Jérôme Sirven
LOCEAN
Université Pierre et Marie Curie
T45 étage 4 CC100
4 place Jussieu
75252 PARIS CEDEX 05
email: js@locean-ipsl.upmc.fr

October 7, 2019

Dr R. Tailleux

Dear R. Tailleux,

you will find here our answer to the comments you made on our article "Generation of Rossby waves off the Cape Verde Peninsula; role of the coastline" which has been submitted to "Ocean science".

We first want to thank you. We apreciated your careful reading of the article and your numerous and interesting suggestions. We explain below how we took them into account. The corresponding modifications on the manuscript are also explicited.

1. You argued that the chlorophyll observations are only a very indirect proxy for dynamical activity and consequently must be completed by Sea Surface Height observations (for example coming from AVISO altimeter data) to convince the reader of the presence of Rossby waves. Consequently we have added a supplementary figure (new figure 2; see below) which is a Hoevmuller diagram of the SSH at different latitudes off the Cape Verde (12°N, 13.5°N, 15°N, and 16.5°N). This figure clearly shows the existence of a wave whose zonal velocity is about 4.5 cm/s, a value which corresponds to the estimation we have previously given from the observations and the theoretical study.

   A figure of the wind stress has also been added (new figure 3; see below). This figure has been drawn from the ERAInterim reanalysis corresponding to the period December 2002-January 2003. The mean wind stress and the wind burst occuring around December 10th are both shown.

   Note that we decided not to add a supplementary figure to document the existence of Kelvin waves propagating along the coast but only to add references about the work of A. Lazar in introduction and conclusion. Here are the changes in the introduction

   Attention is focused on offshore mesoscale activity associated with the upwelling, a recurrent feature of upwelling systems (see Capet et al., 2008 a, b). The alongshore activity, which has received much more attention (see for example Ndoye et al., 2016 and the references herein) is not studied.

and in the conclusion.

The existence of a Chlorophyll signal far from the coast – here extending up to 750 km west to the Cape Verde has to our knowledge never been described. This strongly differs from the coastal signals associated with Kelvin waves which have been previously carefully analyzed (see Ndoye et al. 2016 and the references herein).

Indeed we think that an originality of this paper is to consider signals – and specifically biological signals – far away from the coast. The dynamics of the Kelvin waves, which remains trapped to the coast, have received in this region much more attention and the works of A. Lazar exemplifies this. We thus prefered to emphasize the existence of the Rossby wave and its biological signature.

Just below is given in italic the text which has been introduced about AVISO data on page 4 The corresponding figure is shown at the end of this text.

…To corroborate this hypothesis, we analyzed the Sea Surface Height (hereafter SSH) obtained from AVISO satellite altimeter data for the corresponding period (December 2002 - January 2003). Hoevmuller diagrams are shown at $12°$, $13.5°$, $15°$, *and* $16.5°$ *in figure 1; they clearly confirm the existence of a Rossby wave propagating westwards with a velocity of about* 4.5 *cm s*$^{-1}$. *The amplitude of this wave becomes smaller northwards: it peaked at 13 cm between* $12°$ *and* $13.5°$ *degrees but did not exceed 7 cm at* $16.5°$. *The wavelength is around 700 km, comparable with the extent of the Chlorophyll signal.*

2. Numerical experiments. We have added a new paragraph to respond to your queries. We have chosen a mean state which more or less corresponds to the mean state observed in this region (a dominant wind stress blowing from the north-north-east (see the new figure 3) but which remains simple enough to permit the ulterior analytical work. The reference state has a negligible impact on the solution – only some details are modified – and we decided not to develop this point. However, we had to verify that our results subsist in a more complex (and more realistic) situation than the one where the state is at rest with a unifrom layer. The model has been calibrated so that the Rossby radius has a reasonable value in this area and consequently that the propagation velocity of the waves is realistic. Actually, the phase velocity of the waves is about 4. cm s^{-1}, which corresponds to the phase velocity of the waves shown by AVISO data.

   Just below is given in italic the text of the subsection which has been added in section 2 and the corresponding figure.

