



1 A monthly tidal envelope classification approach for semi-diurnal 2 regimes with variability in S_2 and N_2 tidal amplitude ratios

3 Do-Seong Byun¹, Deirdre E. Hart²

4 ¹Ocean Research Division, Korea Hydrographic and Oceanographic Agency, Busan 49111, Republic of Korea

5 ²School of Earth and Environment, University of Canterbury, Christchurch 8140, Aotearoa New Zealand

6 Correspondence to: Deirdre E. Hart (deirdre.hart@canterbury.ac.nz)

7 **Abstract.** In a world of increasing coastal inundation hazards, an understanding of daily through to monthly tidal envelope
8 characteristics is fundamental to resilient coastal management and development practices. For decades, scientists have
9 described and compared daily tidal forms around the world's coasts based on the four main tidal amplitudes. Our paper builds
10 on this 'daily' method by adjusting the constituent analysis to distinguish the different monthly types of tidal envelope
11 occurring in the semi-diurnal coastal waters around Aotearoa New Zealand. Analyses of tidal records from 23 stations are
12 used, alongside data from the FES2014 tide model database and theoretical experiments, in order to find the key characteristics
13 and constituent ratios of tides that can be used to classify monthly tidal envelopes. The resulting monthly tidal envelope
14 classification approach described (F_M^S) is simple, complementary to the successful and much used daily tidal form factor (F),
15 and of use for coastal flooding, climate change and maritime operation management and planning applications in semi-diurnal
16 regimes.

17

18 1 Introduction

19 Successful human-coast interactions in the world's low-lying areas are predicated upon understanding the temporal and spatial
20 variability of sea levels (Cartwright, 1999; D'Onofrio et al., 1999; Nicholls et al., 2007). This is particularly the case in island
21 nations like Aotearoa New Zealand (ANZ), where over 70% of the population reside in coastal settlements (Stephens 2015).
22 An understanding of tidal water level variations is fundamental to resilient inundation management and coastal development
23 practices in such places (Masselink et al., 2014; Olson, 2012; Pugh, 1996), as well as to accurately resolving non-tidal signals
24 of global interest, such as in studies of sea level change and gravimetry (Egbert et al., 1994; Stammer et al., 2014).

25 In terms of daily cycles, tidal form factors or form numbers (F) based on the amplitudes of the four main tidal constituents
26 (K_1 , O_1 , M_2 , S_2) have been successfully used to classify tidal observations from the world's coasts into four types of tidal
27 regime for nearly a century (Figure 1 a). Originally developed by van der Stok (1897), with a fourth category added by Courtier
28 (1938), these simple and useful form factors comprise the ratio between the combined K_1 and O_1 diurnal amplitudes versus
29 the combined M_2 and S_2 semi-diurnal amplitudes (Table 1). The resulting form factors classify tidal regimes into those which
30 roughly experience one high and one low tide per day (diurnal regimes); or two approximately equivalent high and low tides
31 per day (semidiurnal regimes); or two unequal high and low tides per day (mixed semidiurnal dominant or mixed diurnal
32 dominant regimes) (e.g. Defant 1958).

33 Albeit not part of their original design, some interpretation of the tidal envelope types observed at fortnightly and monthly
34 timescales has accompanied use of daily tidal form classifications (e.g. Pugh, 1996; Pugh & Woodworth, 2014). Whereas the
35 daily tidal form factor identifies the number and form (equal or mixed) of tidal height cycles typical within a lunar day (i.e. 24
36 hours and 48 minutes) at a particular site, a tidal envelope describes the maximum and minimum boundaries of tidal height
37 cycles occurring across a specified timescale at that site. The envelope timescale of interest in this paper is monthly.



38 Tidal envelopes at monthly scales depend on tidal regime. In general, semi-diurnal tidal regimes often feature two spring-neap
39 tidal cycles per synodic (lunar) month (Table 1). These two spring-neap tidal cycles are usually of unequal magnitude, due to
40 the effect of the moon's perigee and apogee, which cycle over the period of the anomalistic month. In contrast, diurnal tidal
41 regimes exhibit two pseudo spring-neap tides per sidereal month. For semi-diurnal regions where the N_2 constituent contributes
42 significantly to tidal ranges, tidal envelope classification should consider relationships between the M_2 , S_2 , and N_2 amplitudes.
43 The waters around ANZ represent one such region: here the daily tidal form is consistently semi-diurnal, but large differences
44 occur between sites within this region in terms of their typical tidal envelope types over fortnightly to monthly timescales.
45 The primacy placed on the four main amplitudes used in daily tidal form calculations has influenced the constituents examined
46 in comparisons between global tide models and satellite altimeter data (e.g. Andersen, 1995; Stammer et al., 2014),
47 emphasizing the importance of daily and spring-neap constituents. Far less attention has been paid to of the importance of
48 other constituents in modern tidal research. More than eighty years after the development of the ever-useful daily tidal form
49 factors, attention to the regional distinction between different tidal envelope types within the semi-diurnal category is also
50 needed, and forms the motivation for this paper. In this first explicit attempt to classify monthly tidal envelope types, we
51 examined the waters around ANZ, a strong semi-diurnal regime with relatively weak diurnal tides (daily form factor $F < 0.15$)
52 and variation in the importance of the S_2 and N_2 amplitude ratios. The result is an approach for classifying monthly tidal
53 envelope types that is transferable to any semi-diurnal regime. As well as providing greater understanding of the tidal regimes
54 of ANZ, we hope that our paper opens the door for new international interest in classifying tidal envelope variability at multiple
55 timescales, work which would have direct coastal and maritime management application including contributing to explanations
56 of the processes behind delta city coastal flooding hazards and their regional spatial variability.

