights and times of the second high water assigned to the upper or lower lunar transit. For arbitrary mixed types, the direct calculation of high and low waters with the HRoI seems not to be possible. In these cases, one needs to first calculate the full curve, either with the harmonic method or maybe with the extended HRoI (as mentioned in Sect. 2), and derive the minima/maxima from the curve.

Authors' response to Referee Comment #2

Dear Reviewer #2.

Thank you very much for your detailed review report. We appreciate that you have taken the time to read the manuscript and to comment on it. Please find below our replies to your comments. The different items from the review report are first cited, followed by our responses.

1) "An important point is mentioned in sentence "the HRoi combines the best from the harmonic and the nonharmonic method." (p 6). I think it could also be written in abstract section to emphasize it from the very start of the paper.

>> The quoted passage is taken from Horn(1960) and might be too subjective to be included in an abstract. However, it is a good idea to specify the nature of the HRoI in the abstract and we will add this information in the revised manuscript.

2) "I'm interested in knowing the time sampling for tide gauges records.
Independently, did you use time series sampling (1) every x (hours? minutes?) or did you use (2) HW, LW recorded time series values? Following the method, keeping with the developments and discussion, it should be answer (1). Could you confirm?"

>> The recorded sampling rate of the tide gauges is 1 minute (for about the last 20 years, depending on the individual tide gauge; previously only high and low water data were available). The presented prediction method uses time series of times and heights of high and low water (and predicts only times and heights of high and low water).

3) „What is the maximum gap you observed in the tide gauges time series? And the longest continuous time series?"

>> The longest continuous time series are from Cuxhaven and Hamburg, each with 115 years and a maximum data gap of half a day, which means that no more than one high or low water is missing at a time. The maximum gaps can be longer than 10 years but the 60% criterion ensures that a sufficient amount of data is available from each tide gauge.

4) „The last sentence, page 10, is important. The fact that this relates to parameters introduced in table 2, the fundamental variables, could be added in a note or in bracket."

>> Thank you for this remark. We will add a note that the parameters, for which we define the ranges on page 11, are related to the parameters in Table 2. Furthermore, we will rework this part of Sect. 4.3 to improve the readability (see also item 5, 6, 16).

5) "Fig. 4 displays the periodogram of lunitidal intervals (L) after normalization. I understand that the normal variable (value) is the maximum value of lunitidal intervals. Fig. 4, I think it is

useful to add in legend, the normalization variable in order to define what is the reference variable used for normalization. It's simple note but it drives the results and plots; Same suggestion for height variable."

>> see answer to comment 16.

6) "I understand from ms, mh, mp, mN' fundamental parameters limits selection specific for this study (p 11) that these expressions are introduced in the text because they are useful for functions arguments development. But, this development is not included in the paper, nor cited/referenced. I'd say that if this part can't be used in the paper to help understanding the discussion, results or development, it could be removed from the text. But, if I'm wrong and if these expressions shouldn't be removed from text, then (1) addition of equations where these parameters are used would be useful for understanding or (2) one sentence could be added to say how it's useful to know the what type of selection have been done on ms, mh, mp, mN'."

>> We will rework this part of Sect. 4.3 (see also item 4, 5, 16) and include information on the calculation of the angular velocities (and add references to Section 2 and Table 2). The definition of the ranges of the linear coefficients m should stay in the manuscript as it sets the limits for the assignment of possible partial tides.

7) P 12: Table 4: To get a short analysis of the percentage presented in this table, what is the importance of the main partial tide? Could you precise in table 4 legend, that the results of the most influencing tidal wave is a synthesis from all the selected tide gauges?

>> The importance of the main partial tide (half synodic month) can be derived from table 3, where the average line intensities from the periodograms are listed. A quantitative statement (in physical units) is difficult, because of the averaging of data from different tide gauges and the normalization of the generalized Lomb Scargle periodogram.
We will make it clear in the legend of table 4 (and table 3) that the results are a synthesis from all the selected tide gauges

8) "P 17: Could you confirm that the results (fig. 7, p17) are residuals representative of both HW, LW? I think yes if I'm referring to fig 8. and 9, later in the paper. Fig. 7 validates the method in the frame of HW, LW prediction. Writing the residual mathematical formula is needed, I think, to sustain the text above fig. 7."

>> Yes, Fig. 7 shows the results for both high water and low water. Residuals are the difference between observed and predicted vertices (high or low water) with the same transit number and event index k. We will add the information on how the residuals are calculated in the revised manuscript.

9) "HW/LW prediction improvement percentage presented in tables could be completed by few words to provide some elements of analysis and understanding, to follow the reassessment."

>> We will add the formula for calculation of the changes and a few sentences about the contents (especially the extreme bins) of Fig. 8 and 9.

10) "For my interest, I'd like to see a result based on harmonic analysis and least squares minimization for the region of interest, in order to be able to compare its capacity to solve tidal dynamics to the HRoi method presented in the paper (for example in section 5). But therefore, I understand that the authors would have to make some other computations using tools and different methods from those which are presented and used here. So it's more a point for future discussion. Is the paper the first publishing for HRoi of investigations on long

period constituents, as it is written in the paper? I'm not aware about the previous HRoi investigations publications for long time period constituents."

>> This is the first publication in which the investigation of long term constituents for the HRoI is described. Older publications on the HRoI, as cited in the manuscript, only present the list of constituents without details about its preparation. We will add a sentence in the conclusions about the need to conduct a comparison study with the harmonic analysis to gain more insight on the relative performances.

11) "P 2: angular velocities: After the word in text "omega", the mathematics notation !i could be introduced, because it's used later, as the first reference in the text."

>> We will add the mathematics notation "omega" after "angular velocities" on page 2.

12) "P 3: - y ˆ to be define in legend (equation 1) (predicted value I think, with y for height or lunitidal interval). I think adding units in equation 1 is needed. Eq.2: symbol L for partial tide: Doodson reference for Eq.1 and Eq. 2 should be cited. They are derived from Laplace and Doodson theory and from harmonic analysis technics. Particularly, Doodson number is in the first column in tables 2 and 3 and is more generally a number currently used in tidal studies."

>> We add some information on variable names and units to better guide the reader and include the references to the alphanumeric Doodson number. The symbol J (not L) is correct in Eq. 2 as the sum runs over the J data points.

13) "P 4: In table 2, I think there is a need to write the thinking who makes you remove the fundamental parameter tau ( ), respective to first letter in Doodson notation? , s, h, p, N' for fundamental parameters to describe tides. to refer to hour angle of "mean" Moon. Just a suggestion: If you think it's relevant, I'd move table 2 in annex for it to play its role of quantitative reference (Tab 2 section 2)."

>> By construction of the method (HRoI) only long-period constituents need to be used, i.e. the parameter belonging to tau is always equal to zero. We will revise the corresponding sentence. We think that Table 2 is more than a quantitative reference, but a central element of the paper that connects our work with previous studies and presents fundamental information for all constituents. Therefore, we would like to keep the table in the main part of the paper instead of moving it to the appendix.

14) "P 5: Could you give a clear distinction between tn symbol used (p 4, p 5) and nt =lunar transit number (p3)? Reading page 5 and section 4.2 (page 7), could you write tn versus nt? It'd ease the reading and ease the comparison withp3, when transit number is introduced. Its formal symbol should be written (first sentence below table 1)."

>> The symbol "tn" is the unit symbol for transit number (such as "h" for hour). The symbol "n_t" is the variable that stands for a value of the transit number (such as "t" for time). We move the introduction of the unit symbol to the sentence with the definition of the transit number. This gives a better distinction between the two notations.

15) "P 7: section 4.1 Data preparation: For both, Â´n lunar transit number", Â´nthe calculation of lunitidal intervals" my opinion is that adding symbols would be benefit for reading. nt and y ˆ (I suggest)."

>> The symbol for the transit number is added in the corresponding sentence. The lunitidal intervals do not have a unique symbol (the symbol "y" can stand for lunitidal intervals or heights) and is therefore not included in the sentence. Instead, we included the symbol "y" in the first sentence in Sect. 4.2.

16) "P 10: May I ask you to add slight modification to Li, Lh expression adding legend and adding units in these 2 expressions. I think it could be good to read the units ex: of angular velocity degree per h (cf table 2)? Degree per tn and L units."

>> The expressions Li and Lh are unitless as the generalized Lomb-Scargle periodogram is normalized to unity. A value of 0 indicates no improvement of the fit and a value of 1 a "perfect" fit (see Zechmeister and Kürster 2009, full reference in the manuscript). We will add this information on page 10.

17) "P 12: Table 4 could be inserted in table 3, by adding: column R (table 4) after column Nh [%] in table 3, column description/name (table 4) in table 3"

>> We think that table 3 and table 4 should be kept separate. Table 3 shows the results from the initial analysis based on the defined rules. Table 4 shows the final results, after manual adjustments have been made to the selection of partial tides. Keeping these two tables (which also belong to different sections) separate makes the procedure more transparent.

18) "Fig. 6 (p 16): I appreciate the synthetic view of figure 6. Suggestion: could you add if possible, the explanation of number above the figure (relating the upper points of partial tides [rank])."

>> The numbers at the top of the figure are just the counts of partial tides (number of circles) in each "column". This is mentioned in the caption.

19) "If possible, it could be interesting to see on map fig.1, the location of Borkum tide gauge, Cuxhaven, Steubenhöft and Emden, Große Seeschleuse tide gauges used in the paper to highlight results"

>> Thanks for this great suggestions. These three stations (and Hamburg) will be highlighted in Fig.1 in the revised manuscript.

Authors' response to Referee Comment #3

Dear Reviewer #3.

Thank you very much for your time and effort to review our manuscript. Please find below our replies to your comments. The different items from the review report are first cited, followed by our responses.

1) "They asserted the slight improvement using the new set of constituents through just one year (2016) verification. I recommend that the authors should conduct additional two year (2017-2018) comparison between prediction and observation to clearly show the improvement."

>> The main focus of the manuscript is the preparation of the list of tidal constituents as used by the operational tidal forecasting service. We think that in this context the verification over one year is sufficient. The old sets of constituents have proven over several decades to deliver good results and no major differences were expected. The presented comparison shows that the new set of constituents can be expected to work equally well or even better because several frequencies in the residuals are now removed (see Fig. 10 and 11). We argue that an additional comparison over two years is not likely to show any significantly different results, but rather inflate the manuscript unnecessarily. Furthermore, quality-checked times and heights of high

and low waters are not yet available from the respective authorities for several tide gauges for the year 2018. This would limit the comparability of the different verifications. We hope that the referee understands our arguments and does not insist on further verification studies.

2) "As the authors mentioned, the HRoI is not widely used in comparison with a 'standard harmonic analysis and prediction (HAP)' method even if it has the better computational efficiency. Is it because that the HRoI is not open to the public or inconvenient to use? Additionally, I wonder if tidal prediction accuracy for the HRoI is better than that of the HAP. Can it predict tides at any time interval like the HAP? I think that the authors need to explain the additional reason why the HRoI is still used at BSH but most of countries have not used it. What are the advantages of using this method?"

>> We are not in a position to judge why the HRoI is not used more widely. The method has been published (as referenced in the manuscript) and it is fairly easy to use.
The original implementation of the HRoI, as described in Sect. 2, uses recorded time series of times and heights of high and low waters in order to predict times and heights of high and low waters. The concept of the HRoI can be generalized to determine the full tidal curve based on equally spaced water level records (e.g. 10 minute intervals). This generalized concept is explained in Müller-Navarra (2013; full reference in the manuscript) and is not subject of the analysis presented in the manuscript.
The characteristics (including advantages) of the HRoI are mentioned in Sect. 1 and 2, but we agree that this aspect is scattered throughout the two sections and should be cleaned up and expanded. In the revised manuscript, we will remove the last paragraph of Sect. 2 and insert it after the second paragraph of Sect. 2. The paragraph will also be expanded to address the advantages in more detail.
The comparison of the HRoI with other methods (e.g. the harmonic method) is not the subject of this paper and would be beyond its scope. Reliable harmonic constants exist only for a few German tide gauges and a comparison study of this kind would need a lot of resources that are not available at present. We agree that this testing is interesting and invite others to use the HRoI for their applications and comparisons.

3) "On p. 2 line 4: 44 angular velocities -> 45 angular velocities (Need to check it)"

>> The list of partial tides published by Horn (1960) consists of 44 angular velocities. These 44 angular velocities are also marked in our Table 2. The sentence in the manuscript is correct.

4) "In Table 3 and Table 4, angular velocity (!) should be expressed more than seven decimal places."

>> We will add one decimal place to the angular velocities in Tables 3 and 4 in the revised manuscript; also to make it consistent with the angular velocities in Table 2 and the operational usage. More decimal places would be beyond the uncertainty estimate which is of the order of 1e-7 degrees/transit number.

5) "On p. 6 line 21: The authors need to explain how to determine the criteria of 60% of high and low waters in more details. It seems to me that the value is low. As shown in Table A1, there are a lot of data sets with more than 90% completeness."