   …**3.3 Numerical set-up of the model**

   In Figure 2 the mean wind for the considered period (December 2002-January 2003) is shown. It exemplifies the situation which is normally found in this *region. The wind regularly blows from the north-north-east with a velocity ranging from 4 to 8 m* s^{-1}*. To take this into account, a constant mean wind stress of amplitude equal to 0.06 N* m^{-2} *(corresponding to a mean wind velocity of about 5 m* s^{-1} *and a value of* $\boldsymbol{\tau}_0$ *equal to* 6×10^{-5} m^2 s^{-2}*) and oriented along a south-south-west direction is applied from rest during four years, until a stationary mean state, which verifies the theoretical relation given in section 3.1, is reached.*

As shown in Figure 2 a wind anomaly was active when the wave of Figure 1 begins to be observed. This anomaly obviously is transient, but to the south of the Cape Verde, it mainly points southwards. To represent this situation in a simplified way, we defined in a first experiment a north-south wind stress anomaly which extends over approximately 500 km and whose maximum is still equal to 0.06 N m^{-2}*. This anomaly is applied during five days (see Fig. 4, first panel). The integration is continued during 45 days, after the anomaly has disappeared.*

To explore the sensibility of the model response to the wind anomaly, others wind anomalies have been applied (see below, in particular Figures 6 to 8). The results obtained for these anomalies are discussed in the next section.

3. You judged that the theoretical analysis is hard to understand. We followed your advice and thus summarized as explained below the major lines of the analysis to guide the reader. We think that a reader who is not interested in the details of the mathematics can now skip some computations.

   In this section, we aim at understanding if the coastline may influence the propagation of the Rossby waves which are created close to the coast and propagate towards the open sea. The impact of the coastline has been investigated by Crépon and Richez (1984), Clarke (1977), and Clarke and Shi (1991) for the Kelvin waves using an analytical approach. Here we focus on the Rossby waves; as we consider an area which extends up to about 1000 km from the coast, we have to generalize the approach followed by Clarke and Shi, which introduced a local system of coordinates dependent of the coastline to study Kelvin waves along an irregular coastline. We try to answer the following questions:

   a) Are there time scales for which the impact of the coastline (small in the numerical experiments) becomes more important ?

   b) A dissymetry between the response north and south of the Cape was visible in the numerical experiments; can this dissymmetry be dependent on the existence of the Cape ?

   The analysis begins by defining and building a system of coordinates that permits to follow the coastline geometry. This procedure is a standard one in mathematics when boundaries are complex; indeed the the boundary conditions can be simply written, which constitutes a substantial advantage. however, it *has a drawback: the differential equations which characerize the problem become slightly more complex because they must include geometrical factors that take into account the deformations associated with the new system of coordinates. This drawback is small in comparison with the advantage.*

When these new equations are established, staightforward calculations are made to obtain a unique partial differential equation (equation (7)), which characterizes the evolution of η *(the thickness of the active layer). This equation is a wave equation. Consequently the ray theory (or equivalently the WKB method) can be applied. When the forcing terms are neglected, this yields a first order nonlinear differential equation (equation (11)). No new ideas are introduced after this. We just rewrite equation (11) by introducing new notations, in order to facilitate its study and the presentation of the results (end of paragraph 4.2). We then describe the results when the tranport along the coast is much larger than the transverse transport (section 4.3).*

We also added the following lines after the system of equations (7)

Note that we use a more complex system than the traditional one which is used for the study of the Kelvin waves and which neglect the variations of the transport perpendicular to the coast :

$$\left\{ \begin{array}{rcl} -FT_Y + a^{-1}\partial_X \eta &=& -b^{-1}\partial_Y \psi \\ \partial_t T_Y + FT_X + b^{-1}\partial_Y \eta &=& a^{-1}\partial_X \psi \\ \partial_t \eta + (C^2/(ab))[\partial_X(bT_X) + \partial_Y(aT_Y)] &=& \partial_t \phi \end{array} \right.$$

We use the complete system (7) because we study Rossby waves far from the coast, for which these hypotheses does not hold. Indeed, the previous simplified system leads to drop in equation (11) below two terms: $(1/b^2)(\partial_Y \theta)^2$ *and* $iF^2\partial_Y(a/(bF^2))\partial_Y\theta$. *These terms include the geometrical factors* a *and* b *due to the coordinates change; and we precisely try to investigate which is the impact of such terms.*

Lastly, note that the time scale we obtain are only dependent of the speed C for a given geometric configuration. The value of C depends on the mean state and on the initial thickness of the model. A larger C leads to shorter time scales.