57 **2 Methodology**

58 **2.1 Study area**

59 Aotearoa New Zealand is a long (1600 km), narrow (≤ 400 km) country situated in the south-western Pacific Ocean and
60 straddling the boundary between the Indo-Australian and Pacific plates. Its three main islands, Te Ika-a-Māui or the North
61 Island, Te Wai Pounamu or the South Island, and Rakiura or Stewart Island, span a latitudinal gradient between about 34° and
62 47° South. The tidal regimes in the surrounding coastal waters are semi-diurnal, with variable diurnal inequalities, and absolute
63 tides that span micro through to macro tidal ranges. Classic spring-neap cycles are present in western areas of ANZ, while
64 eastern areas feature distinct perigean-apogean influences (Byun and Hart, 2015; Heath, 1977, 1985; LINZ, 2017b; Walters et
65 al., 2001).

66 Highly complex tidal propagation patterns occur around ANZ, including a complete semi-diurnal tide rotation: contrary to the
67 southern hemisphere Coriolis Effect, the tide generally circulates around this country in an anti-clockwise direction. This
68 occurs due to the forcing of M_2 and N_2 tides by two amphidromes, situated northwest and southeast of the country, producing
69 trapped Kelvin waves; while the S_2 and K_1 tides exhibit a single wave front generated by an amphidrome to the southeast, plus
70 refraction of a trapped wave around the South Island. Around Te Moana-o-Raukawa or Cook Strait, the waterway between the
71 two main islands, tides travelling north along the east coast run parallel to tides travelling south along the west coast. The
72 pronounced differences between these east/west tidal states, combined with their tidal range differences, together produce
73 marked differences in amplitude and strong current flows through Cook Strait (Heath, 1985; Walters et al., 2001, 2010).

74 **2.2 Data analysis approach**

75 Year-long sea level records were sourced from a total of 23 stations spread around ANZ (Figure 2): eighteen 1 minute-interval
76 records from Land Information New Zealand (LINZ, 2017a); and five 1 hour-interval records from the National Institute of
77 Water and Atmospheric Research (NIWA, 2017). For both the LINZ and NIWA data, years with good quality hourly data



78 were selected for analysis from amongst multi-year records. The 23 tidal records were harmonically analyzed using T_Tide
79 (Pawlowicz et al., 2002) to examine spatial variation in the main tidal constituents' amplitudes, phase-lags, and amplitude
80 ratios, between regions and in comparison with their tidal potential values from Equilibrium Theory (see Table A1 for raw
81 results). An additional set of tidal constituent amplitude data was sourced from Tables 1 and 3 of Walters et al. (2010), derived
82 from 33 records of between 14 and 1900 days length, from around the greater Cook Strait area between ANZ's two main
83 islands, where spring-neap tides reach the strongest in the country.

84 We then classified the monthly tidal envelope types found around ANZ based on detailed examination of constituent ratios
85 produced from the tidal harmonic analysis results, as well as data from the FES2014 tide model (see Carrere et al., 2016 for a
86 full description of this database), and experimental plots of the different tidal envelope types generated from this constituent
87 data. Due to the strong semi-diurnal tidal regimes in the study area, and similar to the approach of Walters et al. (2010), we
88 were able to ignore sidereal (K_1 , O_1) effects and simply consider the effects of spring-neap (M_2 , S_2) and perigean-apogean
89 cycles (M_2 , N_2) in our monthly tidal envelope type characterization.

90 3 Results

91 3.1 Key tidal constituent amplitudes and amplitude ratios

92 In order to better understand the key constituents responsible for shaping tidal height forms around ANZ, we first mapped
93 variability in the amplitudes of the semi-diurnal and diurnal constituents listed in Table 1 (Figure 3) and of the ratio values of
94 the semi-diurnal constituent amplitudes (Figure 4). Table 2 summarizes these data, and contrasts them with those from Defant's
95 (1958) Equilibrium Theory, while Table A1 catalogues the detailed data results.

96 Tidal amplitude ratio comparisons confirmed that the waters around ANZ are dominated by the three astronomical semi-
97 diurnal tides: M_2 , S_2 and N_2 (Table 2), the combination of which can generate fortnightly spring-neap tides (M_2 and S_2) and
98 monthly perigean-apogean tides (M_2 and N_2). Figure 3 reinforces the relatively minor magnitudes of diurnal constituent
99 amplitudes (O_1 , K_1), as well as revealing the stronger west coast amplitudes of the spring-neap cycle generating constituents
100 (M_2 and S_2), the relatively weak S_2 amplitudes overall (half that of Equilibrium Theory), and the more concentric pattern
101 around ANZ of the perigean-apogean cycle generating N_2 amplitude.

