



Variability of distributions of wave set-up heights along a shoreline with complicated geometry

Tarmo Soomere^{1,2}, Katri Pindsoo¹

¹Laboratory of Wave Engineering, Department of Cybernetics, School of Science, Tallinn University of Technology, Akadeemia tee 21, Tallinn, 12618, Estonia

²Estonian Academy of Sciences, Kohtu 6, Tallinn, 10130, Estonia

Correspondence to: Katri Pindsoo (katri.pindsoo@ioc.ee)

Abstract. The phenomenon of wave set-up may substantially contribute to the formation of devastating coastal flooding in certain coastal areas. We study the appearance and properties of empirical probability distributions of the occurrence of different set-up heights in about 80 km long section of coastline near Tallinn in the Gulf of Finland, the eastern Baltic Sea. The study area is often attacked by high waves propagating from various directions and the approach angle of waves varies largely along the shore. The distribution in question is approximated by an exponential distribution with a quadratic polynomial as the exponent. Even though different segments of the study area host substantially different wave regimes, the leading term of this polynomial is usually small (between -0.005 and 0.005) and varies insignificantly along the study area. Consequently, the distribution of wave set-up heights substantially deviates from a Rayleigh or Weibull distribution (that usually reflect the distribution of different wave heights). In about $\frac{3}{4}$ of occasions it is fairly well approximated by a standard exponential distribution. In about 25% of coastal segments it matches a Wald (inverse Gaussian) distribution. This property signals that very high extreme set-up events may in some locations occur substantially more frequently than it could be expected from the probability of occurrence of severe seas.

1 Introduction

The global sea level rise in most of existing projections of climate change (Cazenave et al., 2014) is often associated with major consequences to the coastal zone (Hallegatte et al., 2013). The resulting economic damages to low-lying coastal areas (Darwin and Tol, 2001) may lead to a loss of worldwide welfare by almost 2% by the end of this century (Pycroft et al., 2016). The global sea level rise, however, contributes only a small fraction into the most devastating coastal floodings. These events, additionally to being economically extremely damaging (Meyer et al., 2013), may also lead to massive losses of lives and desertification of entire coastal communities (Dube et al., 2009).

A devastating flooding is usually caused by the interplay of several drivers, with fundamentally different predictability, physical, dynamical and statistical properties. While tides are almost perfectly regular and caused by extra-terrestrial drivers, a reasonable forecast of the impact of low atmospheric pressure (inverted barometric effect), wind-driven surge and wave-induced effects requires a cluster of dedicated atmospheric, ocean circulation and wave models. The resulting high water



levels may be additionally amplified by specific mechanisms and events such as tide–surge interactions (Batstone et al., 2013; Olbert et al., 2013), meteorologically driven long waves (Pattiarachi and Wijeratne, 2014; Pellikka et al., 2014; Vilibic et al., 2014) or seiches (Vilibic, 2006; Kulikov and Medvedev, 2013). On top of that, wave-driven effects at the waterline such as wave set-up and run-up (Stockdon et al., 2006) may greatly contribute to the damaging potential of extreme water
5 levels. These phenomena are driven by momentum carried by waves. They have different time scales and appearance. When a wave crest reaches the shore, the resulting temporary inland movement of the water, with the time scale comparable with the wave period, is termed run-up (see, e.g., Didenkulova, 2009 for an overview and references). In contrast, wave set-up is the increase in the mean water level due to the release of momentum of breaking waves.

Along with contemporary numerical simulations and direct search for worst-case scenarios (e.g., Averkiev and Klevanny, 2010), the use of the probabilistic approach is another classic way to quantify the properties of extreme water levels and
10 related risks. The relevant pool of literature contains substantial amount of work on statistical parameters of water level variations (e.g., Serafin and Ruggiero, 2014; Fawcett and Walshaw, 2016) and extreme water levels and their return periods (e.g., Purvis et al., 2008; Haigh et al., 2010; Arns et al., 2013). Similar probabilistic analysis has been extensively applied to the average and extreme wave properties (e.g., Orimolade et al., 2016; Rueda et al., 2016), wave-driven effects at the
15 waterline (Holland and Holman, 1993; Stockdon et al., 2006), and properties of meteotsunamis (Geist et al., 2014, Bechle et al., 2015). In most occasions the drivers of coastal flooding are neither completely independent nor completely dependent on each other. This feature generates the necessity to consider multivariate distributions of their properties. Most often, the simultaneous occurrence of storm surges and large waves is addressed (e.g., Hawkes et al., 2002; Wadey et al., 2015; Rueda et al., 2016b). A few studies also include an analysis of joint distributions of wave heights, periods and directions (Masina et
20 al., 2015).

Importantly, typical probability distributions of different constituents of extreme water levels may be fundamentally different. The distribution of observed and numerically simulated water levels is usually close to a Gaussian one (Bortot et al., 2000; Johansson et al., 2001; Mel and Lionello, 2014; Soomere et al., 2015). The total water level in semi-sheltered seas with extensive subtidal or weekly-scale variability may contain two components. In the Baltic Sea conditions, one of these
25 (that reflects the water volume of the entire sea) has a classic quasi-Gaussian distribution whereas the other (that reflects the local storm surge) component has an exponential distribution and apparently mirrors a Poisson process (Soomere et al., 2015) similarly to the non-tidal residual in the North Sea (Schmitt et al., 2018). The probabilities of occurrence of different single wave heights are at best approximated either by a Rayleigh (Longuet-Higgins, 1952), Weibull (Forristall, 1978) or a Tayfun distribution (Socquet-Juglard et al., 2005). The probability distribution of run-up heights usually follows the relevant
30 distribution for incident wave heights (Denissenko et al., 2011) but can be approximated by a Rayleigh distribution even if the approaching wave field does not represent a Gaussian process (Denissenko et al., 2013). The empirical probabilities of average or significant wave heights in various offshore conditions usually resemble either a Rayleigh or a Weibull distribution (Muraleedharan et al., 2007; Feng et al., 2014) while Pareto-type distributions are more suitable for the analysis of meteotsunami heights (Bechle et al., 2015).



In this paper we focus on the appearance and properties of empirical distributions of wave-driven local water level set-up. This process, called set-up in what follows, is a classic phenomenon at open ocean coasts. It may often provide as much as 1/3 of the total water level rise during a storm (Dean and Bender, 2006) and significantly contribute to extreme sea level events (Hoeke et al., 2013; Melet et al., 2016, 2018). The impact of this phenomenon *inter alia* contributes to the overall level of danger in the coastal zone because, for example, the baseline level of wave run-up includes the local elevation of water level owing to set-up. The physics of set-up is known for half a century (Longuet-Higgins and Stewart, 1964). Adequate parameterizations of this phenomenon have been introduced more than a decade ago (Stockdon et al., 2006) and many models take it into account to a certain extent (SWAN, 2007; Roland et al., 2009; Alari and Kõuts, 2012; Moghimi et al., 2013).