4. I answer more briefly to the eleven other comments since many of them are strongly linked to the previous major points.

   Abstract line 5. As previously explained, we have now introduced Hoevmuller diagrams from AVISO altimeter data to show that direct observations of the ocean showed the existence of a westward propagating signal. The Abstract has been modified in consequence.

   Page 2, lines 7-9 We added as required "poleward of which Kelvin waves no longer exist".

   Lines 7-8 The well defined "sine like" pattern has been circled as required.

Page 4; the model This point is close to the second point of the main three comments. We constructed a background state which has a non uniform layer because this non uniform layer could have an impact on the Rossby waves propagation (like the β topographic effect). It is slightly more realistic than a uniform layer since the isopycns are not flat in this area. We must avow however that the experiments do not differ much from experiments made with a uniform layer.

Page 5, section 3.2 We introduced as suggested a new subsection named "numerical model setup" and limited the subsection "numerical resolution" to technical points. We do not use one of the calibration methods proposed by Flierl (1981). The climatological observations we used was for the wind stress the climatology of ERA Interim – as explained above a picture of the mean wind in December, 2002 is now shown.

At the open boundaries, we simply assumed that the velocity vanished.

We want to emphasize that the aim of the paper is not to reproduce precisely the climatological characterisitics of the region (the paper of Kanta et al., 2018 shows how a GCM can successfully reproduce the complexity of this region) but to propose a simple model which could explain some observations seen off the Cape Verde.

Page 5, Lines 27-28 As explained above, we tried to improve significantly the observational basis justifying this hypothesis by introducing AVISO altimeter data and adding a figure of the wind stress computed by ERAInterim.

Page 6, line 31 As you suggested, we now have introduced Hoevmuller diagram of the sea surface height which clearly shows the propagation of a wave with a velocity of 4.5 cms^{-1}. More details can be found at the beginning, in our answer to the first comment.

Page 8 The equation is completely justified in Appendix A. The idea is very simple:

$$\partial_t T_X - F T_Y = -a^{-1}\partial_X \eta \Longrightarrow \partial^2_{tt} T_X - F\partial_t T_Y = -a^{-1}\partial^2_{tX}\eta$$

Using the second equation we deduce

$$\partial^2_{tt} T_X + F^2 T_X = -a^{-1}\partial^2_{tX}\eta - Fb^{-1}\partial_Y \eta$$

Doing similar computations from the second equation, we obtain an equation for T_Y of the same type:

$$\partial^2_{tt} T_Y + F^2 T_Y = -b^{-1}\partial^2_{tY}\eta + Fa^{-1}\partial_X \eta$$

We now have just to apply the operator

$$\partial^2_{tt} + F^2$$

to eliminate T_X and T_Y and obtain an equation for η only. In appendix A the computation are done, taking into account our hypothises on the time scale of the motion.

Page 9, line 4 We have modified the sentence as follows *When a and b are close to 1 and the coast has a south north orientation (see Figure 8) the new coordinates system is nearly similar to the original one; indeed F mainly depends on $Y \simeq y$ only and in those regions equation (7) thus simplifies:*

$$\partial_t \left[\eta - R_0^2(\partial^2_{XX}\eta + \partial^2_{YY}\eta)\right] - R_0^2 \left[\partial_Y F \, \partial_X \eta + F^2 \partial_Y (\frac{1}{F^2}) \, \partial^2_{tY}\eta\right] = 0 \quad (1)$$

We recognize the equation characterizing the propagation of waves in the β plane ($\beta = \partial_Y F$) for a shallow water model. For a tilted coast the dependance of the Coriolis parameter as a function of X should be still taken into account.

Page 9, Equation (10) We have not developped this point in the paper but an equation for $\eta_0(X,Y)$ can be established and allows to predict the evolution of the amplitude. It is obtained by putting η in equation (7) and writing all the resulting terms; this leads to an equation which contains η_0 and θ. The hypotheses we made leads to solve the equation for θ which is given (equation (12)) and which contains information about the wave propagation. Once θ is known, $\eta_0(X,Y)$ can be determined using the complete equation. Generally, it is not very interesting except if critical latitude exist (in geometrical optics: focus). The amplitude of the wave increases and in the most simple cases can be represented using Airy functions.