102 In terms of the semi-diurnal constituent amplitude ratios, Figure 4 and Table 2 show that $\frac{a_{S_2}}{a_{M_2}}$ values cover a broad range around
103 ANZ (0.04 to 0.47), with most sites exhibiting relatively smaller values (<0.27 at 22 out of 23 sites) than that of Equilibrium
104 Theory (0.466). In contrast, $\frac{a_{N_2}}{a_{M_2}}$ ratios were found to be more stable around ANZ (values ranging from 0.16 to 0.23) and similar
105 in magnitude to Equilibrium Theory (i.e. 0.191). By grouping the constituent amplitude and amplitude ratio results (Figures 3
106 to 4), we were able to distinguish four distinct monthly tidal envelope regimes around ANZ (Table 2).

- 107 • Firstly, 'spring-neap' type tidal regimes occur where the S_2 tide amplitude is large compared to that of the N_2 (Table
108 2, Figure 3). In these areas there are two spring-neap tides per month with similar ranges, and negligible influence of
109 perigean-apogean cycles. Such a regime occurs in the Kapiti and Cook Strait area (Figure 1), where the N_2 and M_2
110 amplitudes reduce by 75 to 90%, but the S_2 amplitude reduces by only about 30%, compared to on adjacent coasts.
- 111 • In direct contrast, there are 'perigean-apogean' type tidal regimes, in areas where the N_2 amplitude strongly
112 dominates over the S_2 (Table 2, Figure 3). In this type of tidal regime, the M_2 and the N_2 tides combine to produce
113 strong signals over anomalistic timeframes (27.5546 days). Hence the highest tidal ranges in any given month occur
114 in relation to the perigee, when the moon's orbit brings it close to Earth, rather than in line with the moon's phase, as
115 is typical in spring-neap regimes. This type of regime occurs, for example, around the northern Chatham Rise near
116 Kaikoura, and as far north as Castle Point on the east coast of the South Island.



118 The remaining coastal waters around ANZ can be separated into two tidal sub-regions, one with strong spring-neap signals
 119 and the other with strong perigean-apogean signals, but both with overall mixed or *intermediate* monthly tidal envelope types
 120 (Table 2). We distinguished these two envelope types via the combined variability of the ratios of $\frac{a_{S_2}}{a_{M_2}}$ and $\frac{a_{N_2}}{a_{M_2}}$ (i.e. of the
 121 spring-neap cycle; and perigean-apogean cycle forming tides, respectively). By examining these ratios we take account of the
 122 moderating influence of the M_2 tide at both synodic and anomalistic timeframes. In brief, the $\frac{a_{S_2}}{a_{M_2}}$ and $\frac{a_{S_2}}{a_{N_2}}$ ratios vary widely
 123 around ANZ, with highest values in the west, lowest values in the east, and intermediate values to the north and south (Figure
 124 4). By comparison, $\frac{a_{N_2}}{a_{M_2}}$ values are relatively stable and high, except in a relatively small area of central Cook Strait, where this
 125 ratio drops and thus spring-neap cycles predominate (see ‘spring-neap’ type regimes above). The combined variability in these
 126 two ratios means that, except where we find ‘perigean-apogean’ or ‘spring-neap’ type monthly tidal envelope types, spring-
 127 neap tides do occur but the overall monthly envelope shape is fundamentally altered (asymmetrically) due to the perigean-
 128 apogean influence.

- 129
- 130 • In the first of the ‘intermediate’ monthly envelope sub-regions, tides exhibit two dominant, but unequal, spring-neap
 131 cycles per month due to a subordinate, but still influential, perigean-apogean effect. We term this type of monthly
 132 tidal envelope an ‘*intermediate, predominantly spring-neap*’ type regime. Here values of $\frac{a_{S_2}}{a_{N_2}}$ are ≥ 1 , with S_2
 133 amplitudes reaching only around 17 to 50% those of the M_2 constituent (Figures 3 to 4; Table 2). Also in these areas,
 134 values of $\frac{a_{S_2}+a_{N_2}}{a_{M_2}}$ are ≥ 0.45 . This type of tide occurs, for example, at the Westport and Puysegur sites.
- 135
- 136 • In the other ‘intermediate’ monthly envelope sub-region, tides exhibit a mainly perigean-apogean form with a weaker,
 137 but noticeable, spring-neap signal: we term this envelope type as ‘*intermediate, predominantly perigean-apogean*’.
 138 Here values of $\frac{a_{S_2}}{a_{N_2}}$ sit between 0.3 and <1 , while values of $\frac{a_{S_2}+a_{N_2}}{a_{M_2}}$ are 0.3 to 0.4 (Figure 4, Table 2). This type of
 139 tide occurs, for example, at the Auckland and Sumner sites.

140 Figure 5 illustrates the four types of monthly tidal envelope found around ANZ as idealized types, two with stronger spring-
 141 neap signals (hereafter referred to as Types 1 and 2, see Figure 5 a-b) and two with stronger fortnightly perigean-apogean
 142 signals (hereafter Types 3 and 4, see Figure 5 c-d).