10 The contribution from this phenomenon still provides one of the largest challenges in modelling of storm surges and flooding (Dukhovskoy and Morey, 2011; Melet et al., 2013). This feature reflects the intrinsically complicated nature of the formation of this phenomenon. First of all, the set-up height strongly depends on the approach angle of waves at the actual breaker line. This angle is well-defined only if the coastline is almost straight, the nearshore is homogeneous in the alongshore direction and the wave field is monochromatic (Larson et al., 2010; Viška and Soomere, 2013; Lopez-Ruiz et al., 2014; 2015).

15 Generally, this angle is a complicated function of shoreline geometry, nearshore bathymetry, wave properties and (possibly increased) water level. Even if the basic wave properties (height, period and propagation direction) are perfectly forecast or hindcast in a nearshore location, the evaluation of the further propagation of waves is a major challenge because, e.g., refraction properties and the location of the breaking line change along with the local water level. Moreover, accurate wave hindcast and forecast are still a challenge in many water bodies because of insufficient quality of the wind information

20 (Nikolkina et al., 2014).

Several studies have focused on the maxima of set-up heights over certain coastal areas (O’Grady et al., 2015; Soomere et al., 2013) or the maximum contribution from set-up to the local water level extremes (Pindsoo and Soomere, 2016). The problem of evaluation of maximum set-up heights has a relatively simple solution on comparatively straight open ocean coasts. The nearshore of such coasts is usually fairly homogeneous in the alongshore direction and the highest waves tend to approach the shore under relatively small angles. These features make it possible to use simplified schemes for the evaluation of the impact of refraction and shoaling in the nearshore (e.g., Larson et al., 2010). In many occasions it is acceptable to assume that waves propagate directly onshore (O’Grady et al., 2015) or to reduce the problem to an evaluation of the properties of the highest waves that approach the shore from a relatively narrow range of directions (Soomere et al., 2013). In areas with complicated geometry and especially in coastal segments where high waves may often approach under

30 large angles to the shore normal it is necessary to take into account full refraction and shoaling in the nearshore (Viška and Soomere, 2013; Pindsoo and Soomere, 2015).

Even though high storm surges are often associated with severe seas, the formation of high set-up depends on many details. It does not necessarily exhibit its maximum level in the coastal sections that are affected by the highest waves. The maximum storm surge and maximum set-up usually do not occur simultaneously (Pindsoo and Soomere, 2015). In contrary,



in coastal areas with complicated geometry each short segment may have its own ‘perfect storm’ that creates the all-highest sum of storm surge and set-up (Soomere et al., 2013).

5 These observations call for further analysis of the properties of the set-up phenomenon. In this paper we address the basic features of statistical distributions of set-up heights along an about 80 km long coastal section in the vicinity of Tallinn Bay in the Gulf of Finland, the Baltic Sea. The shoreline of this test area has a complicated geometry and contains segments that are open to fairly different directions. The goal is to identify the typical shapes of the distributions of the probability of occurrence of simulated wave set-up heights and to analyse the alongshore variability of these distributions.

The paper is organised as follows. Section 2 introduces the method of evaluation of the maximum set-up height for obliquely approaching waves. It also provides a short overview of the simplified wave model for rapid evaluation of wave time series, the forcing data for this model and the procedure of evaluation of properties of breaking waves. Section 3 presents an analysis of spatial variations of extreme wave set-up heights along the study area. The core result is an estimate of the typical shape of empirical probability distributions of different set-up heights along the coast. These distributions substantially deviate from similar distributions of different wave conditions and exhibit unexpectedly large proportion of high set-up events in comparison with the classic Gaussian, Rayleigh or Weibull statistics. Several implications of the results are discussed in Section 4.

2 Methods and data

2.1 Set-up height for obliquely incident waves

The classic concept of wave set-up (Longuet-Higgins and Stewart, 1962) relates the local increase in the water level with the release of the onshore component of radiation stress in the process of wave breaking. Based on this concept, it has been demonstrated that, in ideal conditions, the maximum set-up height $\bar{\eta}_{\max}$ created by waves propagating directly onshore along a planar impermeable beach is (McDougal and Hudspeth, 1983; Hsu et al., 2006)

$$\bar{\eta}_{\max} = h_b \frac{40\gamma_b^2 - 3\gamma_b^4}{128} \approx \frac{5}{16} \gamma_b H_b, \quad (1)$$

where $\gamma_b = H_b/h_b$ is the breaking index that is assumed to be constant all over the surf zone, h_b is the still water depth at the breaker line and H_b is the wave height at the breaker line (Fig. 1). Expression (1) is used in many engineering applications (Dean and Dalrymple, 1991) and studies into properties of set-up of waves that approach the shore under a relative small angle (see Soomere et al., 2013 and references therein).

If waves approach under a non-negligible angle θ with respect to the shore normal, the situation is much more complicated. Shi and Kirby (2008) argue that the water level set-down at the breaker line is invariant with respect to the approach angle of waves. The average deviation $\bar{\eta}$ of the sea surface from the still water level within the surf zone of an impermeable planar



beach is (Hsu et al., 2006; Shi and Kirby, 2008; the power of γ_b in the first term at the right-hand side of their expression being corrected):

$$\bar{\eta} = \frac{\gamma_b^2 \sin^2 \theta_b}{2h_b(8 + 3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b)}(h^2 - h_b^2) - \frac{3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b}{8 + 3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b}(h - h_b) - \frac{\gamma_b^2}{16}h_b. \quad (2)$$

The last term at the right-hand side of Eq. (2) represents the water level set-down $\bar{\eta}_b$ at the breaker line and θ_b is the wave approach direction at breaking. Here h represents the formal water depth counted from the still water level. The maximum wave set-up $\bar{\eta}_{\max}$ occurs somewhere in the inland where the formal water depth h_{\max} is negative, the actual water depth $d^* = h + \bar{\eta}_{\max} = 0$ and thus $\bar{\eta}_{\max} = -h_{\max}$. For this location, Eq. (2) reduces to:

$$\bar{\eta}_{\max} + h_{\max} = \frac{\gamma_b^2 \sin^2 \theta_b}{2h_b(8 + 3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b)}(h_{\max}^2 - h_b^2) - \frac{3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b}{8 + 3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b}(h_{\max} - h_b) - \frac{\gamma_b^2}{16}h_b + h_{\max} = 0. \quad (3)$$

For shore-normal waves $\theta_b = 0$ and Eq. (3) reduces to a linear equation:

$$-\frac{3\gamma_b^2}{8 + 3\gamma_b^2}(h_{\max} - h_b) - \frac{\gamma_b^2}{16}h_b + h_{\max} = 0. \quad (4)$$

In this case the maximum set-up height $\bar{\eta}_{\max}$ is defined by Eq. (1).