Figure 2. The timeline for each panel has been indicated explicitely.

Sincerely yours,

Jérôme Sirven

The new figures are just below.

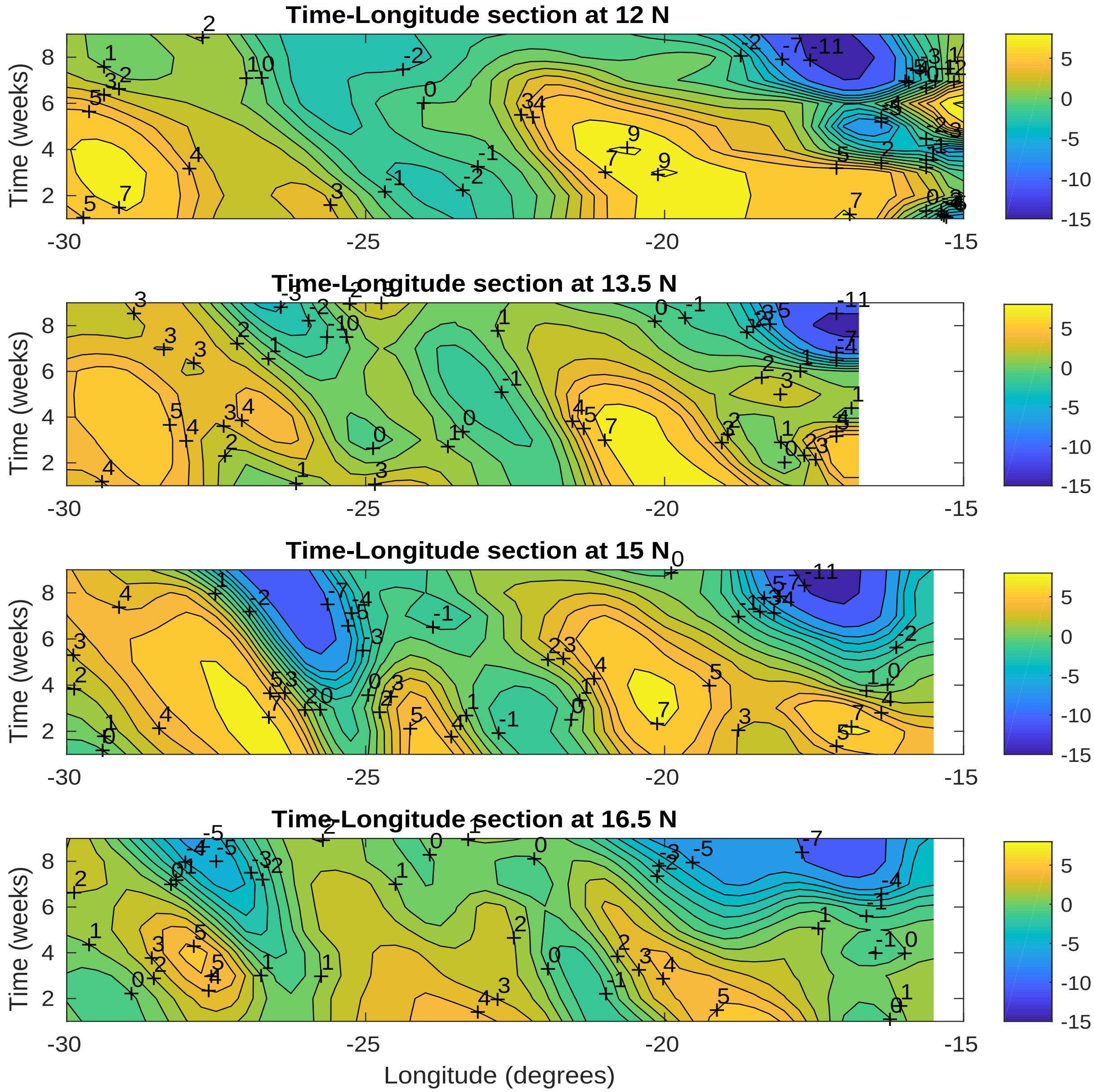

Figure 1: Hoevmuller diagrams of the SSH at latitudes 12°N, 13.5°N, 15°N, and 16.5°N (units: cm). Note the decrease of the amplitude at 16.5°N. The phase velocity of the wave is about 4.5 cm/s.