143 3.2 A monthly tidal envelope factor (F_M^S) for semi-diurnal regimes

144 The four types of monthly tidal envelope types found around ANZ are essentially different combinations of spring-neap and
 145 perigean-apogean signals. Thus, in a similar manner to van der Stok’s (1897) method for calculating *daily* tidal form factors,
 146 a *monthly* tidal envelope factor (F_M^S) may be calculated for semi-diurnal tidal regions, including that of ANZ, according to:

$$147 F_M^S = \frac{a_{M_2} + a_{N_2}}{a_{M_2} + a_{S_2}}, \quad (1)$$

148 which can be further expressed as:

$$149 F_M^S = \frac{1 + \frac{a_{S_2}x}{a_{M_2}}}{1 + \frac{a_{S_2}}{a_{M_2}}}, \quad \text{with } x = \frac{a_{N_2}}{a_{S_2}} \quad (1a)$$

150 for areas characterized by more stable (e.g., lower variability) values of $\frac{a_{S_2}}{a_{M_2}}$ compared to $\frac{a_{N_2}}{a_{S_2}}$, or as:

$$151 F_M^S = \frac{1 + \frac{a_{N_2}}{a_{M_2}}}{1 + \frac{a_{N_2}y}{a_{M_2}}}, \quad \text{with } y = \frac{a_{S_2}}{a_{N_2}} \quad (1b)$$

152 for areas characterized by more stable (e.g., lower variability) values of $\frac{a_{N_2}}{a_{M_2}}$ compared to $\frac{a_{S_2}}{a_{N_2}}$.



153 F_M^S takes into account the roles of the S_2 and N_2 tides in spring-neap and perigean-apogean cycles, while also factoring in the
154 strong M_2 tide influence in both types of cycle. F_M^S may be used to classify the monthly tidal envelope types of any semi-
155 diurnal region (i.e. where $F < 0.25$) based on the analysis of constituent amplitudes and ratios from local data. Below we explain
156 the four steps undertaken to successfully set the boundaries between different monthly tidal envelope types, thereby classifying
157 the region's tides, using our ANZ case study data.

158 **Step 1: Separating regimes dominated by spring-neap versus perigean-apogean signals**

159 Fundamentally, in any semi-diurnal tidal regime ($F < 0.25$) anywhere in the world where $\frac{a_{N_2}}{a_{S_2}} < 1$, spring-neap cycles will be a
160 clear feature of the tidal height records (Table 1). This applies to the waters around ANZ. Thus, we set an initial boundary
161 between different monthly tidal envelope types at $\frac{a_{N_2}}{a_{S_2}} = 1$ (Table 4). That is, regimes where $\frac{a_{N_2}}{a_{S_2}} < 1$ feature stronger spring-
162 neap cycles, while regimes with $\frac{a_{N_2}}{a_{S_2}} > 1$ feature stronger perigean-apogean signals. As summarized in Table 2, compared to
163 areas that experience stronger spring-neap influences, areas of the ANZ coast with stronger perigean-apogean influences are
164 characterized by relatively smaller S_2 amplitudes (2-18 cm), with stronger N_2 amplitudes (10-22 cm).

165 **Step 2: Separating regimes with consistent versus irregular and unequal spring-neap**

166 Tidal regimes with stronger spring-neap signals (i.e. where $\frac{a_{N_2}}{a_{S_2}} < 1$) include places where spring-neap cycles occur as
167 consecutive fortnightly cycles of similar magnitude (hereafter labelled Type 1 or 'spring-neap' type regimes), and places where
168 spring-neap signals dominate but with noticeable variability in the magnitudes of consecutive cycles due to subordinate
169 perigean-apogean influences (hereafter labelled Type 2 or 'intermediate, spring-neap' regimes). In ANZ the strongest spring-
170 neap influence occurs in the greater Cook Strait area, including at Kapiti where harmonic analysis revealed $\frac{a_{N_2}}{a_{S_2}} = 0.35$ (Table
171 A1).

172 To set a boundary between Types 1 and 2 in any semi-diurnal tidal regimes around the world (and between Types 3 and 4 as
173 explained below), it was necessary to take account of the moderating influence of the M_2 amplitude compared to the
174 magnitudes of the S_2 and N_2 amplitudes, since the M_2 constituent influences monthly tidal envelopes at both synodic (spring-
175 neap) and anomalistic (perigean-apogean) timescales (Table 1). In order to do this, experiments were conducted to explore
176 two additional ratios:

- 177 i. the ratio of the 'annual' maximum tidal range to the subsequent tidal range (MTR); and
- 178 ii. the ratio of the 'annual' maximum spring tide range to the subsequent spring tide range (MSR).