For obliquely approaching waves Eq. (3) is a quadratic equation with respect to $q = h_{\max}/h_b$:

$$\frac{\gamma_b^2 \sin^2 \theta_b}{2(8 + 3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b)}(q^2 - 1) - \frac{3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b}{8 + 3\gamma_b^2 - 2\gamma_b^2 \sin^2 \theta_b}(q - 1) - \frac{\gamma_b^2}{16} + q = 0. \quad (5)$$

This equation can be rewritten as

$$8q^2\gamma_b^2 \sin^2 \theta_b + 128q + \gamma_b^2(40 - 3\gamma_b^2) - \gamma_b^2 \sin^2 \theta_b(40 - 2\gamma_b^2) = 0. \quad (6)$$

Equation (6) has two negative solutions for physically reasonable values of γ_b . The physically relevant solution to Eq. (6) must be bounded and should be almost equal to $q \approx -5\gamma_b^2/16$ for very small approach angles $\theta_b \approx 0$. Therefore, the expression

$$q_1 = \frac{-32 + \sqrt{1024 - 2\gamma_b^4 \sin^2 \theta_b [40 - 3\gamma_b^2 - \sin^2 \theta_b(40 - 2\gamma_b^2)]}}{4\gamma_b^2 \sin^2 \theta_b} \quad (7)$$

provides the desired solution. Equation (7) deviates from the expression (30) of Hsu et al. (2006) by reasons discussed by Shi and Kirby (2008). The maximum set-up height for obliquely approaching waves is thus



$$\bar{\eta}_{\max} = -q_1 h_b = -\frac{q_1 H_b}{\gamma_b}. \quad (8)$$

2.2 Wave time series in the nearshore of the study area

We evaluate the shape and parameters of the statistical distribution set-up heights along an about 80 km long coastal segment of Tallinn Bay and Muuga Bay (Fig. 2). The study area is an example of a wave-dominated micro-tidal region. The shoreline of this section of the northern shore of Estonia in the Gulf of Finland, north-eastern Baltic Sea, is locally almost straight for scales up to a kilometre or two. Several relatively straight parts along the Suurupi Peninsula and the area of Saviranna are open to the north. However, on larger scales (from a few kilometres) the coast contains large peninsulas and bays deeply cut into the mainland. The shores of these landforms are open to different directions. As the formation of set-up crucially depends on the wave height and the approach angle, this type of coastal landscape makes it possible to analyse the distribution in question for coastal sections with radically different severity of the wave climate and the associated magnitudes of set-up (Soomere et al., 2013).

The fetch length in the Gulf of Finland is >200 km for western and eastern winds but <100 km for all other wind directions. The all-time highest significant wave height (5.2 m) in the Gulf of Finland has been recorded twice just a few tens of km to the north of the study area (Tuomi et al., 2011). The predominant strong wind directions in this region are south-west and north-north-west. Eastern storms are less frequent but may generate as high waves as the western storms (Soomere et al., 2008). Strong north-north-western storms may generate significant wave heights >4 m in the interior of Tallinn Bay (Soomere, 2005). The varying mutual orientation of high winds, propagation direction of waves and single shoreline segments makes it possible to identify potential alongshore variations in the distributions of set-up heights.

We employ time series of wave properties (significant wave height, wave period and propagation direction) reconstructed using the wave model WAM and one-point high-quality wind information from the vicinity of the study area. The wave model is implemented in a triple nested version with the resolution of the innermost grid about 470 m (Soomere, 2005). The study area is divided into 174 coastal segments with a length of about 500 m (Fig. 2). Each segment corresponds to a nearshore wave model grid cell. The experience with this model in the Baltic Sea and Finnish archipelago indicates that it is important to adequately represent the wave growth in low wind and short fetch conditions (Tuomi et al., 2011; 2012). To meet this requirement, the model uses an increased frequency range of waves up to 2.08 Hz. The ignoring of the presence of sea ice may lead to a certain overestimation of the overall wave energy in the region but apparently does not significantly distort the shape of distributions of set-up heights and the variation in these distributions along the shoreline.

The model is implemented using a simplified scheme that is designed for rapid reconstructions of long-term wave time series. Wave computations are speeded up by replacing long-term calculations of the sea state by an analysis of precomputed maps of wave properties. This simplification relies on a favourable feature of the local wave regime. Namely, wave fields rapidly become saturated and have relatively short memory in the study area (Soomere, 2005). Consequently, a reasonable reproduction of wave statistics is possible by the assumption that an instant wave field in Tallinn Bay is a function of a short



section of the wind dynamics. This assumption justifies the splitting of the calculations into a number of short independent sections with duration of 3–12 hours. The details of the model set-up, used bathymetry, implementation, and validation of the outcome have been repeatedly discussed in the literature (Soomere, 2005; Soomere et al., 2013).

The described approach makes it possible to circumvent one of the major issues of replication of the Baltic Sea wave fields, namely, the frequent inconsistency of different modelled wind data sets (Nikolkina et al., 2014). The quality of wave hindcast primarily depends on the adequacy of the wind information. In particular, wave set-up is intrinsically sensitive with respect to the wave propagation direction. It is therefore crucial to force the wave model with correct information about wind directions. This is an issue in the Gulf of Finland where atmospheric models often fail to reproduce wind directions (Keevallik and Soomere, 2010). To overcome this issue, we use wind data from an offshore location in the central part of this gulf. The wind recordings at Kalbådagrund (59°59' N, 25°36' E, a caisson lighthouse located on the top of an offshore shoal) are known to impeccably represent marine wind properties (Soomere et al., 2008). Even though this site is located at a distance of about 60 km from the study area, it is expected to correctly record wind properties in the offshore that are mostly responsible for the generation of surface waves.

Wind properties at Kalbådagrund were recorded starting from 1981 once in 3 hours. As this resolution of wind measurements was employed for more than two decades, we selected analogous data also from the newer higher-resolution recordings to ensure that the forcing data is homogeneous. The entire simulation interval 1981–2016 contained 103 498 wind measurement instants with a time step of 3 h. In about 9000 cases (less than 10% of the entire set) either wind speed or direction was missing. These time instants were excluded from the further analysis. As some of these instants involved quite strong winds, our analysis may underestimate the highest wave set-up events in some segments of the shore.

2.3 Nearshore refraction and shoaling

The nearshore grid cells selected for the analysis (Fig. 2) are located in ≥ 4 m deep water in order to avoid massive wave breaking in these cells. Some of the cells are located in deeper locations with water depth of 20–27 m. The nearshore of the study area contains various underwater features and bottom inhomogeneities. This means that shoaling and refraction may considerably impact the wave fields even along the relatively short pathways (normally ≤ 1 km in our model setup) from the model grid cells until the breaking line. The predominant storm directions are the south-east, north-north-west and east (Soomere et al., 2008). Consequently, high waves often approach some of the selected grid cells under large angles with respect to the shore normal. Therefore, it is not acceptable to assume that the incidence angles are small. As a result, simplified approaches to replicate the changes in wave properties in the immediate nearshore (Lopez-Ruiz et al., 2014; 2015) and even advanced approximations of refraction and shoaling (Hansen and Larson, 2010) may fail.