>> This selection criterion ensures that only data from tide gauges that record both high and low water are included in the analysis. Some tide gauges fall dry at low water and do not record meaningful tidal data during this time (and no low water height and time is included in the quality-checked time series). These tide gauges

have a data completeness of 50% at most. The threshold at 60% is rather conservative in this regard.

6) "On p. 7 line 12: What is 'tidal events'?"

>> High water and low water are referred to as "tidal events". We will clarify the language in the revised manuscript.

7) "On p. 19 lines 1 and 2: in the residua -> in the residual (?); the two residua -> the two residual (?)"

>> Yes, this is a typo. The sentence should read "The change of constituents has an influence on the remaining periodicities in the residuals." This will be corrected in the revised manuscript.

8) "In Figure 7: The authors need to explain how to determine a bin width for time and height differences."

>> The number and the width of the bins are chosen in such a way that the central bin is centred at the origin.

9) "The authors need to use the subscript in expressing name of tidal constituents throughout the manuscript. That is, Sa -> S_a (subscript a)"

>> We followed the naming scheme of the "Standard list of Tidal Constituents" by the IHO which does not use subscripts. We will add this information on page 6, line 2 in the revised manuscript, but prefer to keep the naming as it is if the reviewer does not have any objections.

Authors' response to Editor Comment #1

Dear Phil,

Thank you very much for your additional comments to our manuscript. Please find below our replies to your comments. We will upload the revised manuscript as soon as possible.

1) "One is that paper does have the feel of a highly-technical internal report and it might help to have an introductory paragraph in Section 6 (perhaps) to show that you know that there have been other methods for analysing HL waters in the past."

>> We will add some more references to other methods of tidal predictions in the revised manuscript. Thank you for your literature suggestions.

2) "Another is the comment by R1 about comparing the method used here to more standard harmonic methods, which you replied to in your paragraph (3) saying this was work in progress. But surely a tidal agency like the BSH is called on to produce hourly (or similar) tidal values for use in science or practical applications and you must have those data sets to hand. As regards the present paper it would not take much work to make a comparison for one or two places (say Cuxhaven). Last year I picked up a leaflet at the BSH which says'complete predictions of water level curves at Cuxhaven have been available on the internet since May 2010'."

>> The tidal information service from BSH does not provide hourly predictions on an operational basis (yet). As we start to have 19 years of 1-minute data from more and

more tide gauges, we are currently setting up the programs to use this high resolution data in our routine tidal predictions. For previous years, only the HL water recordings were saved for most tide gauges. As the request for a comparison with the more widely used harmonic methods has been expressed in all review reports, we will include a short comparison for two stations (probably Cuxhaven and Hamburg) in the revised manuscript.

The leaflet that you are referring to probably covers the water level and storm surge forecasting service (not the tidal information service). These water level curves (with and without surge) are produced by different methods, in which the tidal data calculated with the HRoI (times and heights of high and low waters) is used as an input.

3) "I understand the method for a particular station of course, but the rankings must be different for different stations so I was unclear how you arrived at the final choice. Could you make that clearer?"

>> The rankings displayed in Table 4 are a synthesis from the data of all analysed tide gauges. This will be mentioned in the caption of the table and will be made clearer in the corresponding paragraph. The goal of Table 4 is to produce one comprehensive list that reflects the information from all tide gauges in the area under investigation. A tailored analysis for an individual tide gauge is of course possible (and needed), if the general list does not lead to good results.

4) Most of the other remarks have been directly incorporated into the revised manuscript (thanks for all the details). Here are the answers to your questions:

4a) "I suspect that when most agencies produce tidal constants for a particular year they do not remove big storms; they are part of the sea level climatology, leading inevitably to ambiguity as to what defines the tide. So, in your case does this storm surge removal make any difference to the results?"

>> We try to predict water levels considering past long-term meteorological conditions (as good as this is possible). A single extreme event, like a severe storm surge, is not representative of the tidal behaviour at a site (and cannot be forecasted in the framework of tidal predictions). The model function (sum of harmonics) is also not made to properly account for these extreme events and the least squares method is likely (depends on the number of storm surges) to give results that lead to slightly higher heights at all times, if the storms are nor removed. We do not have numbers at hand on how much this storm surge removal influences the results (this will also depend on the number of extreme events in an individual time series). Part of this topic is the fundamental question on how to define the (astronomical) tide.

4b) "Tables 5 and 6 - is it necessary to have gauge number in these tables"

>> The numbers are necessary, because the short names for the tide gauges are not always unique, e.g. Borkum (Fischerbalje) vs. Borkum (Südstrand).

4c) "Figure 8-11. It might be good to make 8 and 10 into 8(a,b) and 9 and 11 into a new 9(a,b)."

>> We would like to keep these figures separated, as they cover slightly different aspects of the residual analysis.

Reassessment of long-period constituents for tidal predictions along the German North Sea coast and its tidally influenced rivers

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Abstract. The Harmonic Representation of Inequalities is a ~~method~~ procedure for tidal analysis and prediction that combines aspects of the non-harmonic and the harmonic method. With this technique, the deviations of heights and lunitidal intervals, especially of high and low waters, from their respective mean values are represented by superpositions of long-period tidal constituents. This ~~study~~ article documents the preparation of a constituents list for the operational application of the Harmonic Representation of Inequalities. Frequency analyses of observed heights and lunitidal intervals of high and low water from 111 tide gauges along the German North Sea coast and its tidally influenced rivers have been carried out using the generalized Lomb-Scargle periodogram. One comprehensive list of partial tides is realized by combining the separate frequency analyses and by applying subsequent improvements, e.g. through manual inspections of ~~long-time~~ long time series data. The new set of 39 partial tides largely confirms the previously used set with 43 partial tides. Nine constituents are added and 13 partial tides, mostly in close neighbourhood of strong spectral components, are removed. The effect of these changes has been studied by comparing predictions with observations from 98 tide gauges. Using the new set of constituents, the standard deviations of the residuals are reduced on average by 2.41% (times) and 2.30% (heights) for the year 2016. The new set of constituents ~~is~~ will be used for tidal analyses and predictions starting with the German tide tables for the year 2020.

1 Introduction

Tidal predictions for the German Bight are calculated at the Federal Maritime and Hydrographic Agency (*Bundesamt für Seeschifffahrt und Hydrographie*, BSH) and are published in official tide tables each year. The preparation of tidal predictions has a long tradition at BSH and its predecessor institutions: the first tide tables by the German Imperial Admiralty were issued for the year 1879.

Since 1954, a method named Harmonic Representation of Inequalities (HRoI) ~~is~~ has been used at BSH to calculate tidal predictions for tide gauge locations along the German North Sea coast and its tidally influenced rivers (Horn, 1948, 1960; Müller-Navarra, 2013). This technique allows analysing the deviations of times and heights, especially at high and low water, from their respective mean values. In contrast to the widely used harmonic method (e.g. Parker, 2007, and references therein) , the HRoI utilizes only long-period partial tides. This reduction in frequency space allows for a computationally efficient way to calculate times and heights of high and low water. Other techniques for tidal analysis of high and low waters are described, e.g., in Doodson (1951) and Foreman and Henry (1979) ; these two methods additionally consider diurnal and semi-diurnal

constituents. The HRoI has proven to be especially useful for predicting semi-diurnal tides in shallow waters where the harmonic method would need a large number ($\gtrsim 60$) of constituents or could even fail to produce adequate results. The fundamentals of the HRoI are summarized in Sect. 2 for completeness.

An important aspect of tidal prediction is the selection of relevant partial tides (angular velocities, ω) to be included in the underlying analysis of water level records. While it is possible to determine these partial tides individually for each single tidal analysis, it is desirable in an operational service to have one comprehensive set of constituents that can be used for all tide gauges under investigation. Horn (1960) presented a list of 44 angular velocities that were used with the HRoI. This selection of partial tides was probably utilized until the year 1969 when the set was slightly modified (compare Tab. 2 in Sect. 2). To our knowledge, no documentation exists of the methods and specific water level records that were used to prepare these lists of angular velocities.

The objective of this work is to review the set of partial tides used with the HRoI by determining the most important long-period constituents for applications in the German Bight. Therefore, we perform a spectral analysis of water level observations from 111 tide gauges. The available tide gauge data is presented in Sect. 3. The analysis of high and low water time series is described in Sect. 4. In Sect. 5, tidal predictions based on an existing list of partial tides and predictions based on the new set are compared with observed water levels. The article closes with a comparison of predictions made with the HRoI and the harmonic method for two sites (Sect. 6) and the conclusions (Sect. 7).

2 Harmonic Representation of Inequalities

The Harmonic Representation of Inequalities (HRoI) is a derivative of the non-harmonic method by essentially translating it into an analytical form. The non-harmonic method has been used for a long time, e.g. by Lubbock (1831) for the analysis of tides in the port of London. With the non-harmonic method, the times of high and low ~~water~~ waters are calculated by adding mean lunitidal intervals and corresponding inequalities to the times of lunar transits. Likewise, the heights of high and low ~~water~~ waters are determined by adding corresponding inequalities to the respective mean heights. The inequalities are corrections for the relative positions of earth, moon and sun (e.g. semi-monthly, parallactic, declination).

The original implementation of the HRoI, as introduced by Horn (1948, 1960), can be used to calculate vertices of tide curves, i.e. high water time, high water height, low water time and low water height. In this form the method is tailored to semi-diurnal tides. Müller-Navarra (2013) shows how the HRoI may be generalized to predict tidal heights at equidistant fractions of the mean lunar day. This generalization allows the determination of the full tidal curve at a chosen sampling interval. Here, we focus only on the application of calculating the times and heights of high and low ~~water~~ waters.

According to Horn (1960) , the HRoI combines the best from the harmonic and the non-harmonic method: the analytical procedure of the first method, and the principle of calculating isolated values directly which is characteristic for the second. The strength of the HRoI lies in the prediction of times and heights of high and low water when the full tidal curve is considerably non-sinusoidal. This is frequently the case in shallower waters, such as the German Bight, and in rivers. As the HRoI uses only

Table 1. The high and low waters are classified into four types (event index k).

k	description
1	high water assigned to upper transit
2	low water assigned to upper transit
3	high water assigned to lower transit
4	low water assigned to lower transit

observed times and/or heights of high and/or low waters, the method can also be applied when a record of the full tidal curve is not available (e.g. historic data) or when a tide gauge runs dry around low water (e.g. analysis of only high waters).

Let $(t_j, h_j), j = 1, ..., J$, be a time series of length J of high and low water heights h_j recorded at times t_j. All times need to be given in UTC. The HRoI method is based on the assumption that the variations of the individual heights and lunitidal intervals around their respective mean values can be described by sums of harmonic functions. The lunitidal interval is the time difference between the time t_j and the corresponding lunar transit at Greenwich. As a general rule, the daily higher high water and the following low water are assigned to the previous upper lunar transit, and the daily lower high water and the following low water are assigned to the previous lower transit. For example, in the year 2018, the mean lunitidal interval for high (low) water was determined to be 9 h 4 min (16 h 5 min) for Borkum and 15 h 22 min (22 h 32 min) for Hamburg. See Fig. 1 in Sect. 3 for the locations of these two sites.

A convenient method to organize high and low waters of semi-diurnal tides is the lunar transit number n_t (Müller-Navarra, 2009). It counts the number of upper lunar transits (unit symbol: tn) at the Greenwich meridian since the transit on December 31, 1949, which has been arbitrarily set to $n_t = 0$ tn. A lower transit always has the same transit number as the preceding upper transit. Each high and low water is uniquely identified by using the number n_t of the assigned lunar transit and an additional event index k as defined in Tab. 1. The differentiation between upper and lower transit allows for changes in the Moon's declination which alternately advance and retard times, and increase and decrease the heights of successive tides (diurnal inequality).

A full tidal analysis with the HRoI comprises the investigation of eight times time series (heights and lunitidal intervals of the four event types listed in Tab. 1). Each time series is described by a model function $\hat{y}$ of the following form:

$$\hat{y}(n_t) = a_0 + \sum_{l=1}^{L} \left[a_l \cos(\omega_l n_t) + a_{l+L} \sin(\omega_l n_t) \right].\tag{1}$$

The parameters $a_l, l = 0, ..., 2L$ are determined from a least-squares fit, i.e.

$$\chi^2 = \sum_{j=0}^{J} (y_j - \hat{y}_j)^2 \to \min \quad ,\tag{2}$$

where y_j are the observed heights or lunitidal intervals. The angular velocities ω_l [°/tn] are taken from a previously defined set of L partial tides. In Tab. 2, we list two sets of partial tides that have been used in the past at BSH and the new set that is the result of this work.

Table 2. Sets of angular velocities that have been used with the HRoI. See Sect. 2 for a description of the columns.