179 We determined that the monthly tidal envelope boundary (F_M^S) between spring-neap (Type 1) and intermediate, spring-neap
180 dominant (Type 2) regimes would occur at point where the MTR and MSR tidal range ratios exhibited the same value. 360
181 days of synthetic tidal range data were generated under conditions of $F = 0.25$ (the boundary between 'semi-diurnal' and 'mixed,
182 mainly semi-diurnal' type daily forms); and $a_{M_2} = 3a_{S_2}$; $a_{K_1} = a_{O_1}$; and $g_{M_2} = g_{S_1} = g_{K_1} = g_{O_1} = 0^\circ$ (a subset of the
183 assumptions employed by Courtier, 1938). When $F = 0.25$, calculations revealed that the MTR value was 0.795. When the MSR
184 ratio was set to the same value (0.795), calculations revealed that the F_M^S value was 0.795 (Figure A1). Based on this value, a
185 review of our observation records revealed that the Kapiti site, with its F_M^S value of 0.79, exhibited the only completely spring-
186 neap dominated site amongst our ANZ records. Hence we found that the boundary between monthly tidal envelope Types 1
187 and 2 in ANZ would sit somewhere between the tidal regimes of Kapiti and the site with the next strongest spring-neap
188 influence, Manukau, where $F_M^S = 0.93$.



190 **Step 3: Separating regimes with ‘perigeen-apogean’ and ‘intermediate, perigeen-apogean dominated’ monthly tidal**
191 **envelopes**

192 Amongst our case study sites, areas with the most extreme perigeen-apogean signal typically exhibited $\frac{a_{S_2}}{a_{M_2}}$ values of about
193 0.04 to 0.05 (Table A1). In order to determine the boundary between ‘perigeen-apogean’ and ‘intermediate, perigeen-apogean
194 dominant’ regimes (i.e. Types 3 and 4), we conducted 31 experiments each generating 360 days of synthetic tidal ranges based
195 on the fixed condition of $\frac{a_{S_2}}{a_{M_2}} = 0.05$, but with $\frac{a_{N_2}}{a_{S_2}}$ values ranging from 3 to 6 at intervals of 0.01 using Eq. (1a). Examining the
196 shapes of the resultant monthly tidal envelopes, we were able to set a boundary value between Types 3 and 4 regimes at
197 $F_M^S = 1.15$ in NZ waters (Table 4, Figure 6).

198 In summary, Figure 7 illustrates the classification of monthly tidal envelope types in the waters around ANZ using F_M^S . We
199 find the west coast is characterized by Type 2 monthly tidal envelopes, with two unequal spring-neap cycles per month. Type
200 1 monthly tidal envelopes, with their defined spring-neap tides, are only found in the Cook Strait area. This area’s defined
201 spring-neap tides were explored in detail by Walters et al. (2010) - Figure 6 includes a re-analysis of their data using the F_M^S
202 ratios. In contrast, the central east coast shows Type 4, perigeen-apogean tidal envelopes, which is unusual for semi-diurnal
203 regimes internationally (i.e. Figure 1c). Type 3 or intermediate, perigeen-apogean dominated monthly tidal envelopes are found
204 in the rest of the waters surrounding ANZ.

205 **4 Discussion and conclusion**

206 Daily tidal water level variations are a key control on shore ecology; access to marine environments via boat and shipping
207 infrastructure such as ports, jetties and wharves; drainage links between the ocean and coastal hydrosystems such as lagoons
208 and estuaries; and the duration and frequency of opportunities to access the intertidal zone for recreation and food harvesting
209 purposes. Fortnightly and monthly tidal envelope variations, such as those associated with spring-neap and perigeen-apogean
210 cycles, have similar moderating roles on human usage of intertidal and shoreline environments, and additionally these medium
211 term variations in tide levels are important factors in coastal inundation risks (Menéndez & Woodworth, 2010; Stephens 2015;
212 Stephens et al., 2014; Wood, 1978, 1986;). High perigeen-spring tides, for example, interact with extreme weather events
213 (including low pressures, strong winds and extreme rainfall) to produce significant coastal inundation in low-lying coastal
214 settlements such as on deltas (Hart et al., 2015).

215 In a world of rising sea levels, and coastal inundation hazard cascades (Menéndez and Woodworth, 2010), having common
216 ways of describing different types of tidal envelope is essential for living safely and productively in coastal cities. This paper
217 has employed observations from ANZ, FES2014 tidal data, and theoretical experiments, to demonstrate a simple approach to
218 classifying different monthly tidal envelope types, applicable to semi-diurnal regions anywhere. The result is a widely
219 applicable monthly tidal envelope factor (F_M^S) for classifying semi-diurnal regimes based on the amplitudes and amplitude
220 ratios of three key constituents operating at synodic anomalistic timescales (M_2 , S_2 , and N_2).

221 At a very basic level, in any semi-diurnal tidal regime anywhere in the world where the value of $\frac{a_{N_2}}{a_{S_2}} < 1$, then spring-neap
222 cycles will be clearly visible in tidal height records, either as consecutive fortnightly cycles of similar magnitude (Type 1), or
223 as a dominant signal with noticeable variability in the magnitudes of consecutive fortnightly cycles, due to a subordinate
224 perigeen-apogean influence (Type 2). Conversely, in semi-diurnal areas of the world’s oceans where $\frac{a_{N_2}}{a_{S_2}} > 1$, then perigeen-
225 apogean cycles will be clearly visible, either as singularly evident monthly cycles (Type 4), or as a dominant influence with
226 subordinate spring-neap signals (Type 3). As illustrated in Sect. 3.2, quantitatively determining the actual boundaries between
227 monthly tidal envelope Types 1 versus 2, and Types 3 versus 4 regimes at a local scale involves analysis of observational data,
228 taking into account the moderating influence of the M_2 amplitude compared to the magnitudes of the S_2 and N_2 amplitudes.