For this reason we calculate the joint impact of shoaling and refraction of approaching waves in the framework of the linear wave theory using the technique developed in (Viška and Soomere, 2013; Soomere et al., 2013). As usual, it is assumed that the numerically evaluated wave field for each time instant is monochromatic and characterised by the numerically simulated



significant wave height H_0 , peak period and mean approach direction θ_0 with respect to the onshore-directed normal to the shoreline. These properties are evaluated at the centre of each selected grid cell. Similarly, it is assumed that the nearshore seabed from the centre of each grid cell until the waterline is plane with isobaths strictly parallel to the shoreline. Finally, we assume that breaking waves are long waves. Then the wave height H_b at the breaking line can be found as the smaller real solution of the following algebraic equation of 6th order (Viška and Soomere, 2013; Soomere et al., 2013):

$$\frac{H_b^5 g}{H_0^4 \gamma_b} \left(1 - \frac{g H_b \sin^2 \theta_0}{\gamma_b c_{f0}^2} \right) = c_{g0}^2 (1 - \sin^2 \theta_0). \quad (9)$$

Here c_g is the group speed, c_f is the phase speed and the subscripts “0” and “b” indicate the relevant value at the centre of the particular wave model grid cell and at the breaker line, respectively. The set of assumptions is completed with the common notion that the breaking index is $\gamma_b = H_b/h_b = 0.8$ (Dean and Dalrymple, 1991).

Part of the introduced assumptions such as the plane seabed and dry coast without any vegetation, monochromatic wave fields and a constant value of the breaking index for windseas as well as the ignoring of the wave period (or steepness) in the calculations are not fully realistic and may lead to certain overestimation of the set-up heights. As we are first of all interested in the basic features of the distributions of different set-up heights, we expect that the use of this set of assumptions makes it possible to highlight the features of question in a more contrast manner than they would appear for real wave fields. In other words, we make an attempt to isolate the worst possible scenarios for wave set-up for generally realistic wave conditions.

Several earlier studies of extreme set-up heights (Soomere et al., 2013; Pindsoo and Soomere, 2015) have taken into account only waves that approached the coast under angles $\pm 15^\circ$ with respect to the shore normal. This assumption is often valid on the open ocean coasts where waves usually approach the shore under relatively small angles. However, it may fail in semi-sheltered basins with short fetch and thus is abandoned in this paper.

3 Results

3.1 Maximum set-up heights

The phenomenon of wave set-up is only profound if large waves propagate towards the shore. This is usually the case on open ocean coasts where swells almost always create a certain set-up. The situation may be different in sheltered sea areas with complicated geometry where intense swells may be infrequent and wind waves may often propagate from the nearshore towards the open sea. The depicted features are typical for the study area. The wind regime of the study area is a superposition of four wind systems (Soomere et al., 2008). The most frequent wind direction is from south-west (that is, from the mainland to the sea). The proportion of wave fields that propagate onshore is 40–70% along the entire study area



(Fig. 3). The statistical properties of set-up heights discussed below thus represent 40 000–70 000 examples of wave fields in each coastal segment. The only exception is grid cell 107 (Fig. 2) between Viimsi Peninsula and the island of Aegna that is sheltered for almost all directions.

We start from a comparison of maximum set-up heights evaluated using the above-described approach and a simpler method (Soomere et al., 2013) that took into account wave fields that propagated almost directly onshore ($\pm 15^\circ$ with respect to the shore normal). The results of the two methods differ insignificantly (by less than 0.1 m) in about 80% of the coastal segments (Fig. 4). The alongshore variations in the maxima of set-up heights evaluated from Eq. (8) are considerably smaller than the variations of values estimated using a simpler method.

The largest differences become evident in segments that are sheltered from predominant storm directions; often in these sections of deeply cut bays where waves usually approach under a relatively large angle. Interestingly, estimates based on Eq. (8) are remarkably (by up to 50%) higher in these sections than those derived using the simplified method. This difference stresses the potential of generation of remarkable set-up heights even by obliquely approaching high waves and indicates that the simpler method substantially underestimates the maximum set-up height in such segments. It is also likely that refraction and diffraction may redirect wave energy so that even beaches that are seemingly well sheltered geometrically may at times receive remarkable amounts of wave energy. In other words, the impact of refraction often overrides here the effect of geometric blocking of waves by changing orientation of the coastline (cf Caliskan and Valle-Levinson, 2008). The differences in maxima of set-up heights for such (seemingly geometrically sheltered) sections are often 0.2–0.3 m and reach up to 0.5 m. Such a strong impact of refraction is not usual but also not unique in coastal areas. It is usually thought to be responsible for a local increase in wave heights not only in the Baltic Sea (Soomere, 2003) but also in extreme ocean conditions (Babanin et al., 2011). The major differences in the two sets of estimates of set-up heights clearly signal that the use of simplified methods and taking into account only waves that propagate almost onshore may lead to substantial underestimation of wave-driven hazards.

In contrary, the simpler method overestimates the maximum set-up height in a few locations at headlands that are fully open to the Gulf of Finland (Fig. 4). A likely reason for such sporadic overestimation is that the formation of set-up is sensitive with respect to the approach angle of waves. The magnitude of set-up rapidly decreases with an increase in the approach angle. This decrease is ignored in (Soomere et al., 2013) where the height of set-up created by waves that approached under angles of 10–15 degrees with respect to shore normal was overestimated. This feature additionally stresses the importance of correct evaluation of refraction and shoaling.

Some differences of the latter set of results from those presented in (Soomere et al., 2013; Pindsoo and Soomere, 2015) stem from the different time intervals used in the calculations. Simulations for 1981–2012 indicate that the maximum set-up heights in coastal areas open to the east were mostly created in the 1980s (Soomere et al., 2013) even though the maximum wave heights occurred starting from the mid-1990s. This feature may be related to a change in the strong wind directions so that eastern storms became weaker for about two decades.



However, there is increasing evidence that this process has reversed and strong eastern storms have returned to the area. The first evidence of this change is that the all-time highest significant wave height 5.2 m was recorded in the Gulf of Finland for the second time during an extreme eastern storm on 29–30 November 2012 (Pettersson et al., 2013). Pindsoo and Soomere (2015) observed that many new all-time highest set-up events apparently occurred in coastal locations open to the east. This process evidently continues and has led to generation of all-highest simulated waves in a number of locations at the eastern Viimsi Peninsula near Leppneeme (Fig. 5). These aspects will be addressed in more detail elsewhere.