Doodson	m_s	m_h	m_p	$m_{N'}$	ω [°/h]	ω [°/tn]	name	set 1[a]	set 2[b]	this work
ZZZZAZ	0	0	0	1	0.0022064	0.0548098		x	x	x
ZZZAZZ	0	0	1	0	0.0046418	0.1153082			x	
ZZZBZZ	0	0	2	0	0.0092836	0.2306165				x
ZZAYZZ	0	1	-1	0	0.0364268	0.9048862			x	
ZZAZZZ	0	1	0	0	0.0410686	1.0201944	Sa	x	x	x
ZZBXZZ	0	2	-2	0	0.0728537	1.8097724		x	x	x
ZZBZZZ	0	2	0	0	0.0821373	2.0403886	Ssa	x	x	x
ZAXZZZ	1	-2	0	0	0.4668792	11.5978420		x	x	x
ZAXAZZ	1	-2	1	0	0.4715211	11.7131503	MSm	x	x	x
ZAYXZZ	1	-1	-2	0	0.4986643	12.3874200				x
ZAYZZZ	1	-1	0	0	0.5079479	12.6180365				x
ZAYAAZ	1	-1	1	1	0.5147961	12.7881545				x
ZAZYYZ	1	0	-1	-1	0.5421683	13.4681129		x	x	
ZAZYZZ	1	0	-1	0	0.5443747	13.5229227	Mm	x	x	x
ZAZZYZ	1	0	0	-1	0.5468101	13.5834211		x	x	
ZAZZZZ	1	0	0	0	0.5490165	13.6382309		x	x	x
ZAZZAZ	1	0	0	1	0.5512229	13.6930407		x	x	x
ZAZAZZ	1	0	1	0	0.5536583	13.7535391				x
ZABYZZ	1	2	-1	0	0.6265120	15.5633115		x	x	
ZABBAZ	1	2	2	1	0.6426438	15.9640460				x
ZBVBZZ	2	-4	2	0	0.9430421	23.4263005		x		
ZBWZZZ	2	-3	0	0	0.9748271	24.2158785		x	x	x
ZBXZYZ	2	-2	0	-1	1.0136894	25.1812631		x	x	x
ZBXZZZ	2	-2	0	0	1.0158958	25.2360729	MSf	x	x	x
ZBXZAZ	2	-2	0	1	1.0181022	25.2908827		x	x	
ZBXAZZ	2	-2	1	0	1.0205376	25.3513811		x	x	
ZBYZZZ	2	-1	0	0	1.0569644	26.2562673				x
ZBZXZZ	2	0	-2	0	1.0887494	27.0458453		x	x	x
ZBZYZZ	2	0	-1	0	1.0933912	27.1611535		x	x	x
ZBZZYZ	2	0	0	-1	1.0958266	27.2216520		x	x	
ZBZZZZ	2	0	0	0	1.0980330	27.2764618	Mf	x	x	x

Doodson	m_s	m_h	m_p	$m_{N'}$	ω [°/h]	ω [°/tn]	name	set 1[a]	set 2[b]	this work
					continued from previous page					
ZBZZAZ	2	0	0	1	1.1002394	27.3312716		x	x	x
ZCVAZZ	3	-4	1	0	1.4874168	36.9492232	Sν2	x	x	x
ZCWYZZ	3	-3	-1	0	1.5192018	37.7388011		x	x	
ZCXYYZ	3	-2	-1	-1	1.5580641	38.7041858		x	x	
ZCXYZZ	3	-2	-1	0	1.5602705	38.7589956	SN	x	x	x
ZCXYAZ	3	-2	-1	1	1.5624769	38.8138054		x	x	
ZCXZZZ	3	-2	0	0	1.5649123	38.8743038		x	x	x
ZCXAZZ	3	-2	1	0	1.5695541	38.9896120	MStm	x	x	x
ZCZWZZ	3	0	-3	0	1.6331241	40.5687675		x		
ZCZYZZ	3	0	-1	0	1.6424077	40.7993844	Mfm	x	x	x
ZDUZZZ	4	-5	0	0	1.9907229	49.4519514		x	x	x
ZDVZZZ	4	-4	0	0	2.0317915	50.4721458	2SM	x	x	x
ZDXXZZ	4	-2	-2	0	2.1046452	52.2819182		x	x	
ZDXZZZ	4	-2	0	0	2.1139288	52.5125347	MSqm	x	x	x
ZDXZAZ	4	-2	0	1	2.1161352	52.5673444				x
ZDZZZZ	4	0	0	0	2.1960661	54.5529235		x	x	x
ZETAZZ	5	-6	1	0	2.5033126	62.1852961		x	x	x
ZEVYZZ	5	-4	-1	0	2.5761662	63.9950685	2SMN	x	x	x
ZEVZZZ	5	-4	0	0	2.5808080	64.1103767		x		
ZEVAZZ	5	-1	1	0	2.5854499	64.2256849				x
ZEXYZZ	5	-2	-1	0	2.6583035	66.0354573		x	x	
ZFTZZZ	6	-6	0	0	3.0476873	75.7082187		x	x	x
ZFVZZZ	6	-4	0	0	3.1298246	77.7486076		x	x	x
ZHRZZZ	8	-8	0	0	4.0635830	100.9442917		x	x	x
Number of partial tides in set of constituents:								44	43	39

[a] set 1 was probably used until the year 1969, see also Tab. 3 in Horn (1960). [b] set 2 was probably used from 1970 until 2019, see also appendix E in Goffinet (2000), Tab. 5 in Müller-Navarra (2013) includes $\omega = 23.4263005°$/tn but this angular velocity has never been included in calculations for BSH tide tables or tide calendars.

All tidal constituents considered here have angular velocities that are linear combinations of the rate of change of four fundamental astronomical ~~variables~~arguments: the mean longitude of the moon (s), the mean longitude of the sun (h), the mean longitude of the lunar perigee (p) and the negative of the longitude of the moon's ascending node (N'). The second to fifth column in Tab. 2 give the respective linear coefficients m. The two other arguments that one encounters using the harmonic method can be effectively neglected: the coefficients for the rate of change of the mean lunar time and of the mean longitude of the solar perigee are always equal to zero, because only long-period ~~times are considered~~constituents need to be considered, and the time series are too short to resolve differences due to the variations of the solar perigee. The angular velocities in the sixth

and seventh column are given in degrees per hour and in degrees per transit number ~~(unit symbol: tn)~~, respectively. ~~They~~ The conversion between these two units is $1°/\text{tn} \cdot \tau\left[\frac{\text{h}}{\text{tn}}\right] = 1°/\text{h}$ with the length of the mean lunar day $\tau = 24.84120312\,\frac{\text{h}}{\text{tn}}$. The angular velocities are calculated using the expressions for the fundamental astronomical arguments as published by the International Earth Rotation and Reference Systems Service (2010, Sect. 5.7). The ~~first column is the~~ alphabetical Doodson number ~~and the~~ is given in the first column (Doodson, 1921; Simon, 2013) . The eighth column states the commonly used names[1]. A mark in one of the last three columns indicates whether the angular velocity is included in the respective constituents list for usage with the HRoI.

~~According to Horn (1960) , the HRoI combines the best from the harmonic and the non-harmonic method: the analytical procedure of the first method, and the principle of calculating isolated values directly which is characteristic for the second. The strength of the HRoI lies in the prediction of times and heights of high and low water when the full tidal curve is considerably non-sinusoidal. This is frequently the case in shallow waters and rivers.~~

3 Tide gauge data

The tide gauges at the German coast and in rivers are operated by different federal and state authorities. These agencies provide BSH with ~~quality checked~~ quality-checked water level records of high and low waters (times and heights). Table A1 in the appendix lists 137 German tide gauges which deliver water level observations on a regular basis and for which tidal predictions were published in BSH tide tables (*Gezeitentafeln*) or tide calendar (*Gezeitenkalender*) for the year 2018 (Bundesamt für Seeschifffahrt und Hydrographie, 2017a, b). For the analysis presented in Sect. 4, all data until the year 2015 is considered that was systematically archived in electronic form at the BSH tidal information service (as of August 2018). The data periods are given in the fourth and fifth column in Tab. A1 and cover $22 - 27$ years for most gauges. Much longer time series ~~are~~ were readily available for tide gauges at Cuxhaven (BSH gauge number ~~DE~~506P) and Hamburg (~~DE~~508P) for which data since the year 1901 is used. We are aware that the tidal regime can change over such a long time, but include all available data in the analysis to maximize the achievable spectral resolution.

Only tide gauges with more than 19 years of data are included in order to cover the period of rotation of the lunar node (18.6 years) in the frequency analysis. In addition, we use only tide gauges where more than 60% of high and low waters are recorded during the gauge's data period. This criterion excludes gauges for which no low water observations are available. The 111 gauges that ~~fulfill~~ fulfil these two criteria are marked in the column labelled "used for analysis" in Tab. A1. The locations of ~~the~~ all tide gauges are shown on the map in Fig. 1.

4 Analysis of high water and low water time series

The following analysis is applied to the water level records of all 111 tide gauges that are marked in the seventh column of Tab. A1 in the appendix.

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[1] see, e.g., the IHO Standard List of Tidal Constituents: https://www.iho.int/mtg_docs/com_wg/IHOTC/IHOTC_Misc/TWCWG_Constituent_list.pdf
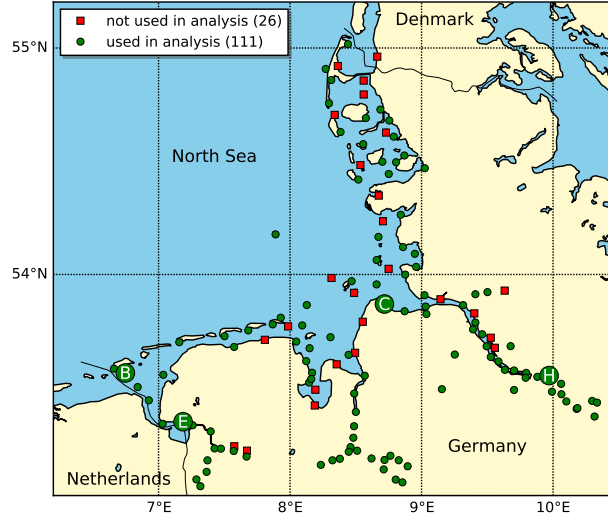
Figure 1. ~~Locations~~ The locations of all tide gauges in the German Bight from ~~Table~~ Tab. A1. Some of the tide gauges mentioned in the ~~German Bight~~ text are highlighted: Borkum, Fischerbalje (B); Emden, Große Seeschleuse (E); Cuxhaven, Steubenhöft (C); Hamburg, St. Pauli (H).

4.1 Data preparation

Data preparation includes the assignment of lunar transit numbers n_t and the calculation of lunitidal intervals as described in Sect. 2 for each record of high or low water. The lunar transit times are calculated ~~using~~ following the algorithm by ~~Meeus (1998) and the lunar coordinates from~~ Meeus (1998, Chap. 15) with the modification of direct calculation of lunar coordinates using the periodic terms given in the work by Chapront-Touzé and Chapront (1991).

The observed water levels include extreme events, such as storm surges. These events are not representative for the tidal behaviour at the site of a tide gauge and ~~need to be~~ are removed from the data set. We apply a 3-sigma-clipping separately for the eight times series analysed with the HRoI (see Sect. 2). Only those data points are used in the analysis, for which the height and the lunitidal interval are within the range of three times the respective standard deviation.

4.2 Frequency analysis

The observed heights and lunitidal intervals ~~are~~ (y) can be understood as being functions of the assigned transit number (n_t). We calculate periodograms for the heights and tidal intervals using the corresponding frequency scale tn^{-1}.