229 Figure 1b illustrates the division of the semi-diurnal areas of the world's oceans into those where spring-neap cycles are the
230 main monthly tidal envelope influence versus those where the perigean-apogean signal is paramount, while Figure 1c illustrates
231 areas of the world's oceans where spring-neap signals are minor compared to 'perigean-apogean' influences in the monthly
232 tidal envelope. The potentially predictable but relatively lower frequency tidal water level fluctuations such as those in our
233 perigean-apogean monthly envelope classes are an important cause and moderator of coastal inundation hazards in different
234 locations around the world (e.g. Wood 1978, 1986; Stephens 2015).

235 The simple approach to classifying monthly tidal envelope types in semi-diurnal regions demonstrated in this paper
236 complements the existing, commonly used way of describing daily tidal forms based on the amplitudes of the four key, diurnal
237 (K_1 , O_1) and semi-diurnal (M_2 , S_2) constituents (e.g. Defant 1958). We hope that our work inspires other efforts to study tidal
238 height variations at timescales greater than daily, work which could draw renewed attention to the fundamental role of tidal
239 water levels in shaping coastal environments, including in hazards such as low-frequency coastal flooding.

240 **Data Availability**

241 The tidal data used in this paper are available from LINZ (2017a; 2017b), NIWA (2017) and Walters et al. (2010). Details of
242 the FES2014 tide model database are found in Carrere et al. (2016) and via
243 <https://www.aviso.altimetry.fr/en/data/products/auxiliary-products/global-tide-fes.html>). Appendix 1 contains the data
244 produced from analysis of these primary resources in this paper.

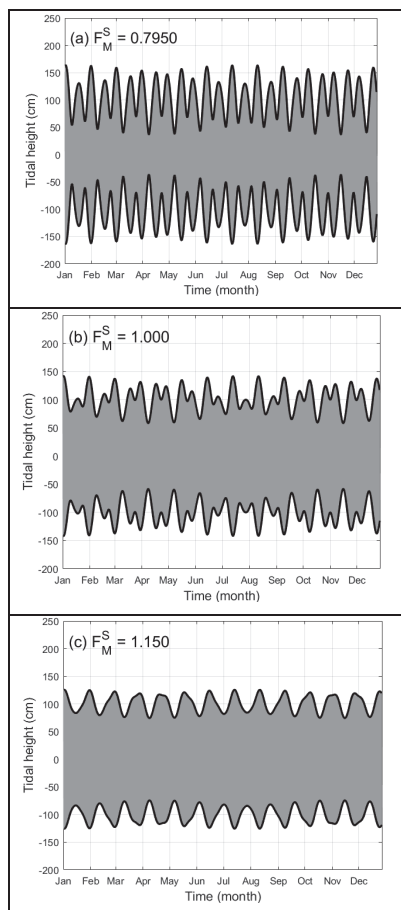
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Appendix 1

Table A.1. Values for 5 tidal harmonic constants, tidal ranges, form factors (F), and monthly tidal envelope factor (F_M^S) for a semi-diurnal regime used in classifying tidal envelope forms, for 23 stations around New Zealand, and compared to values derived from Equilibrium Theory

Station name (record used)	M_2		S_2		N_2		K_1		O_1		F	F_M^S	Type
	α_i (cm)	g_i (deg)	α_i (cm)	g_i (deg)	α_i (cm)	g_i (deg)	α_i (cm)	g_i (deg)	α_i (cm)	g_i (deg)			
Equilibrium Theory	-	-	-	-	-	-	-	-	-	-	0.68	-	-
Kapiti (2011)	55	280	26	336	9	277	2	195	2	18	0.05	0.790	1
Manukau (2011)	109	297	29	332	20	287	6	17	1	287	0.05	0.935	2
Westport (2015)	113	309	29	348	23	287	2	198	3	40	0.04	0.958	2
Charleston (2015.7.-2016.1)	106	319	27	344	22	304	3	6	3	243	0.05	0.962	2
Puysegur Point (2012)	78	350	19	13	17	335	3	316	4	245	0.07	0.979	2
North Cape (2010)	80	230	15	279	16	209	8	10	2	351	0.11	1.011	3
Boat Cove, Raoul Island (2012)	50	208	9	287	10	176	5	43	3	44	0.14	1.017	3
Dog Island (2011)	91	33	18	57	21	6	2	119	4	60	0.06	1.028	3
Auckland (2011)	112	216	17	275	22	192	7	356	2	324	0.07	1.039	3
Fishing Rock, Raoul Island (2011)	52	206	8	283	11	178	5	35	2	41	0.12	1.050	3
Lofton Point (2011)	70	195	9	262	14	168	6	352	2	328	0.1	1.063	3
Tauranga (2011)	70	211	9	277	14	186	5	360	1	330	0.08	1.063	3
Korotiti Bay (2011)	78	207	11	265	16	181	6	349	1	317	0.08	1.056	3
Moturiki (2011)	73	189	10	265	15	156	5	173	1	136	0.07	1.060	3
Green Island (2011)	73	81	10	91	17	50	3	93	4	44	0.08	1.084	3
Port Chalmers (2011)	77	112	9	112	17	89	3	270	3	247	0.07	1.093	3
Summer (2011)	84	136	6	151	18	109	5	273	3	245	0.09	1.133	3
Gisborne (2010)	64	176	5	251	14	148	4	336	1	275	0.07	1.130	3
Napier (2011)	64	167	4	240	14	138	3	298	2	221	0.07	1.147	3
Kaikoura (2011)	65	146	3	171	14	117	4	275	4	233	0.12	1.162	4
Oxwanga, Chatham Island (2011)	48	149	2	224	10	119	2	246	2	179	0.08	1.160	4
Castlepoint (2011)	63	159	3	225	14	129	3	280	3	219	0.09	1.167	4
Wellington (2011)	49	148	2	352	11	116	2	268	3	219	0.1	1.176	4
Overall range	48-113	33-350	2-29	13-352	9-23	6-335	2-8	6-360	1-4	40-351	0.04-0.14	0.79-1.176	1-4