3.2 Frequency of occurrence of set-up heights

The empirical distributions of occurrence of set-up heights are clearly non-Gaussian in many locations of the study area (Fig. 6, 7). Even though the replication of wave propagation directions by the wave model and the impact of refraction may suffer from insufficient resolution of both the wind information and wave model, the presented distributions exhibit a similar shape for the entire study area. The appearance of these distributions in semilogarithmic (log-linear) coordinates is often clearly concave upwards. This concave appearance also clearly differs from all usual distributions of the magnitude of wave phenomena such as the classic (Rayleigh) distribution of single wave heights (Longuet-Higgins, 1952), the Tayfun distribution of the heights of largest waves, the Weibull family of distributions for the occurrence of various wave conditions, or the Rayleigh distribution for run-up of (narrow-banded) Gaussian wave fields (Didenkulova et al., 2008). Therefore, none of these distributions can be used for the approximation of set-up heights. The frequently occurring concave upwards shape signals that even the exponential distribution (that describes, e.g., storm surges in the study area, Soomere et al., 2015) is not suitable for their description. Therefore the background process of formation of high set-up events is not a Poisson one (that would lead to a basically linear shape of the distribution in question in log-linear coordinates). To further explore the shape of the distributions of set-up heights and their possible variations along the shoreline we assume that these distributions belong to the family of general exponential distributions. The overall appearance of empirical distributions in log-linear coordinates (Fig. 7) suggests that their shape can be, as a first approximation, matched with a quadratic polynomial $ap^2 + bp + c$, where p is the probability of a particular set-up height.

Such an approximation is, however, not straightforward because of several reasons. Firstly, the number of nonzero points of the distributions in Fig. 7 is highly variable along the study area depending on the typical magnitude of the set-up. Secondly, the relevant empirical distributions have gaps for some value(s) of the set-up height. A natural reason for this feature is that we are looking at very low probabilities (of an order of 0.001%) of occurrence of relatively high set-up events, down to the level of a few events in 1981–2016. Thirdly, a few locations host several outliers – unexpectedly high set-up events. To eliminate the impact of these aspects on the results, we performed the analysis on this approximation in three ways. Firstly, we used all data points in Fig. 7 starting from the height of 0.01 m to evaluate the coefficients a , b and c . Secondly, we used for the same purpose only set-up heights from 0.01 m up to 0.4 m (Fig. 7) This approach was not applicable in some locations where set-up heights did not reach the level of 0.4 m. However, it made it possible to check whether the shape of



the distribution is governed by the majority of events or it is steered by the presence of a few very large set-up heights. Thirdly, we evaluated these coefficients starting from the height of 0.01 m up to the first gap in the empirical distribution (the lowest set-up height that did not occur in 1981–2016). Doing so made it possible to roughly evaluate the effect of possible gaps in the relevant distribution.

- 5 The particular values of the coefficients a , b and c depend to some extent on the chosen approximation (Fig. 6) but the shape of the polynomial and the approximate distribution is invariant with respect of the particular choice. All exponential distributions match generally well the data points corresponding to the largest set-up heights. The resulting theoretical distributions differed to some extent from each other but the differences were mostly insignificant and the relevant lines were located almost in the centre of the 95% confidence intervals of each other (Fig. 6).
- 10 The coefficients a at the leading term of the approximating polynomial (Fig. 8) are mostly very small, in the range of $(-0.005, 0.005)$ whereas their 95% confidence interval normally include the zero value. This feature indicates that in most occasions the hypothesis that the parameter a can be set to zero in this approach and distributions of set-up heights can be reasonably approximated with an exponential distribution. In this case the parameter b characterises the vulnerability of the particular coastal segment with respect to the set-up phenomenon similarly to the analysis of storm-driven high water levels
- 15 (Soomere et al., 2015). A few outliers of the parameter a in relatively sheltered coastal segments were exclusively negative and reached the values down to -0.08 (Fig. 8). These values correspond to distributions with convex upwards shape in semilogarithmic coordinates and are thus qualitatively similar to the family of Gaussian or Weibull distributions. Importantly, in about a quarter of coastal segments of the study area the parameter a is positive and its 95% confidence intervals to not include the zero value. In other words, in this location the leading term of the quadratic polynomial is
- 20 positive at a 95% significance level. This feature corresponds to the concave-up appearance of the relevant distributions of set-up heights. This appearance first of all means that very large set-up events are systematically much higher and/or occur much more frequently than one could expect from the classic Gaussian or Weibull-type statistics. The described features indicate that the empirical distribution of set-up heights can be adequately approximated using an inverse Gaussian (Wald) distribution with a probability density function (Folks and Chhikara, 1978)

25
$$P = \sqrt{\frac{\lambda}{2\pi\alpha^3}} \exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right). \quad (9)$$

For a certain set of parameters λ (the shape parameter) and μ (the mean) a part of the graph of this function has a concave upward shape in semilogarithmic coordinates (Fig. 7) and thus fairly well approximates the empirical distributions of wave set-up in the relevant locations.

- All coefficients of the quadratic approximation of the exponent vary insignificantly along the study area. This is remarkable
- 30 because the shape of the relevant Weibull distribution for different wave conditions largely varies along the study area (Soomere, 2005). The variations of the leading coefficient a are uncorrelated with the values of maximum set-up heights



along the study area. It is thus likely that the presence of locations where an inverse Gaussian distribution governs the properties of set-up heights is connected with a specific match of the directional structure of winds and the orientation of the coastline. This feature also signals that the basic features of the distribution in question are only weakly (if at all) connected with to the properties of local wave climate. This conjecture is supported by comparatively small variations in the values of other parameters in the polynomial approximation (Fig. 7b,c). The values of c are all positive and mostly in the range of 2.5–4 (Fig. 8). The values of parameter b are, as expected, almost everywhere negative, concentrated around -0.2 and typically vary between -0.1 and -0.4 . A few locations with positive values of this parameter correspond to large negative values of a .

4 Discussion and conclusions

The performed analysis first of all reveals that numerical estimates of maxima of wave set-up heights are relatively sensitive with respect of how the impact of radiation stress and the transformation of wave properties in the nearshore are evaluated. The magnitude of the related effects substantially depends on the appearance of bathymetry. The impact of refraction can easily override the purely geometric effects of shoreline orientation changes and redirect substantial levels of wave energy into seemingly sheltered shore sections. This feature calls for the necessity of using high-resolution information about wind (incl. wind directions) and bathymetry together with advanced methods for the evaluation of propagation and impact of radiation stress in the nearshore in operational and hindcast models of coastal flooding.

The core message of the analysis is that the empirical probability distribution of different set-up heights can be usually fairly well approximated by a standard exponential distribution $\exp(-\lambda x)$. When the exponent function of the general exponential distribution is approximated using a quadratic function, the coefficient at its leading term does not differ from zero at a significance level of 95% for more than $\frac{3}{4}$ of the coastal segments of the study area. As this area contains a variety of sections open to different directions and with radically different wave properties, it is likely that the qualitative shape of the distribution in question only weakly depends on the properties of local wave climate.