The ~~tidal events~~ occurrences of high and low waters are irregularly spaced in time. Additionally, there are many longer data gaps which cannot be interpolated. This excludes the fast Fourier transform (FFT) as a spectral analysis technique. Instead, we use the generalized Lomb-Scargle periodogram as defined by Zechmeister and Kürster (2009), including their normalization if not mentioned otherwise. The frequency scale covers the range from ~~0.0001~~ 0.0001 to $2\,\text{tn}^{-1}$ with an interval
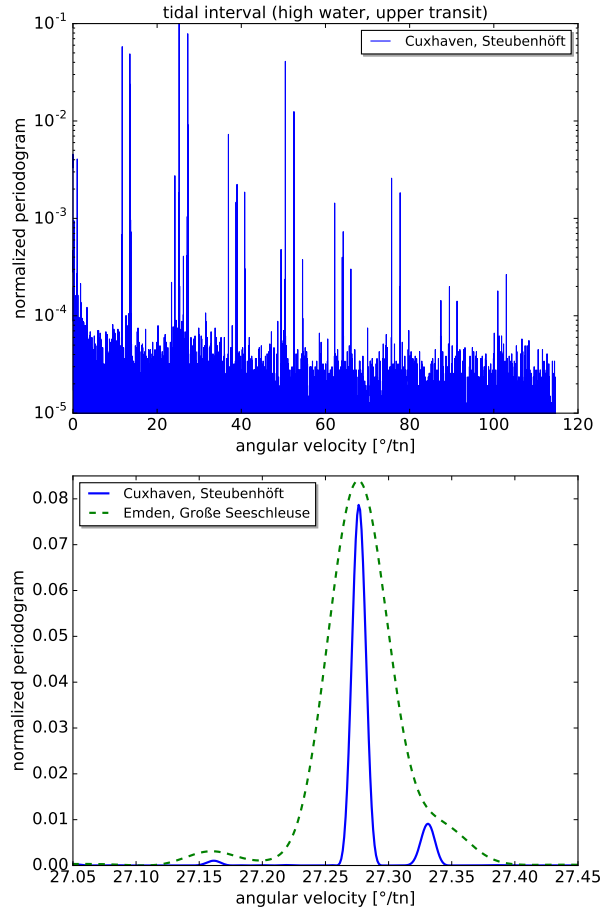
Figure 2. *Upper panel:* Normalized periodogram of the lunitidal intervals of high waters (assigned to upper lunar transits) for the tide gauge Cuxhaven. Notice the upper part of the logarithmic scale ~~which~~ is truncated at 0.1 for better visibility of weak lines. *Lower panel:* Zoom into the region with the spectral line corresponding to half a tropical month (Mf) at 27.2764618°/tn. The longer time series for Cuxhaven leads to narrower spectral lines (solid blue curve) compared to Emden (dashed green line).

of ~~0.01999~~0.01999 tn^{-1} (100 000 points in the periodogram). This corresponds to approximately $0.0057 - 114.5916°$/tn or $0.0002 - 4.6130°$/h. The upper limit corresponds to twice the mean sampling interval (Nyquist criterion).

    Artefacts from spectral leakage pose a major problem when identifying peaks in a periodogram. They arise from the finite length of the time series~~(e.g. Press et al., 1992)~~. This effect can be reduced by applying an ~~appodization function~~apodization
5 function, i.e. multiplying the data with a suitable window function, that smoothly brings the recorded values to zero at the beginning and the end of the sampled time series (e.g. Press et al., 1992; Prabhu, 2014) . We apply a Hanning window to the data which gives a good compromise between reducing side lobes and preserving the spectral resolution.

8
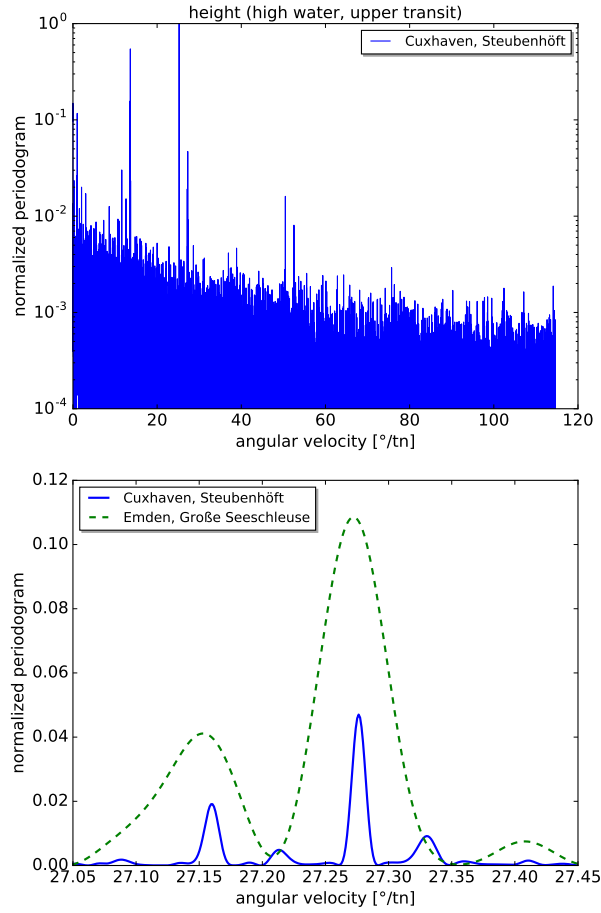
Figure 3. *Upper panel:* Normalized periodogram of the heights of high waters (assigned to upper lunar transits) for the tide gauge Cuxhaven. Notice the logarithmic scale. *Lower panel:* Zoom into the region with the spectral line corresponding to half a tropical month (Mf) at 27.2764618°/tn. The longer time series for Cuxhaven leads to narrower spectral lines (solid blue curve) compared to Emden (dashed green line).

For each tide gauge, periodograms are calculated for the eight time series that are analysed with the HRoI. In the upper panels of Fig. 2 and 3, we show periodograms of the lunitidal intervals and heights (of high waters assigned to an upper transit, event index $k = 1$) for the tide gauge Cuxhaven. Cuxhaven (together with Hamburg) provides by far the longest ~~times~~ time series that is used in the analysis (compare Tab. A1). In these figures, the vertical axis is normalized to the strongest peak and the

5 horizontal axis is converted to degrees per transit number for better comparison with Tab. 2. The periodogram for the lunitidal intervals reveals many more strong spectral lines above the noise floor as compared to the periodogram for the heights. A ~~frequency depended~~ frequency-dependent noise level can clearly be seen in Fig. 3 (noise level increases towards lower angular velocities). The lower panels of Fig. 2 and 3 show a small extract of the respective upper periodograms. Additionally, data for

tide gauge Emden is included for illustration of the differences in spectral line width. The time series from Emden is about four times shorter than the one from Cuxhaven. This leads to broader spectral lines in the periodogram and it can be expected that some weaker lines are unresolvable.

4.3 Identifying relevant partial tides

5 We aim to find all local maxima in a periodogram that are above a noise threshold. This threshold is calculated in a two-step process that is described in the following.

In the first step, the strongest spectral lines are removed from the periodogram. The values above the 99.5th percentile are removed from the data set and a histogram is calculated from the remaining values p (100 bins with central values x_{bin}). The histogram shows an exponential trend from a ~~high~~ large number of data points with low periodogram values to a few points that

10 fall into the bins at the upper end. An exponential curve $y_{\mathrm{bin}} = a \cdot \exp(-x_{\mathrm{bin}}/b)$ is fitted to the histogram, with fit parameters a and b. The process of removing data points above the 99.5th percentile from the periodogram is repeated until the ratio $\max(p)/b$ falls below the value of 30. This value is based on experience.

In the second step, the noise threshold is determined using a set of remaining points in the periodogram that represent a continuum above the noise level. The result is illustrated in Figs. 4 and 5 for lunitidal intervals and heights at the tide gauge

15 Borkum. For this procedure, the periodogram is split into 25 sections with the same number of data points. The data point at the 99.5th percentile is selected in each section and an exponential function is fitted to these 25 points. The fit is repeated after a 1-sigma-clipping. The noise threshold corresponds to the resulting exponential function plus one standard deviation (solid red line in Figs. 4 and 5).

~~The noise threshold functions~~ In preparation of the following combined evaluation of the results from all tide gauges~~are~~

20 ~~averaged~~, the noise threshold functions from the different periodograms are averaged; separately for lunitidal intervals ~~, $L_{\mathrm{i}}(\omega)$,~~ ~~and heights, $L_{\mathrm{h}}(\omega)$:~~ (L_{i}) and heights (L_{h}):

$$L_{\mathrm{i}}(\omega) = 0.0004816 \cdot \exp(-0.0101045\,\mathrm{tn}/^{\circ} \cdot \omega) \quad ,$$

$$L_{\mathrm{h}}(\omega) = 0.0024472 \cdot \exp(-0.0149899\,\mathrm{tn}/^{\circ} \cdot \omega) \quad .$$

These two functions represent mean lower intensity boundaries for the selection of significant peaks. The expressions L_{i} and

25 L_{h} are unitless, due to the normalization of the Lomb-Scargle periodogram (Zechmeister and Kürster, 2009) .

~~A second selection criterion is~~ In addition to the intensity of a local maximum, the number of ~~occurrences of the local maxima in the periodograms and their~~ its occurrences in the different periodograms and its assignment to the partial tides ~~having~~ determine the inclusion into the list of constituents for the HRoI. The local maxima must match the theoretically expectable partial tides that have well known angular velocities computable from the linear combinations of the rate of change

30 of the four fundamental astronomical arguments s, h, p and N' (see Sect. 2). The angular velocities of 1268 ~~potential~~ partial tides have been precalculated using again the expressions for the fundamental astronomical arguments as published by the International Earth Rotation and Reference Systems Service (2010, Sect. 5.7) . The ranges of the linear coefficients m ~~for the fundamental variables~~ are chosen based on experience (compare Tab. 2):
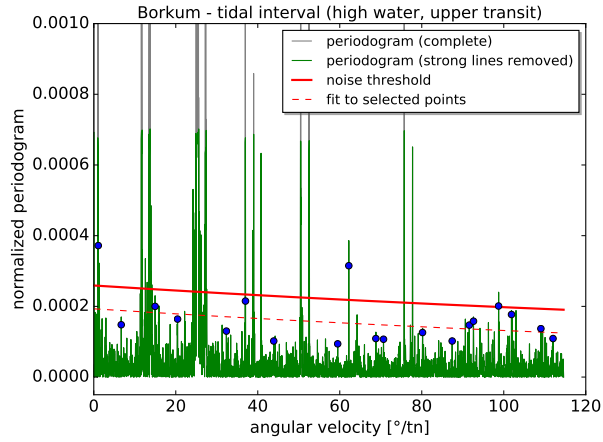
Figure 4. Determination of the noise threshold for the tidal interval (high water, upper transit) at tide gauge Borkum, Fischerbalje: the ~ The strongest lines are removed from the periodogram (grey vs. green lines; first step as described in Sect. 4.3) and an exponential function (dashed red curve) is fitted to selected points (blue; second step as described in Sect. 4.3). The noise threshold (thick red line) is shifted up by one standard deviation. See text for more details.
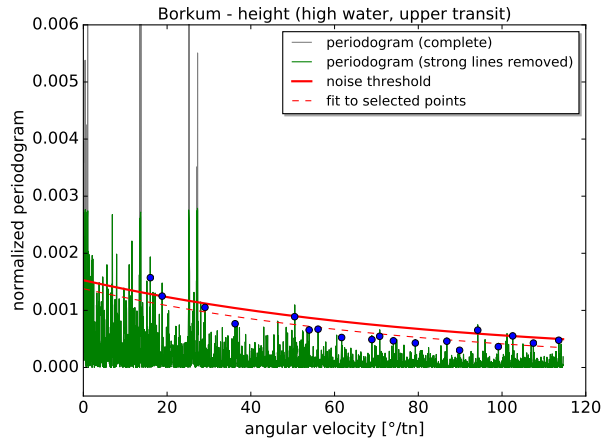
Figure 5. Same as Fig. 4 but for the heights at tide gauge Borkum, Fischerbalje.

$$m_s = 0, ..., 8$$

$$m_h = -8, ..., 3$$

$$m_p = -2, ..., 3, \quad \text{if } ((m_s = 0 \text{ and } m_h \geq 0)$$

$$\text{or } (m_s > 0 \text{ and } m_h \geq -m_s - 1))$$

5 $\quad m_{N'} = -1, 0, 1, \quad \text{if } ((m_s = 0 \text{ and } m_h = 0$

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and $m_p = 0$ and $\cancel{m'_N \geq 0}\underset{\sim\sim\sim}{m_{N'} \geq 0}$)

or ($m_s \neq 0$ or $m_h \neq 0$

or $m_p \neq 0$))

A partial tide from the precalculated list is assigned uniquely to the closest peak in the periodogram if the difference in angular velocity is less than half the spectral resolution. The spectral resolution r is defined as

$$r = 360°/T \quad , \tag{3}$$

with T being the length of the time series in transit numbers. For example, the spectral resolution of a time series of 19 years is

$$r = \frac{360°}{19\,\text{yr} \cdot 365.25\,\text{d/yr} \cdot \tau} = 0.05°/\text{tn} \quad , \tag{4}$$

where $\tau = 1.03505013\,\text{d/tn}$ is the length of the mean lunar day.

For each identified partial tide, we calculate (i) the percentage of periodograms in which the partial tide has been detected, separately for lunitidal interval (N_i) and height (N_h); and (ii) the average intensity in the periodograms, separately for lunitidal interval (I_i) and height (I_h). In order to be considered relevant, a partial tide with angular velocity ω must meet the following criteria: $N_i \geq 33\%$ and $I_i(\omega) > L_i(\omega)$, or $N_h \geq 33\%$ and $I_h(\omega) > L_h(\omega)$. All partial tides that meet these selection criteria are listed Tab. 3.

Table 3. The most important partial tides that were identified in the periodograms, based on the combined evaluation of data from all tide gauges. See Sect. 4.3 for information on selection criteria and I_i, I_h, N_i and N_h.