247 **Figure A1.** Monthly tidal forms at the boundaries of the (a) ‘spring-neap’ versus ‘intermediate, spring-neap dominant’ tidal forms; at
248 the (b) ‘intermediate, spring-neap dominant’ versus ‘intermediate, perigean-apogean dominant’ tidal forms; and at the (c)
249 ‘intermediate, perigean-apogean dominant’ versus ‘perigean-apogean’ tidal forms, produced using the conditions summarized in
250 Table A1.

251



252 **Author contribution**

253 Both authors conceived of the idea behind this paper. DH produced the initial manuscript draft. D-SB analyzed the tidal data
254 and wrote the results sections. Both authors worked on and finalized the full manuscript.

255 **Competing interests**

256 The authors declare that they have no conflict of interest.

257

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- 322
- 323



324 **Table 1. Tidal constituent pairs associated with different monthly tidal envelope contributors and their intervals, including their**
325 **cycle types and controlling factors**

Constituent pairs	Interval (days)	Cycle type	Control
M ₂ , S ₂	14.7653	spring-neap	Moon phase, i.e. the axial alignment of Moon and Sun relative to Earth during the synodic month.
M ₂ , N ₂	27.5546	perigean-apogean	Relative distance between the Moon and Earth throughout the Moon's orbit over the anomalistic month.
K ₁ , O ₁	13.6608	tropic-equatorial	Changes in the Moon's declination during the sidereal month.

326 *Note.* With monthly tidal envelope characterization, the N₂ is considered in addition to the constituents included in daily tidal form
327 classification (e.g. Defant 1958).

328



Table 2. Comparison of tidal constituent amplitudes, amplitude ratios (including daily tidal form factor, F , and monthly tidal envelope factor, F_M) and ranges between the four distinct types of monthly tidal envelope found in the 23 case study semi-diurnal tide regimes of Aotearoa New Zealand, and compared to Equilibrium Theory amplitude ratios

Monthly tidal envelope description	$F \left(\frac{K_1 + O_1}{M_2 + S_2} \right)$	Amplitudes (cm) of						Amplitude ratios of					Example sites	
		M_2	S_2	N_2	O_1	K_1	O_1	$\frac{S_2}{M_2}$	$\frac{N_2}{M_2}$	$\frac{S_2 + N_2}{M_2}$	$F_M \left(\frac{M_2 + N_2}{M_2 + S_2} \right)$	$\frac{K_1}{M_2}$		$\frac{O_1}{M_2}$
n/a	0.68 mixed diurnal	-	-	-	-	-	-	0.47	0.19	0.66	-	0.584	0.415	Equilibrium Theory*
Spring-neap type	0.05 semi-diurnal	55	26	9	2	2	0.47	0.16	2.89	0.64	0.79	0.04	0.04	Kapiti
Intermediate, spring-neap dominant	0.04 to 0.07 semi-diurnal	78 to 113	19 to 29	17 to 23	1 to 4	2 to 6	0.24 to 0.27	0.18 to 0.22	1.12 to 1.45	0.45 to 0.46	0.93 to 0.98	0.02 to 0.06	0.01 to 0.05	Māpūkai, Westport, Charleston, Pigeon Point, North Cape, Boat Cove and Fishing Rock (Raoul Island), Dog Island, Auckland, Loffin Point, Tauranga, Korōtiti Bay, Moturiki, Green Island, Port Chalmers, Sumner, Gisborne, Napier, Kaikoura, Otago, Castlepoint, Wellington
Intermediate, perigeon-apogean dominant	0.06 to 0.14 semi-diurnal	50 to 112	4 to 18	10 to 22	1 to 4	2 to 8	0.06 to 0.2	0.2 to 0.23	0.29 to 0.94	0.28 to 0.43	1.01 to 1.15	0.02 to 0.10	0.01 to 0.06	
Perigeon-apogean type	0.08 to 0.12 semi-diurnal	48 to 65	2 to 3	10 to 14	2 to 4	2 to 4	0.04 to 0.05	0.21 to 0.22	0.18 to 0.21	0.25 to 0.27	1.16 to 1.18	0.04 to 0.06	0.04 to 0.06	

Note. Data sourced from this research, and from Defant (1958)* (for tidal station details, including tidal phase-lags, see Appendix 1 Table A1).