Another important message is that the basic shape of this distribution function is concave upwards in a log-linear plot in a substantial number of coastal segments. The local shape of the relevant empirical distributions of wave set-up heights can be adequately approximated with a family of inverse Gaussian (Wald) distributions. Even though the absolute values of the coefficients at the leading term of such a quadratic approximation are relatively small, approximations with other classic distributions such a Rayleigh or Weibull ones (Fig. 7) are evidently inappropriate. As the coefficient at the linear term of this quadratic approximation is relatively small (Fig. 8b), the use of a Lévy distribution might also be appropriate.

This result is intriguing because sensible approximations of inverse Gaussian (Wald) distributions are scarce in descriptions of geophysical phenomena and we have established no arguments in favour of this distribution for wave set-up heights. Perhaps the most well-known example of the use of a Wald distribution is to describe the time a Brownian motion (with positive drift) takes to reach a fixed positive level. Other examples include statistical properties of soil phosphorus (Manunta et al., 2002), long-distance seed dispersal by wind (Katul et al., 2005) or some models of failure (Park and Padgett, 2005).



Finally, we note that the presented results do not require any modification of the classic estimates of extreme values of set-up heights and their return periods based, for example, on the block maximum method. Namely, the limiting distributions of independent block maxima follow either a Gumbel, Weibull or Frechet distribution notwithstanding the distribution of the underlying values (Coles, 2004). This general theorem is obviously valid also for any time series that follows an inverse Gaussian distribution as well. A delicate implication from the match of statistics of set-up heights with an inverse Gaussian distribution is that set-up events with heights close to extreme ones may be much more frequent than their estimates based on classic Gaussian or Weibull statistics and also clearly more frequent than similar estimates for Poisson processes. This increase in the probability of large wave set-up events is balanced by a similar decrease in the relative number of events with an average magnitude compared to normally distributed events. The described features basically indicate that the frequency and role of close-to-extreme set-up events (and their contribution to associated damages and economic losses) may be underestimated based on observations of similar events of average height. In particular, severe set-up events may occur substantially more frequently than it could be expected from the probability of occurrence of severe seas.

Code availability: From the authors on request (Matlab scripts).

Data availability: Kalbadagrund wind data are applicable from the Finnish Meteorological Institute.

15 **Sample availability:** not applicable.

Authors team: Tarmo Soomere and Katri Pindsoo

Author contribution. T. Soomere designed the study, derived the equations and approximations used in the paper, produced Figure 6, compiled the introduction and discussion, checked the consistency of the results, and polished the entire text. K. Pindsoo developed the scripts, ran the simulations, produced most of graphics, drafted the body parts of the manuscript.

20 **Competing interests:** The authors declare that they have no conflict of interest.

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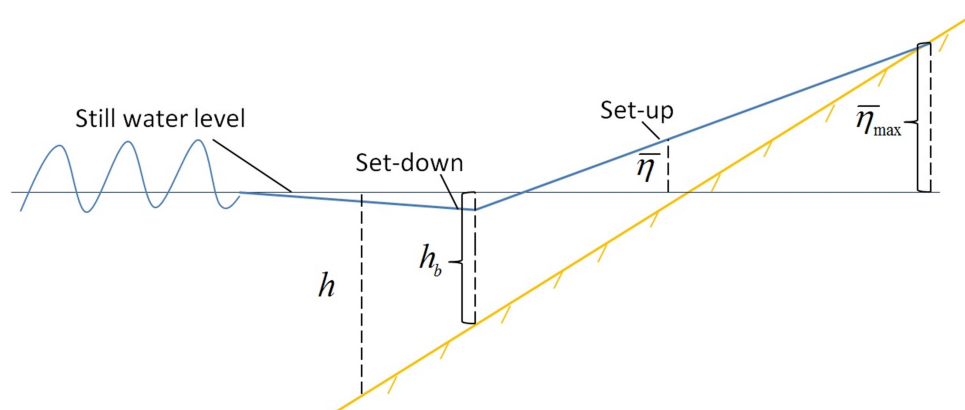


Figure 1: Scheme of the cross-section of a coastal area hosting wave set-up.

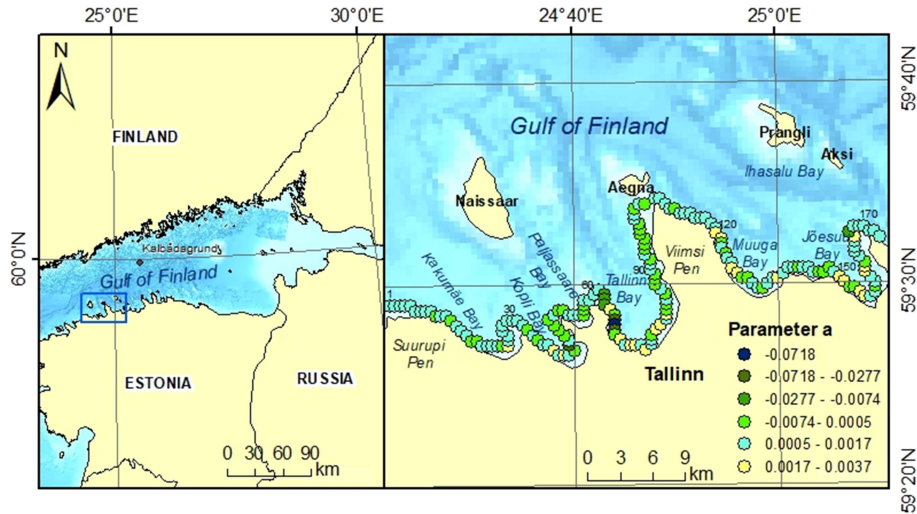


Figure 2: Study area (box in the left panel) in the vicinity of Tallinn Bay. Small circles along the shoreline in the right panel indicate the nearshore grid cells of the wave model WAM with a resolution of about 470 m. Colour code indicates the coefficient of the leading term in the approximation of the exponent of empirical distribution of set-up heights in single locations. The selected grid cells are numbered consecutively from the west to the east.