Doodson	ω [°/tn]	I_i [-]	I_h [-]	N_i [%]	N_h [%]
ZZZZAZ	~~0.054810~~ 0.0548098	0.0086	0.0102	78	45
ZZZBZZ	~~0.230616~~ 0.2306165	0.0019	0.0085	29	47
ZZAXZZ	~~0.789578~~ 0.7895780	0.0009	0.0088	16	34
ZZAZZZ	~~1.020194~~ 1.0201944	0.0070	0.0367	85	84
ZZBZZZ	~~2.040389~~ 2.0403886	0.0013	0.0068	26	56
ZAXZZZ	~~11.597842~~ 11.5978420	0.0009	0.0034	69	35
ZAXAZZ	~~11.713150~~ 11.7131503	0.0234	0.0024	100	10
ZAYXZZ	~~12.387420~~ 12.3874200	0.0006	0.0031	2	65
ZAYZZZ	~~12.618036~~ 12.6180365	0.0007	0.0042	34	21
ZAYAAZ	~~12.788154~~ 12.7881545	0.0005	0.0031	1	45
ZAZYZZ	~~13.522923~~ 13.5229227	0.0112	0.0119	99	90
ZAZZZZ	~~13.638231~~ 13.6382309	0.0105	0.0297	97	87
ZAZAZZ	~~13.753539~~ 13.7535391	0.0006	0.0016	63	2
ZABBAZ	~~15.964046~~ 15.9640460	0.0010	0.0032	1	70
ZBWZZZ	~~24.215878~~ 24.2158785	0.0029	0.0081	83	7
ZBXZZZ	~~25.236073~~ 25.2360729	0.4550	0.0706	100	92
ZBYZZZ	~~26.256267~~ 26.2562673	0.0034	0.0079	55	9
ZBZYZZ	~~27.161154~~ 27.1611535	0.0008	0.0021	43	29
ZBZZZZ	~~27.276462~~ 27.2764618	0.0382	0.0070	100	85
ZCVAZZ	~~36.949223~~ 36.9492232	0.0037	0.0013	99	25
ZCXYZZ	~~38.758996~~ 38.7589956	0.0009	0.0014	89	9
ZCXAZZ	~~38.989612~~ 38.9896120	0.0010	0.0004	96	0
ZCZYZZ	~~40.799384~~ 40.7993844	0.0008	0.0000	93	0
ZDUZZZ	~~49.451951~~ 49.4519514	0.0004	0.0000	38	0
ZDVZZZ	~~50.472146~~ 50.4721458	0.0196	0.0025	100	73
ZDXZZZ	~~52.512535~~ 52.5125347	0.0059	0.0015	99	15
ZDZZZZ	~~54.552924~~ 54.5529235	0.0003	0.0003	43	0
ZETAZZ	~~62.185296~~ 62.1852961	0.0006	0.0000	93	0

Doodson	ω [°/tn]	I_i [-]	I_h [-]	N_i [%]	N_h [%]
ZEVYZZ	~~63.995068~~ 63.9950685	0.0003	0.0001	61	0
ZEVAZZ	~~64.225685~~ 64.2256849	0.0004	0.0000	78	0
ZFTZZZ	~~75.708219~~ 75.7082187	0.0014	0.0003	98	2
ZFVZZZ	~~77.748608~~ 77.7486076	0.0010	0.0003	97	5
ZHRZZZ	~~100.944292~~ 100.9442917	0.0002	0.0000	36	0
ZHTZZZ	~~102.984681~~ 102.9846805	0.0002	0.0002	37	0

4.4 Adjustment of constituent list and ranking

In this section, we describe adjustments made to the list of partial tides based on manual inspections of certain periodograms and other considerations for an operational application. These adjustments lead to the set of partial tides in Tab. 4.

5   The periodograms calculated from longer time series offer a higher spectral resolution and contain more spectral information compared to the periodograms of shorter time series. ~~For example, this can be seen~~ This is demonstrated in the lower panels of Fig. 2 and 3 with periodograms based on time series from tide gauge Cuxhaven (115 years) and Emden (27 years). The higher information content from longer water level records needs to be appreciated and incorporated adequately. Therefore, the periodograms of Cuxhaven and Hamburg have been inspected manually to find partial tides that appear in the data of these

10   two tide gauges and might not be detectable in other periodograms. Six partial tides with the following Doodson numbers were identified and added to the list: ZAZZAZ (ZAZZZZ), ZBXZYZ (ZBXZZZ), ZBZXZZ, ZBZZAZ (ZBZZZZ), ZCXZZZ and ZDXZAZ (ZDXZZZ). The Doodson numbers in parenthesis are partial tides from Tab. 3 that differ only by $\Delta m_{N'} = \pm 1$. For these pairs, long time series are needed to clearly see two separate spectral lines in the periodograms.

The noise in the periodograms increases towards lower angular velocities and the identification of partial tides below 1°/tn

15   becomes less clear. For this reason, and after inspecting several periodograms manually, the partial tide ZZAXZZ is considered to be a misidentification and has been removed from the list. ~~The other way round~~ Conversely, the partial tide ZZBXZZ has been added to the list, because of its importance for tide gauges located upstream in the Elbe river. Finally, we decided to cut the list after the eighth synodic month to keep the range of angular velocities consistent with previously used lists of partial tides (compare Tab. 2).

20   The final set of ~~long-term~~ long-period partial tides from our analysis is listed in Tab. 4. In the last column, each partial tide is assigned a number R indicating its overall importance (in decreasing order). The rank R is based on the combined evaluation of data from all tide gauges and is calculated by the following procedure:

$$R_i = \mathrm{rank}(\mathrm{norm}(I_i(\omega) - L_i(\omega)) \cdot N_i)$$

$$R_h = \mathrm{rank}(\mathrm{norm}(I_h(\omega) - L_h(\omega)) \cdot N_h)$$

25   $$R = \mathrm{rank}((3R_i + R_h)/4) \tag{5}$$

where the function norm() returns normalized values in the range [0,1] and the function rank() returns the position of a list element, if the list were sorted in increasing order. In Eq. 5, the results from lunitidal intervals are weighted ~~higher~~ stronger because the noise level is lower in the respective periodograms.

Table 4. The modified and adopted new list of long-period partial tides. The rank R indicates the importance of a partial tide for tidal analysis, based on the combined evaluation of data from all tide gauges.

Doodson	ω [°/tn]	description / name	R
ZZZZAZ	~~0.054810~~ 0.0548098	lunar nodal precession	6
ZZZBZZ	~~0.230616~~ 0.2306165	half lunar apsidal precession	13
ZZAZZZ	~~1.020194~~ 1.0201944	tropical year / Sa	7
ZZBXZZ	~~1.809772~~ 1.8097724		31
ZZBZZZ	~~2.040389~~ 2.0403886	half tropical year / Ssa	17
ZAXZZZ	~~11.597842~~ 11.5978420		14
ZAXAZZ	~~11.713150~~ 11.7131503	MSm	8
ZAYXZZ	~~12.387420~~ 12.3874200		34
ZAYZZZ	~~12.618036~~ 12.6180365		19
ZAYAAZ	~~12.788154~~ 12.7881545		39
ZAZYZZ	~~13.522923~~ 13.5229227	anomalistic month / Mm	3
ZAZZZZ	~~13.638231~~ 13.6382309	tropical month	4
ZAZZAZ	~~13.693041~~ 13.6930407		38
ZAZAZZ	~~13.753539~~ 13.7535391		21
ZABBAZ	~~15.964046~~ 15.9640460		36
ZBWZZZ	~~24.215878~~ 24.2158785		11
ZBXZYZ	~~25.181263~~ 25.1812631		35
ZBXZZZ	~~25.236073~~ 25.2360729	half synodic month / MSf	1
ZBYZZZ	~~26.256267~~ 26.2562673		12
ZBZXZZ	~~27.045845~~ 27.0458453		33
ZBZYZZ	~~27.161154~~ 27.1611535		15
ZBZZZZ	~~27.276462~~ 27.2764618	half tropical month / Mf	2
ZBZZAZ	~~27.331272~~ 27.3312716		27
ZCVAZZ	~~36.949223~~ 36.9492232	Sν2	10
ZCXYZZ	~~38.758996~~ 38.7589956	SN	16
ZCXZZZ	~~38.874304~~ 38.8743038		24
ZCXAZZ	~~38.989612~~ 38.9896120	MStm	22
ZCZYZZ	~~40.799384~~ 40.7993844	Mfm	23
ZDUZZZ	~~49.451951~~ 49.4519514		29
ZDVZZZ	~~50.472146~~ 50.4721458	fourth synodic month / 2SM	5
		continued on next page	

Doodson	ω [°/tn]	description / name	R
continued from previous page			
ZDXZZZ	52.5125347	MSqm	9
ZDXZAZ	52.5673444		37
ZDZZZZ	54.5529235		30
ZETAZZ	62.1852961		25
ZEVYZZ	63.9950685	2SMN	28
ZEVAZZ	64.2256849		26
ZFTZZZ	75.7082187	sixth synodic month	20
ZFVZZZ	77.7486076		18
ZHRZZZ	100.9442917	eighth synodic month	32

The rank R can be used to select a sublist of partial tides when performing a tidal analysis of water levels with less than 18.6 years of data. This is important, because not all partial tides can be resolved against each other for shorter time series. The minimum difference in angular velocity is given by the resolution criterion (Eq. 3). Figure 6 illustrates the resolvable partial tides as a function of the length of the data record. Note that the high-ranked partial tide representing the tropical month (which occurs at $R = 4$ in the list) cannot be included for time series shorter than about nine years. For a tidal analysis of time series shorter than nine years, it is therefore often better to perform a reference analysis: 19 years of data are used from a different tide gauge with a similar tidal behaviour (e.g. similar course of the semi-monthly inequality) and the results are translated to the original gauge by applying the respective differences of the mean lunitidal intervals and mean heights. This way, nodal corrections can be avoided which come with their own assumptions and approximations (e.g. Godin, 1986; Amin, 1987).

5 Comparison of predictions with observations: two different lists of constituents for the HRoI

For verification of the new constituent list, tidal predictions based on an existing list of partial tides and based on the new set are compared with observations. The predictions are made for the year 2016 and are compared with tide gauge observations from the same year.

5.1 Tidal analysis and prediction

We calculate tidal predictions (times and heights of high and low waters) with the HRoI using (i) the 43 partial tides from "set 2" in Tab. 2 and (ii) the 39 partial tides derived from our analysis. The data and software are otherwise identical. The predictions are based on amplitudes a_m (see Eq. 1) that are determined from a tidal analysis of water level records from 1995 to 2013 (19 years). The analysis is applied in two iterations with a 3-sigma-clipping in between. Data are only used from tide gauges that have delivered enough observations in this time period to include all partial tides in the analysis. Additionally, the tide gauges must have deliver observations for the year 2016. The 98 tide gauges

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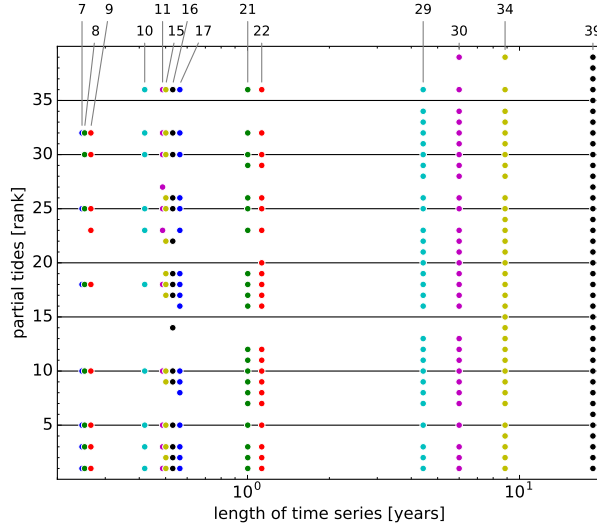
Figure 6. The partial tides (identified by their rank R from Tab. 4) that can be resolved as a function of the (minimum) length of the time series. If two partial tides cannot be resolved against each other, the one with the lower rank is dropped. Note the logarithmic time axis from 0.2 to 20 years. The numbers at the top are the counts of partial tides.

that ~~fulfill~~ fulfil these criteria are marked in the column "used for verif." in Tab. A1. ~~The~~ All tide gauge data is prepared ~~and filtered~~ as described in Sect. 4.1, including the removal of extreme events from the observations that are used for comparison.

5.2 Evaluation of residuals

In ~~Fig. 7, we show histograms~~ this section, we present results from the analysis of the residuals ~~(observation − prediction)~~ in the
5   following order: the distributions of residuals for the tide gauge Cuxhaven, the means and standard deviations for some major ports, the changes in the standard deviations for all tide gauges, and the changes in the remaining frequencies. The residuals are the differences between the observed and the predicted vertices (times and heights of high and low waters) with the same assigned transit number and event index k.