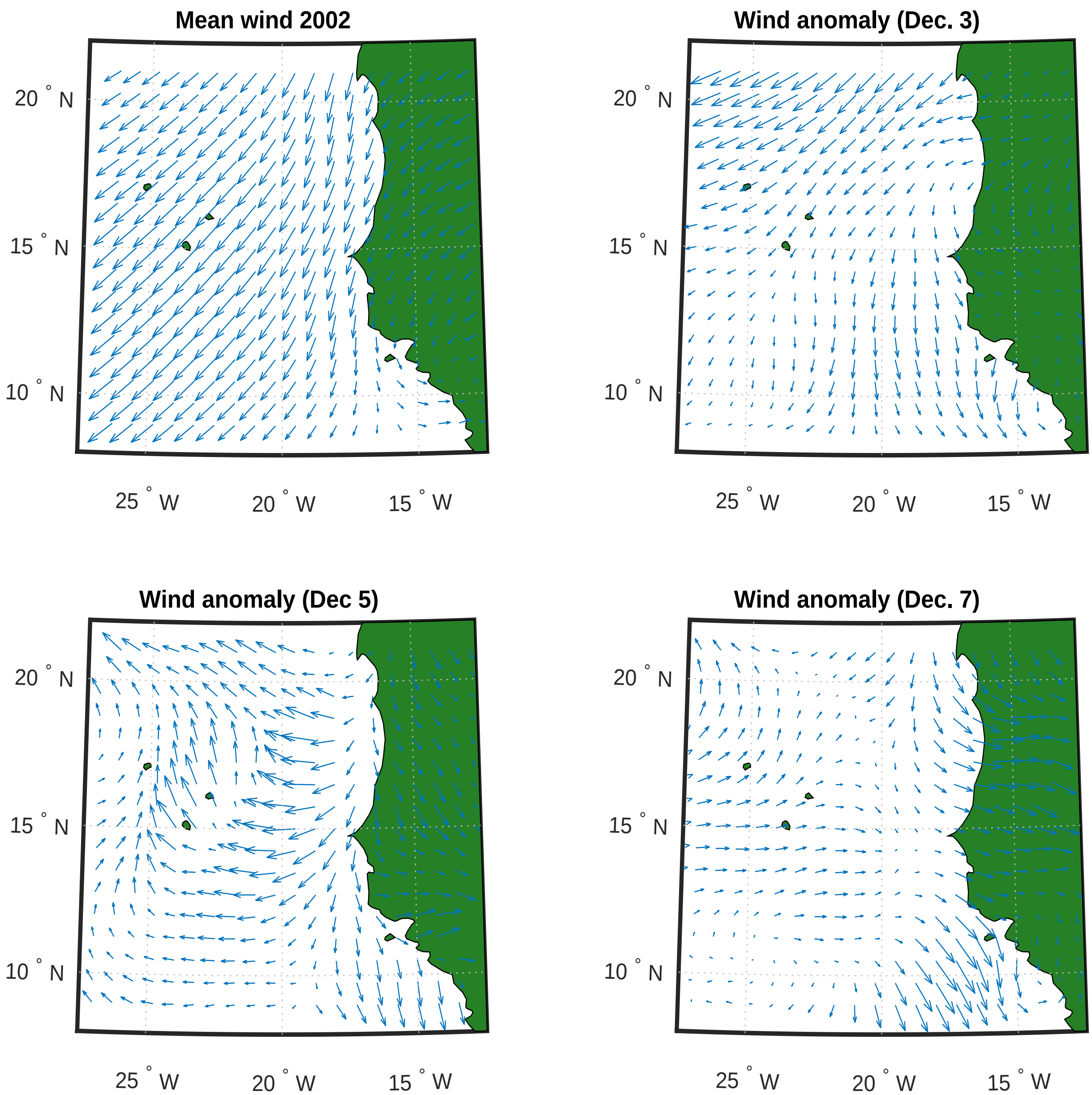

Figure 2: Mean wind velocity in December 2002 (maximum of about $\simeq$ 8 m s^{-1}) and wind velocity anomaly in December 3, 5, and 7 (maximum of about $\simeq$ 4 m s^{-1}).