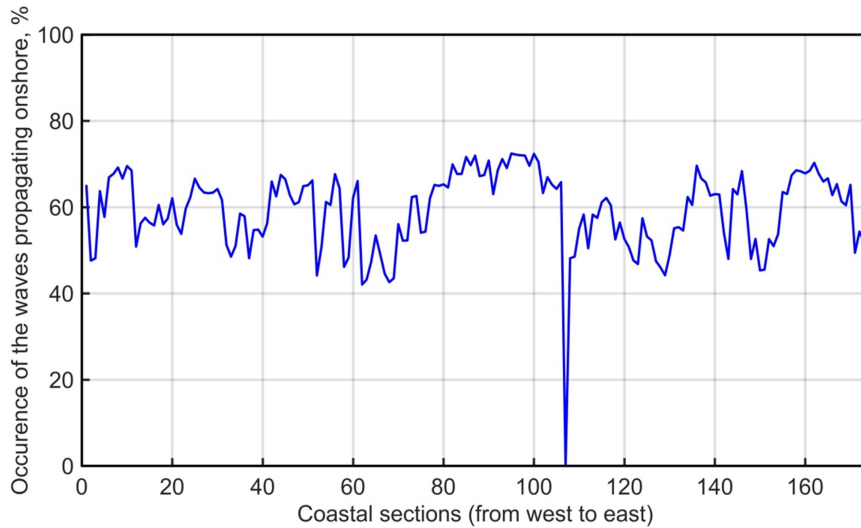


Figure 3: The percentage of occurrence of waves that propagate onshore and produce elevated wave set-up events along the study area.

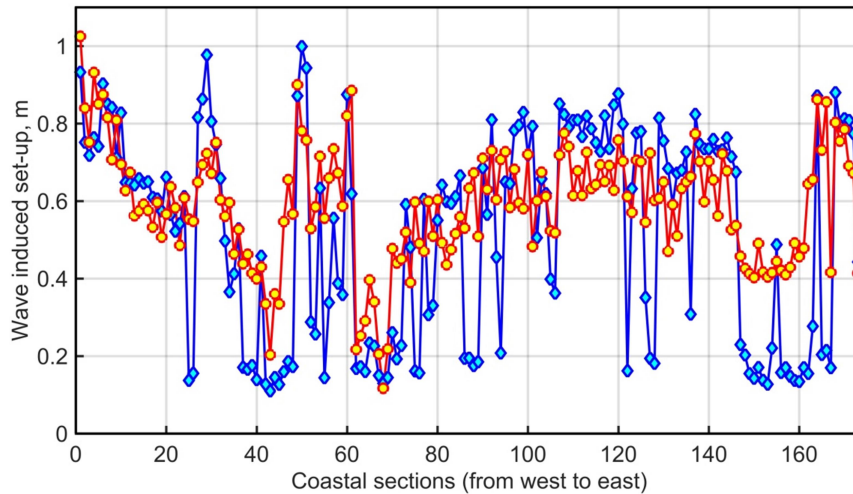
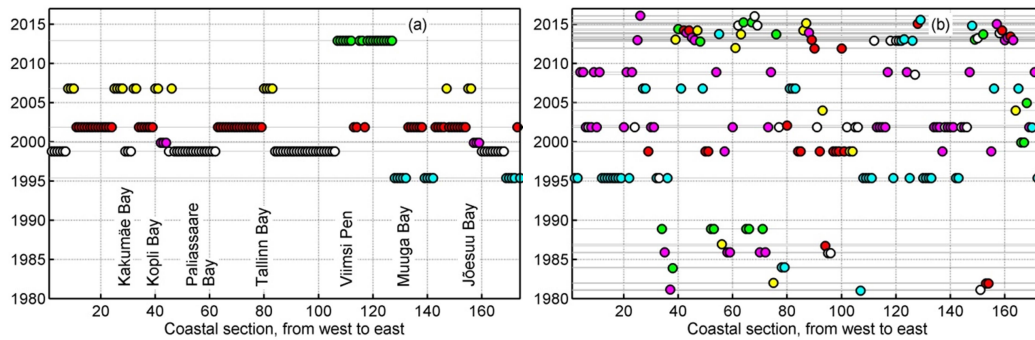
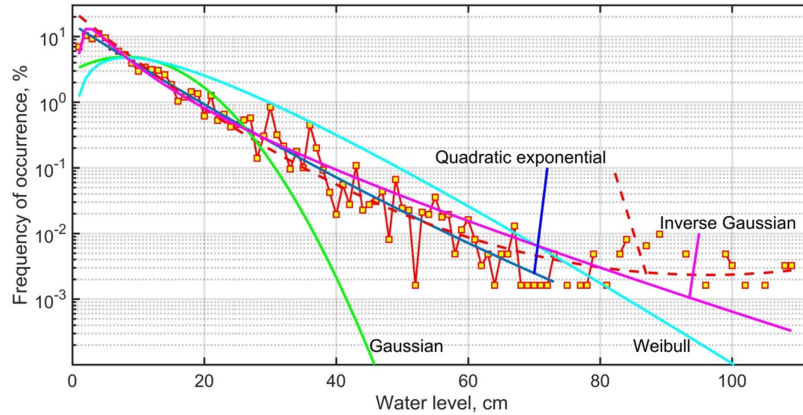


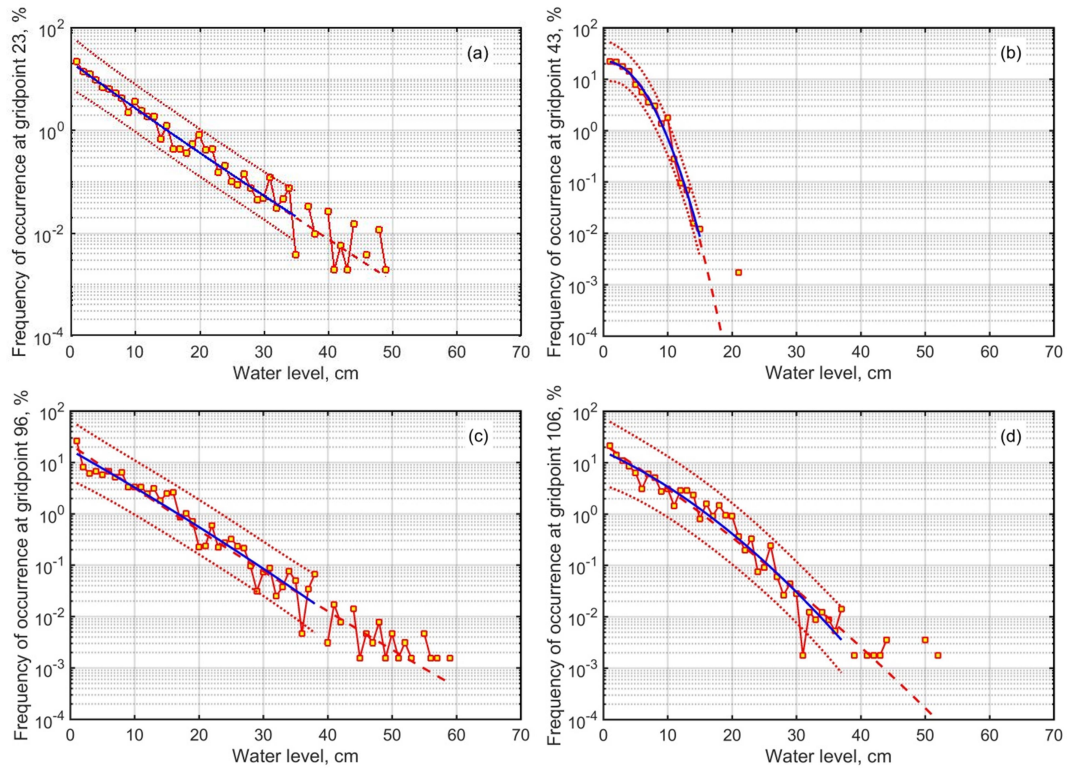
Figure 4: Maximum set-up heights evaluated using all onshore-propagating wave fields and Eqs. (1–8) (red circles) and similar heights evaluated using only those waves that approach the shore at an angle $\pm 15^\circ$ from the onshore normal (blue diamonds).