Figure 7 shows histograms of the residuals for the tide gauge Cuxhaven. The ~~left and the right panel~~ panels on the left and on
10   the right display histograms for the times and heights, respectively. Each panel contains one histogram for the tidal prediction with 43 partial tides (red) and one histogram for the tidal prediction with 39 partial tides (yellow). Using the new set of partial tides, the standard deviation of the residuals ~~has decreased~~ decreases from 9.6 min to 9.0 min for the times and from 0.28 m to 0.27 m for the heights.

In the same way as for Cuxhaven, residuals are calculated for the data of all 98 tide gauges included in the verification. The
15   mean values μ and standard deviations σ of the residuals for eleven major ports are ~~presented~~ summarized in Tab. 5 for the times and in Tab. 6 for the heights. Based on the results from all tide gauges, the average standard deviation of the residuals is
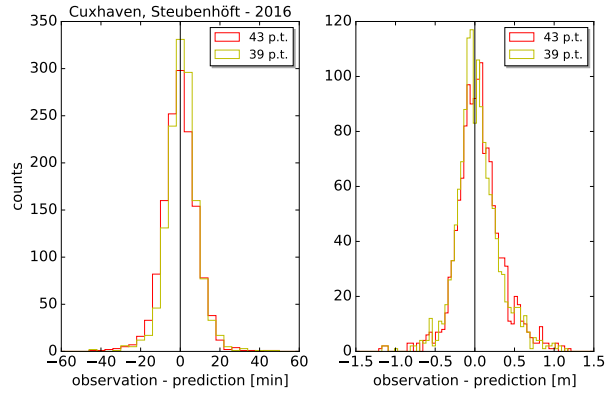
Figure 7. Histograms of the residuals for ~~the two~~ tide gauge Cuxhaven and year 2016. The different colours indicate predictions based on the different sets of partial tides ~~for tide gauge Cuxhaven and year 2016.~~ (red: predictions with 43 partial tides; yellow: predictions with 39 partial tides). Left: time differences with a bin width of 4 min. Right: height differences with a bin width of 0.04 m.

Table 5. Residuals of predicted and observed times of high and low water: mean μ and standard deviation σ in minutes.

gauge number	gauge name	43 partial tides		39 partial tides	
		μ	σ	μ	σ
101P	Borkum	-2.7	11.2	-2.1	11.0
103P	Bremerhaven	-6.4	10.4	-4.6	10.1
111P	Norderney	1.5	10.9	0.7	10.6
502P	Bremen	-7.3	10.8	-5.5	10.5
505P	Büsum	3.4	17.5	4.6	17.4
506P	Cuxhaven	-0.1	9.6	1.0	9.0
507P	Emden	-9.2	13.8	-8.2	13.3
508P	Hamburg	-10.1	10.4	-7.7	10.3
509A	Helgoland	-2.4	7.8	-2.6	7.7
510P	Husum	-5.1	12.0	-4.3	11.7
512P	Wilhelmshaven	-3.2	10.0	-2.7	9.6

13.2 min or 0.28 m, respectively, using the set of 39 tidal constituents. In most cases, the new set of ~~tidal constituents~~ partial tides gives small improvements in μ and σ.

The percentage changes $\Delta\sigma$ of the standard deviations ~~for all 98 tide gauges~~ are presented in the histograms in Fig. 8 ~~for the timesand in~~ (times) and Fig. 9 ~~for the heights~~ (heights) for all 98 tide gauges. The percentage change is calculated in the

Table 6. Residuals of predicted and observed heights of high and low water: mean μ and standard deviation σ in meters.

| gauge number	gauge name	43 partial tides		39 partial tides	
		μ	σ	μ	σ
101P	Borkum	0.05	0.24	0.03	0.24
103P	Bremerhaven	0.06	0.28	0.04	0.28
111P	Norderney	0.03	0.25	−0.01	0.24
502P	Bremen	0.01	0.27	−0.01	0.26
505P	Büsum	0.04	0.29	0.01	0.28
506P	Cuxhaven	0.05	0.28	0.04	0.27
507P	Emden	−0.01	0.28	−0.02	0.27
508P	Hamburg	−0.08	0.33	−0.12	0.31
509A	Helgoland	0.03	0.25	0.02	0.24
510P	Husum	0.02	0.29	0.00	0.28
512P	Wilhelmshaven	0.05	0.27	0.03	0.27

following way:

$$\Delta\sigma[\%] = 100\% \cdot \frac{\sigma_{39\text{ p.t.}} - \sigma_{43\text{ p.t.}}}{\sigma_{43\text{ p.t.}}} \quad , \tag{6}$$

where $\sigma_{39\text{ p.t.}}$ and $\sigma_{43\text{ p.t.}}$ are the standard deviations of the residuals using the predictions with 39 partial tides and 43 partial tides, respectively. The average reductions of the standard deviations ~~in the residuals~~ are 2.41% (times) and 2.30% (heights).

5 The two tide gauges with the largest improvements in this study, both with regard to times and heights, are Holmer Siel (BSH gauge number 649B) at the North Frisian coast and Bremervörde (687P) in the river Oste. Further tide gauges with improved standard deviations are located all around the area of investigation. Regarding the times (Fig. 8), the four tide gauges with increased standard deviations are Westerland (620P) and Hörnum (624P) located at the North Frisian island of Sylt, and Dove-Elbe (727P) and Bunthaus (729P) located upstream the river Elbe. Regarding the heights (Fig. 9), the five tide gauges

10 with the largest positive changes are also located upstream the river Elbe, namely Geesthacht (732D), Altengamme (732A), Zollenspieker (731P), Illmenau (730A) and Fahrenholz (730C). The water levels in this part of the Elbe are partly influenced by river discharge, which can lead to deviations from the tidal predictions.

The change of constituents has an influence ~~of~~ on the remaining periodicities in the ~~residua~~residuals. Periodograms are calculated for the two ~~residua~~ sets of residuals (times and heights) for each tide gauge. The 98 periodograms of each type are

15 averaged. The resulting mean periodograms are shown in Fig. 10 and 11. In both figures, the strongest peaks are located at very low angular velocities ($\lesssim 1°/$tn). As mentioned before, the unambiguous identification of partial tides at these periods is difficult and consequently no ~~considerable~~ major improvements are achieved in reducing the (average) residual periodicities in this range. Four further strong peaks are visible in both figures at about 15, 25, 52 and 64°/tn for the prediction with 43 partial tides. These peaks ~~could clearly be~~ are clearly reduced with the new predictions (39 partial tides).
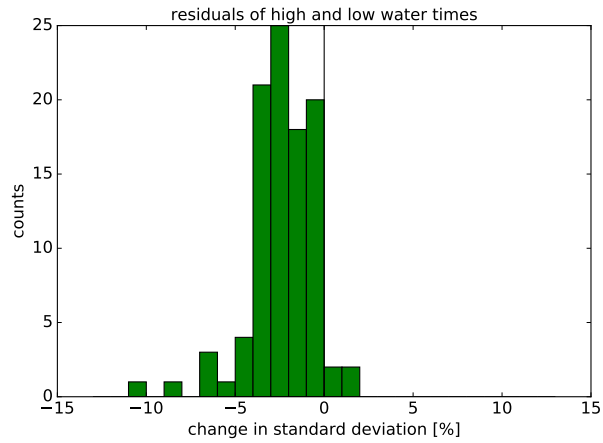
Figure 8. Histogram of the change in the standard deviation of the residuals of high and low water times for all 98 tide gauges.
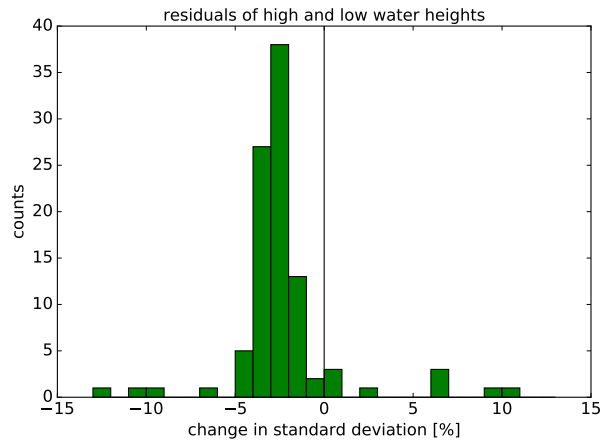

Figure 9. Histogram of the change in the standard deviation of the residuals of high and low water heights for all 98 tide gauges.

6 Comparison of predictions with observations: the HRoI and the harmonic method

The harmonic method is the most widely used technique for tidal predictions. The following comparison of predictions calculated with the HRoI and with the harmonic method shall demonstrate the respective capabilities. The comparison is done for the two tide gauges at Cuxhaven, Steubenhöft, and Hamburg, St. Pauli. The first site is located at the mouth of the river Elbe into the North Sea, while the second is about 100 km upstream in the river Elbe. The predictions are compared with tide gauge observations from the year 2016.

21
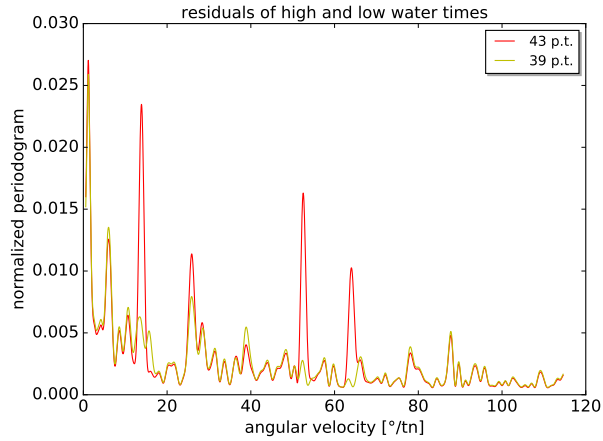
Figure 10. Mean periodogram of residual high ~~or~~ and low water times for all tide gauges used in the verification. The different colours indicate predictions based on the different sets of partial tides (red: predictions with 43 partial tides; yellow: predictions with 39 partial tides)
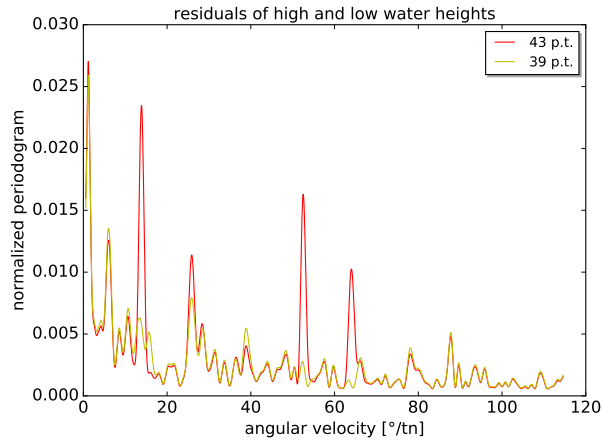
Figure 11. ~~Mean periodogram of residual~~ Same as Fig. 10, but for the high ~~or~~ and low water heights~~for all tide gauges used in verification~~.

6.1 Tidal analysis and prediction

The predictions with the HRoI (39 partial tides) are the same as in Sect. 5. The harmonic analysis is based on continuous observations from the years 1996-2014 at 10-minute intervals. The harmonic constituents (amplitudes H and phases g) and the constant vertical offset Z_0 are determined from a least-squares fit with the following model function:

$$\hat{y}_{\text{harm}} = Z_0 + \sum_{l=1}^{L}\left[H_l \cdot \cos(V_l(t_0) + \omega_l t - g_l)\right] \quad . \tag{7}$$

5

We use the 68 partial tides (with angular velocities ω_l) from Foreman (1977) . These are also the default constituents in the Matlab packages t_tide and UTide (Pawlowicz et al., 2002; Codiga, 2011) which have become widely accepted standard implementations of the harmonic method. Since the data records exceed 18.6 years, we add the partial tide with the angular velocity of the lunar node and omit nodal corrections. This gives a total of $L = 69$ constituents. The time t is referenced to the

5   midpoint t_0 of the time series and the astronomical argument $V_l(t_0)$ is calculated for each partial tide using the expressions for the fundamental astronomical arguments as published by the International Earth Rotation and Reference Systems Service (2010, Sect. 5.7). The analysis is applied in two iterations with a 3-sigma-clipping in between to remove outliers in the observations.

We show in Figs. 12 and 13 the predictions and observations for Cuxhaven and Hamburg, respectively. Only ten days in June 2016 are shown from the complete time series for better visibility of the individual high and low waters. The two curves

10   in each figure are the observed water levels (dark blue) and the harmonic prediction (light green). The high and low waters are marked separately for observations (red circles), vertices determined from the harmonic prediction (green squares) and predictions made with the HRoI (yellow triangles).
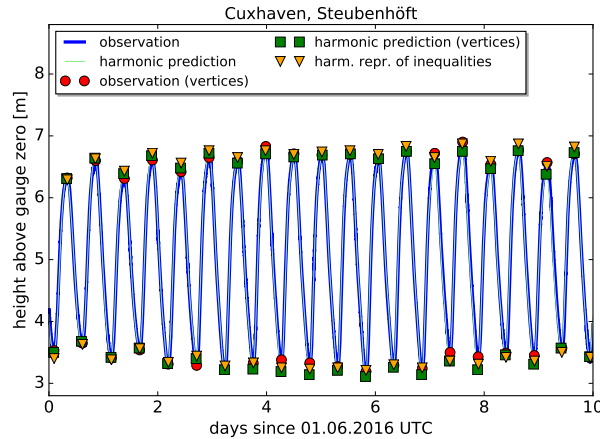
Figure 12. Observations and two predictions for the tide gauge Cuxhaven, Steubenhöft. The first 10 days of June 2016 are shown.

6.2   Evaluation of residuals

As in Sect. 5, the residuals are the differences between the observed and the predicted vertices (times and heights of high and

15   low waters) with the same assigned transit number and event index k. We calculate the means and the standard deviations of the residuals regarding times and heights. The results are shown in Tab. 7. The differences for the heights are within a few centimetres. For the times, the standard deviations are approximately $4 - 5$ minutes larger in the case of the harmonic method. The residuals for the times are also shown in Fig. 14, where the curves for the harmonic method (blue) suggest that long-period periodicities could be present in the residuals which are not covered by the predictions. Based on the calculated parameters,

20   the deviations of the harmonic prediction from the observations (and also from the prediction made with the HRoI) are larger
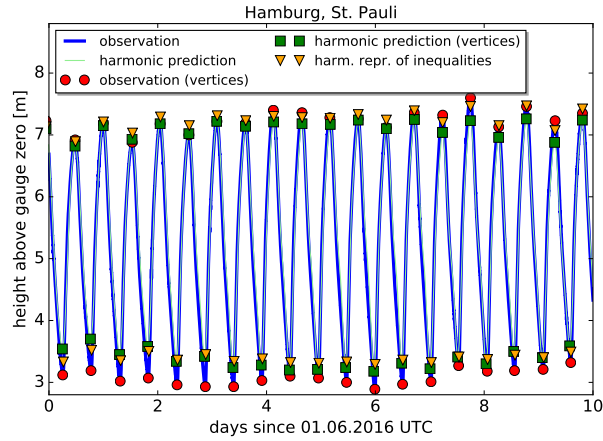
Figure 13. Observations and two predictions for the tide gauge Hamburg, St. Pauli. The first 10 days of June 2016 are shown.

Table 7. Residuals of predicted and observed times and heights of high and low water: mean μ and standard deviation σ.

	times [min]			
gauge name	HRoI (39 p.t.)		harmonic pred.	
	μ	σ	μ	σ
Cuxhaven	1.0	9.0	12.0	12.9
Hamburg	−7.7	10.3	11.8	15.1

	heights [m]			
gauge name	HRoI (39 p.t.)		harmonic pred.	
	μ	σ	μ	σ
Cuxhaven	0.03	0.27	0.05	0.29
Hamburg	−0.11	0.31	−0.10	0.37

for Hamburg as compared to Cuxhaven. This supports the assumptions that the application of the HRoI is especially useful for tide gauge locations in rivers.

7   Conclusions

Time series of high and low water records from 111 German tide gauges were analysed to determine important long-period
5   partial tides. ~~An application is the usage of these constituents in tidal analyses and predictions with the HRoI.~~ Generalized
Lomb-Scargle periodograms were calculated from lunitidal intervals and heights for all tide gauges, and spectral peaks were
identified in these periodograms above noise thresholds. The separate analyses of lunitidal intervals and heights were combined

24
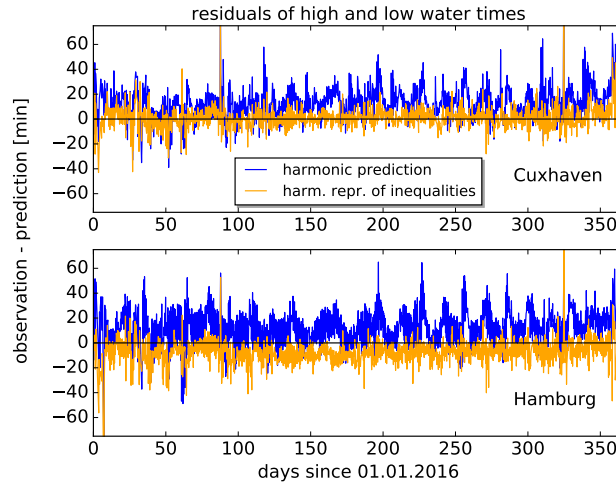
Figure 14. The residuals of high and low water times for the tide gauges Cuxhaven (upper panel) and Hamburg (lower panel).

to realize one comprehensive list of partial tides~~for applications in the German Bight~~. An application is the usage of these constituents in tidal analyses and predictions with the HRoI.

The new set of 39 partial tides largely confirms the previously used set with 43 partial tides. It can be seen from Tab. 2, that nine new constituents were added and 13 constituents were removed. Many of the removed angular velocities are close to strong partial tides, such as the anomalistic month (Mm), the half synodic month (MSf) and the half tropical month (Mf). The removed constituents might have been artefacts from spectral leakage, which are most prominent in the proximity of strong spectral lines ~~, that~~ and which were misidentified as true signals in previous studies. The unambiguous identification of partial tides is very difficult at angular velocities below approximately 1°/tn because the noise levels in the periodograms increase towards lower angular velocities and the results from different tide gauges are less consistent.

The verification based on observations from 98 tide gauges in the year 2016 suggests that the usage of the new constituents list ~~leads~~ can lead to slightly better predictions. In particular, the average standard deviations of the residuals are lower and ~~residual periodicities could be reduced around four frequencies~~ four frequencies were reduced.

This study presents for the first time a thorough investigations of the long-period constituents used with the HRoI. The new list of constituents ~~is~~ will be used in tidal analyses and predictions with the HRoI for German tide gauges starting with the BSH tide tables for the year 2020.

In future work, extensive comparison of the HRoI with the common harmonic method might provide more insights into the capabilities of both tidal analysis techniques. The German Bight would be an ideal area of investigation with its large network of tide gauges located both at the open North Sea and far within tidally influenced rivers.

Appendix A: Table of tide gauges

25

Table A1. 137 German tide gauges which deliver water level observations on a regular basis and for which tidal predictions were published in BSH tide tables (*Gezeitentafeln*) or tide calendar (*Gezeitenkalender*) for the year 2018. The data from the tide gauges are observed times and heights of high and low water. The tide gauges are operated by different federal and state agencies which provide tide gauge records to BSH. Abbreviations in the third column correspond to the following agencies: E: Emden Waterways and Shipping Authority (Wasserstraßen- und Schifffahrtsamt Emden, WSA Emden), BH: WSA Bremerhaven, B: WSA Bremen, C: WSA Cuxhaven, T: WSA Tönning, HPA: Hamburg Port Authority, W: WSA Wilhelmshaven, HU: Landesbetrieb für Küstenschutz, Nationalpark und Meeresschutz Schleswig-Holstein (LKN-SH Husum), H: WSA Hamburg, L: WSA Lauenburg, N: Niedersächsischer Landesbetrieb für Wasserwirtschaft, Küsten- und Naturschutz (NLWKN), M: WSA Meppen.

BSH gauge number	gauge name	auth.	data period [start/end date]	data period [years]	completeness of data [%]	used for analysis	used for verif.
DE101P	Borkum, Fischerbalje	E	02.01.1963-31.12.2015	53.0	62	x	x
DE103P	Bremerhaven, Alter Leuchtturm	BH	01.11.1965-31.12.2015	50.2	62	x	x
DE111P	Norderney, Riffgat	E	01.01.1964-31.12.2015	52.0	67	x	x
DE502P	Bremen, Oslebshausen	B	01.01.1950-31.12.2015	66.0	99	x	x
DE504A	Brunsbüttel, Mole 1	C	01.08.2010-31.12.2015	5.4	95		
DE505P	Büsum	T	01.01.1963-31.12.2015	53.0	63	x	x
DE506P	Cuxhaven, Steubenhöft	C	01.01.1901-31.12.2015	115.0	99	x	x
DE507P	Emden, Große Seeschleuse	E	01.01.1989-31.12.2015	27.0	99	x	x
DE508P	Hamburg, St. Pauli	HPA	01.01.1901-31.12.2015	115.0	100	x	x
DE509A	Helgoland, Binnenhafen	T	01.01.1989-31.12.2015	27.0	99	x	x
DE510P	Husum	T	01.01.1989-31.12.2015	27.0	98	x	x
DE512P	Wilhelmshaven, Alter Vorhafen	W	01.01.1973-31.12.2015	43.0	98	x	x
DE613C	Hojer, Schleuse	HU	01.01.1999-31.12.2015	17.0	100		
DE617P	List, Hafen	T	01.01.1986-31.12.2015	30.0	98	x	x
DE618P	Munkmarsch	HU	16.01.1989-31.12.2015	27.0	49		
DE620P	Westerland	HU	01.01.1986-31.12.2015	30.0	94	x	x
DE622P	Amrum Odde	HU	17.04.1996-07.12.2015	19.6	40		
DE623A	Rantumdamm	HU	08.01.1996-31.12.2015	20.0	89	x	
DE624P	Hörnum, Hafen	T	01.01.1989-31.12.2015	27.0	99	x	x
DE628A	Osterley	HU	09.04.1997-11.11.2015	18.6	40		
DE629B	Föhrer Ley Nord	HU	27.04.1994-11.11.2015	21.5	46		
DE631P	Amrum, Hafen (Wittdün)	T	01.01.1989-31.12.2015	27.0	96	x	
DE632P	Föhr, Wyk	HU	01.01.1989-31.12.2015	27.0	100	x	x

BSH gauge number	gauge name	auth.	data period [start/end date]	data period [years]	completeness of data [%]	used for analysis	used for verif.
DE635P	Dagebüll	T	01.01.1989-31.12.2015	27.0	99	x	x
DE636F	Hooge, Anleger	HU	01.01.1989-31.12.2015	27.0	93	x	x
DE637A	Strand, Hamburger Hallig	HU	02.05.1989-31.12.2015	26.7	79	x	
DE637P	Gröde, Anleger	HU	01.01.1989-31.12.2015	27.0	48		
DE638P	Schlüttsiel	HU	01.01.1989-31.12.2015	27.0	96	x	x
DE642C	Rummelloch, West	HU	14.06.1994-08.12.2015	21.5	47		
DE645P	Süderoogsand	HU	23.03.1993-20.11.2015	22.7	62	x	
DE647A	Pellworm, Anleger	T	01.03.1996-31.12.2015	19.8	88	x	
DE649B	Holmer Siel	HU	01.01.1994-31.12.2015	22.0	89	x	x
DE649P	Nordstrand, Strucklahnungshörn	HU	01.01.1989-31.12.2015	27.0	96	x	x
DE653P	Südfall, Fahrwasserkante	HU	25.03.1993-02.12.2015	22.7	67	x	
DE655D	Tümlauer Hafen	HU	24.09.2001-31.12.2013	12.3	93		
DE658B	Linnenplate	HU	11.04.2001-05.12.2013	12.7	62		
DE664P	Eidersperrwerk, AP	T	01.01.1989-31.12.2014	26.0	97	x	
DE666P	Blauort	HU	12.01.1989-31.12.2015	27.0	82	x	x
DE667B	Meldorf - Sperrwerk, AP	HU	04.01.1994-31.12.2015	22.0	71	x	
DE669P	Deichsiel	HU	01.01.1989-31.12.2013	25.0	96	x	
DE673P	Trischen, West	HU	18.03.1989-25.11.2015	26.7	61	x	
DE675C	Mittelplate	HU	01.01.1992-25.11.2015	23.9	55		
DE675P	Friedrichskoog, Hafen	HU	01.01.1989-31.12.2015	27.0	100	x	x
DE676P	Zehnerloch	C	01.01.1989-14.11.2015	26.9	98	x	x
DE677C	Scharhörnriff, Bake A	C	01.01.2001-31.12.2015	15.0	96		
DE677P	Scharhörn, Bake C	C	01.01.1989-31.12.2015	27.0	98	x	x
DE678W	Neuwerk	HPA	01.01.1994-31.12.2015	22.0	46		
DE681A	Neufeld, Hafen	HU	01.01.1994-31.12.2015	22.0	78	x	x
DE681P	Otterndorf	C	01.01.1989-31.12.2015	27.0	79	x	
DE682P	Osteriff	C	01.01.1989-31.12.2015	27.0	89	x	
DE683P	Belum, Oste	C	01.01.1989-31.12.2015	27.0	96	x	x
DE685P	Hechthausen, Oste	C	01.01.1989-31.12.2015	27.0	95	x	x
DE687P	Bremervörde, Oste	C	01.01.1989-31.12.2015	27.0	94	x	x
DE688P	Brokdorf	H	01.01.1989-31.12.2015	27.0	98	x	x
DE690P	Stör - Sperrwerk, AP	HU	01.01.2000-31.12.2015	16.0	87		
DE691R	Kasenort, Stör	HU	01.01.1989-31.12.2015	27.0	99	x	x