5 Figure 5: Six storms that caused the highest waves in different coastal sections of the study area in 1981–2016. Notice the cluster of green circles along the eastern coast of Viimsi Peninsula in an autumn storm of 2013 (b) 58 storms that caused the highest wave set-up in these sections in January 1981–May 2016. The set-up heights are evaluated similarly to the procedure in (Pindsoo and Soomere, 2015) using only waves approaching at an angle $\pm 15^\circ$ with respect to the shore normal.



5 **Figure 6:** Simulated distributions of various set-up heights (red squares) in the easternmost coastal segment of the study area and the relevant normal Weibull and inverse Gaussian (Wald) distributions evaluated using the method of moments. Blue line: interpolation of the empirical distribution in semilogarithmic coordinates with a quadratic function (equivalently, the formal local exponential distribution with a general quadratic exponent) from the set-up height of 0.01 m until the first gap in the empirical distribution; red dashed line: similar interpolation using all data points.



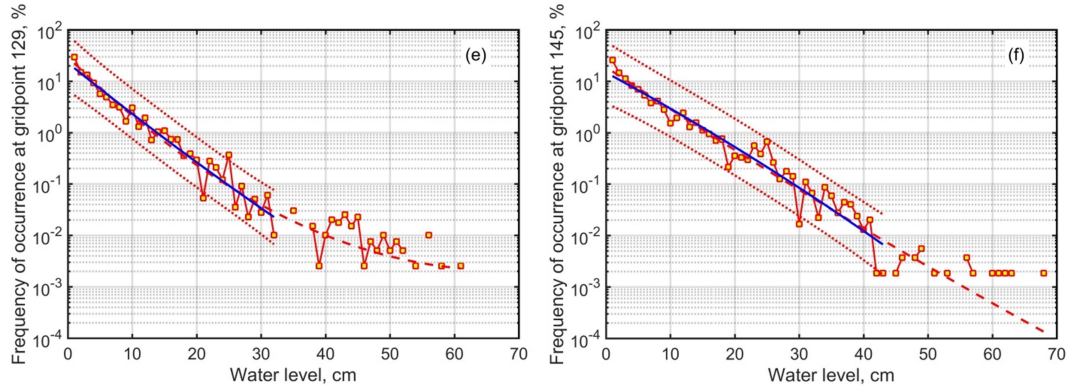
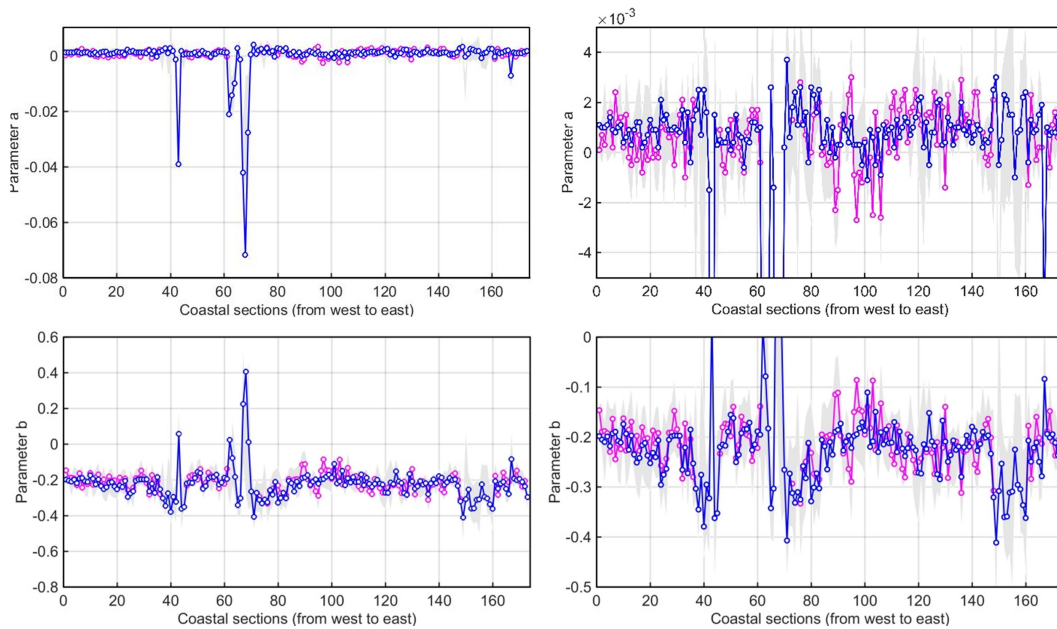
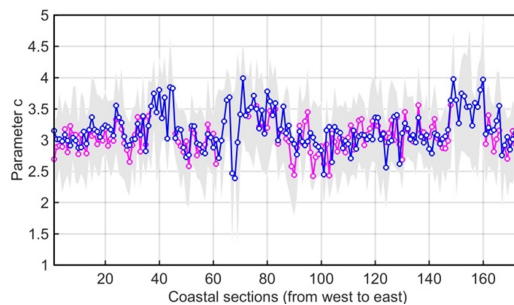


Figure 7: Simulated distributions of various set-up heights (red squares) at various locations of the Tallinn Bay area open to different directions. Blue line: interpolation with a quadratic function from the set-up height of 0.01 m until the first gap in the empirical distribution; red dotted lines: its 95% confidence intervals; red dashed line: similar interpolation using all data points.

5 The interpolating lines evaluated using only the data points from 0.01 m to 0.4 m are fully masked by blue lines.





5 **Figure 8:** Alongshore variation of the coefficients a, b, c of the quadratic approximation $ap^2 + bp + c$ of the exponent of empirical distributions of set-up heights in the Tallinn Bay area. For the parameters a, b , also more detailed alongshore variation is presented in graphs with vertically stretched scales. Blue line: the respective parameter calculated for the range of set-up heights from 0.01 m until the first gap in the empirical distribution; the grey area marks the 95% confidence interval of this value, pink line describes the values of the relevant parameter for the range of set-up heights from 0.01 m until 0.04 m.