BSH gauge number	gauge name	auth.	data period [start/end date]	data period [years]	completeness of data [%]	used for analysis	used for verif.
DE692P	Itzehoe, Stör	H	01.01.1989-31.12.2015	27.0	97	x	
DE693P	Breitenberg, Stör	H	01.01.2000-31.12.2015	16.0	95		
DE695P	Glückstadt	H	01.01.1989-31.12.2015	27.0	95	x	x
DE697P	Krautsand	H	01.01.1989-31.12.2015	27.0	89	x	x
DE698P	Kollmar (Kamperreihe)	H	01.01.1989-31.12.2015	27.0	98	x	x
DE700R	Krückau - Sperrwerk, BP	H	01.01.2000-31.12.2015	16.0	89		
DE703P	Grauerort	H	01.01.1989-31.12.2015	27.0	98	x	x
DE704R	Pinnau - Sperrwerk, BP	H	01.01.2000-31.12.2015	16.0	93		
DE706P	Uetersen, Pinnau	H	01.01.1989-31.12.2015	27.0	92	x	x
DE709P	Stadersand, Schwinge	H	01.01.1989-31.12.2015	27.0	98	x	x
DE711P	Hetlingen	H	01.01.1989-31.12.2015	27.0	94	x	x
DE712P	Lühort, Lühe	H	01.01.1989-31.12.2015	27.0	97	x	x
DE714P	Schulau	H	01.01.1989-31.12.2015	27.0	97	x	x
DE715P	Blankenese, Unterfeuer	HPA	01.01.1989-31.12.2015	27.0	98	x	x
DE717P	Cranz, Este - Sperrwerk, AP	H	01.01.1989-31.12.2015	27.0	83	x	x
DE718P	Buxtehude, Este	H	01.01.1989-31.12.2015	27.0	86	x	x
DE720P	Seemannshöft	HPA	01.01.1989-31.12.2015	27.0	100	x	x
DE724P	Harburg, Schleuse	HPA	01.01.1989-31.12.2015	27.0	100	x	x
DE727P	Dove - Elbe, Einfahrt	HPA	01.01.1989-31.12.2015	27.0	99	x	x
DE729P	Bunthaus	HPA	01.01.1989-31.12.2015	27.0	100	x	x
DE730A	Ilmenau - Sperrwerk, AP	L	01.01.1989-31.12.2015	27.0	98	x	x
DE730C	Fahrenholz, Ilmenau	L	01.01.1989-31.12.2015	27.0	93	x	x
DE730P	Over	L	01.01.1989-31.12.2015	27.0	98	x	x
DE731P	Zollenspieker	L	01.01.1989-31.12.2015	27.0	98	x	x
DE732A	Altengamme	L	01.01.1989-31.12.2015	27.0	96	x	x
DE732D	Geesthacht, Wehr UP	L	01.01.1989-31.12.2015	27.0	98	x	x
DE734P	Alte Weser, Leuchtturm	BH	01.01.1989-31.12.2015	27.0	99	x	x
DE735A	Spieka Neufeld	N	01.01.1989-31.12.2015	27.0	50		
DE735B	Wremertief	N	01.01.1994-31.12.2015	22.0	38		
DE737P	Dwarsgat, Unterfeuer	BH	01.01.1989-31.12.2015	27.0	99	x	x
DE737S	Robbensüdsteert	BH	01.01.1989-31.12.2015	27.0	96	x	x
DE738P	Fedderwardersiel	N	01.01.1989-31.12.2015	27.0	49		
DE741A	Nordenham, Unterfeuer	BH	01.01.1989-31.12.2015	27.0	99	x	x

BSH gauge number	gauge name	auth.	data period [start/end date]	data period [years]	completeness of data [%]	used for analysis	used for verif.
DE741B	Rechtenfleth	BH	01.01.1993-31.12.2015	23.0	99	x	x
DE743P	Brake	B	01.01.1989-31.12.2015	27.0	97	x	x
DE744A	Elsfleth Ohrt	B	01.01.1989-31.12.2015	27.0	96	x	x
DE744P	Elsfleth	B	01.01.1975-31.12.2015	41.0	80	x	x
DE745P	Huntebrück, Hunte	B	01.01.1989-31.12.2015	27.0	98	x	x
DE746P	Hollersiel, Hunte	B	01.01.1989-31.12.2015	27.0	98	x	x
DE747P	Reithörne, Hunte	B	01.01.1989-31.12.2015	27.0	98	x	x
DE748P	Oldenburg - Drielake, Hunte	B	01.01.1989-31.12.2015	27.0	97	x	x
DE749P	Farge	B	01.01.1989-31.12.2015	27.0	99	x	x
DE750A	Wasserhorst, Lesum	B	01.01.1989-31.12.2015	27.0	90	x	x
DE750B	Ritterhude, Hamme	B	01.01.1989-31.12.2015	27.0	91	x	x
DE750C	Niederblockland, Wümme	B	01.01.1989-31.12.2015	27.0	91	x	x
DE750D	Borgfeld, Wümme	B	01.01.1989-31.12.2015	27.0	90	x	x
DE750P	Vegesack	B	01.01.1975-31.12.2015	41.0	80	x	x
DE751P	Bremen, Wilhelm-Kaisen-Brück	B	01.01.1989-31.12.2015	27.0	99	x	x
DE752P	Bremen, Weserwehr	B	01.01.1989-31.12.2015	27.0	98	x	x
DE754P	Wangerooge, Langes Riff, (Nord)	W	01.01.1976-31.12.2015	40.0	71	x	x
DE756P	Wangerooge, Ost	W	01.05.1976-31.12.2015	39.7	58		x
DE760P	Mellumplate, Leuchtturm	W	01.01.1989-31.12.2015	27.0	99	x	x
DE761P	Schillig	W	01.01.1989-31.12.2015	27.0	92	x	x
DE764B	Hooksielplate	W	01.01.1989-31.12.2015	27.0	93	x	x
DE766P	Voslapp	W	01.01.1989-31.12.2015	27.0	95	x	x
DE769P	Wilhelmshaven, Ölpier	W	01.01.1989-31.12.2015	27.0	97	x	x
DE770P	Wilhelmshaven, Neuer Vorhafen	W	01.01.1989-31.12.2015	27.0	97	x	x
DE773P	Arngast, Leuchtturm	W	15.05.2001-31.12.2015	14.6	88		
DE776P	Vareler Schleuse	N	01.01.1989-31.12.2015	27.0	49		
DE777P	Wangerooge, West	W	01.01.1976-31.12.2015	40.0	73	x	x
DE778P	Harlesiel	N	01.01.1989-31.12.2015	27.0	57		
DE779P	Spiekeroog	E	01.01.1989-31.12.2015	27.0	98	x	x
DE781P	Langeoog	E	01.01.1989-31.12.2015	27.0	98	x	x
DE782P	Bensersiel	N	01.01.1989-31.12.2015	27.0	100	x	x
DE796C	Leybucht, Leyhörn	N	01.01.1992-31.12.2015	24.0	100	x	x
DE798P	Borkum, Südstrand	E	02.01.1989-31.12.2015	27.0	95	x	x

BSH gauge number	gauge name	auth.	data period [start/end date]	data period [years]	completeness of data [%]	used for analysis	used for verif.
~~DE~~799G	Dukegat	E	01.01.1989-31.12.2015	27.0	95	x	x
~~DE~~799P	Emshörn	E	01.01.1989-31.12.2015	27.0	99	x	x
~~DE~~802P	Knock	E	01.01.1989-31.12.2015	27.0	99	x	x
~~DE~~803P	Pogum, Ems	E	01.01.1989-31.12.2015	27.0	98	x	x
~~DE~~805P	Terborg, Meßstelle, Ems	E	01.01.1989-31.12.2015	27.0	98	x	x
~~DE~~806P	Leerort, Ems	E	01.01.1989-31.12.2015	27.0	97	x	x
~~DE~~808A	Leda - Sperrwerk, Unterpegel	E	01.01.1989-31.12.2015	27.0	98	x	x
~~DE~~810A	Nortmoor, Altarm Jümme	N	01.01.2000-31.12.2015	16.0	85		
~~DE~~810B	Detern, Jümme	N	01.01.2000-31.12.2015	16.0	86		
~~DE~~810P	Westringaburg, Leda	N	01.01.1989-31.12.2015	27.0	84	x	x
~~DE~~812P	Dreyschloot, Leda	E	01.01.1989-31.12.2015	27.0	89	x	x
~~DE~~813P	Weener, Ems	E	01.01.1989-31.12.2015	27.0	98	x	x
~~DE~~814B	Rhede, Ems	M	01.01.1989-31.12.2015	27.0	94	x	x
~~DE~~814P	Papenburg, Ems	E	01.01.1989-31.12.2015	27.0	98	x	x
~~DE~~816P	Herbrum, Hafendamm, Ems	M	01.01.1989-01.11.2015	26.8	82	x	
					Number of tide gauges	111	98

References

Amin, M.: A method for approximating the nodal modulations of real tides, International Hydrographic Review, 64, 103–113, 1987.

Bundesamt für Seeschifffahrt und Hydrographie: Gezeitentafeln 2018. Europäische Gewässer, BSH, 2017a.

Bundesamt für Seeschifffahrt und Hydrographie: Gezeitenkalender 2018. Hoch- und Niedrigwasserzeiten für die Deutsche Bucht und deren

5   Flussgebiete, BSH, 2017b.

Chapront-Touzé, M. and Chapront, J.: Lunar Tables and Programms from 4000 B.C. to A.D. 8000, Willmann-Bell, 1991.

Codiga, D. L.: Unified Tidal Analysis and Prediction Using the UTide Matlab Functions, Technical report 2011-01, Graduate School of
    Oceanography, University of Rhode Island, 2011.

Doodson, A. T.: The Harmonic Development of the Tide-Generating Potential, Proceedings of the Royal Society of London. Series A, 100,

10   305–329, https://doi.org/10.1098/rspa.1921.0088, 1921.

Doodson, A. T.: The analysis of high and low waters, International Hydrographic Review, 28, 13–77, 1951.

Foreman, M. G. G.: Manual for Tidal Heights Analysis and Prediction, Pacific Marine Science Report, 77-10, 1977.

Foreman, M. G. G. and Henry, R. F.: Tidal Analysis Based on High and Low Water Observations, Pacific Marine Science Report, 79-15,
    1979.

15 Godin, G.: The use of nodal corrections in the calculation of harmonic constants, International Hydrographic Review, 63, 143–162, 1986.

Goffinet, P.: Qualitätssteigerung der Seevermessung und Navigation durch neuartige Beschickungsverfahren, Phd thesis, Universität Han-
    nover, 2000.

Horn, W.: Über die Darstellung der Gezeiten als Funktion der Zeit, Deutsche Hydrographische Zeitschrift, 1, 124–140, 1948.

Horn, W.: Some recent approaches to tidal problems, International Hydrographic Review, 37, 65–84, 1960.

20 International Earth Rotation and Reference Systems Service: IERS Conventions (2010), Verlag des Bundesamts für Kartographie und
    Geodäsie, Frankfurt am Main, 2010.

Lubbock, J. W.: On the tides in the port of London, Philosophical Transactions of the Royal Society of London, 121, 349–415,
    https://doi.org/10.1098/rstl.1831.0022, 1831.

Meeus, J.: Astronomical Algorithms, Willmann-Bell, 1998.

25 Müller-Navarra, S.: Zur automatischen Scheitelpunktbestimmung gemessener Tidekurven in der Deutschen Bucht, Hydrologie und Wasser-
    bewirtschaftung, 53, 380–388, 2009.

Müller-Navarra, S.: Gezeitenvorausberechnungen mit der Harmonischen Darstellung der Ungleichheiten (On Tidal Predictions by Means of
    Harmonic Representation of Inequalities), Berichte des Bundesamtes für Seeschifffahrt und Hydrographie, 50, 2013.

Parker, B. B.: Tidal Analysis and Prediction, U.S. Department of Commerce, NOAA Special Publication NOS CO-OPS 3, 2007.

30 Pawlowicz, R., Beardsley, B., and Lentz, S.: Classical tidal harmonic analysis including error estimates in MATLAB using T_TIDE, Com-
    puters & Geosciences, 28, 929–937, https://doi.org/10.1016/S0098-3004(02)00013-4, 2002.

Prabhu, K. M. M.: Window Functions and Their Applications in Signal Processing, CRC Press, 2014.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P.: Numerical Recipes in FORTRAN, Cambridge University Press, 1992.

Simon, B.: Coastal Tides, Institut océanographique, 2013.

35 Zechmeister, M. and Kürster, M.: The generalised Lomb-Scargle periodogram. A new formalism for the floating mean and Keplerian peri-
    odograms, A&A, 496, 577–584, https://doi.org/10.1051/0004-6361:200811296, 2009.