330 **Table 3. Monthly tidal envelope factor (F_M^S) for classifying different monthly tidal types in Aotearoa New Zealand’s semi-diurnal**
 331 **tidal regime**

F_M^S	Type	Name	Description of monthly tidal envelope
< 0.795	1	Spring-neap type	Two similar magnitude spring-neap cycles.
> 0.795 and < 1.0	2	Intermediate, spring-neap dominated	Two unequal spring-neap cycles.
> 1.0 and < 1.15	3	Intermediate, perigeen-apogean dominated	A strong perigeen-apogean cycle plus two weaker spring-neap cycles.
> 1.15	4	Pperigeen-apogean type	A distinct perigeen-apogean cycle.

332
 333

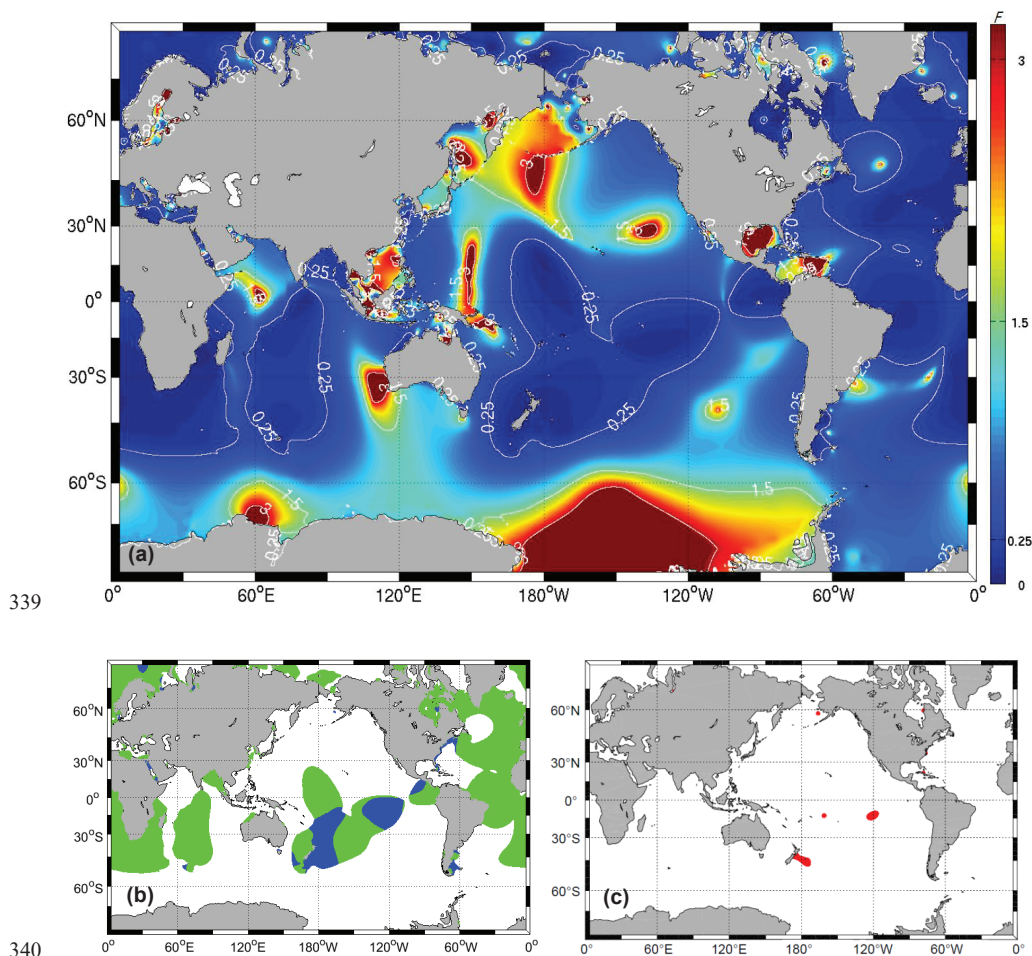


334 **Table 4. Typical (representative) input and boundary values for classifying monthly tidal envelope types around the Aotearoa New**
 335 **Zealand coast**

F_M^S boundary case	Eq. (1)		F_M^S	Notes
	Given value	x or y		
Type 1 vs. Type 2	$\frac{a_{S_2}}{a_{M_2}} = 0.47$	$x = 0.36$	0.8	When $MSR \approx MTR$ with $F=0.25$, then the value of F_M^S : $a_{S_2} = 2.78 a_{N_2}$ $a_{S_2} = a_{N_2}$ $a_{S_2} = 0.249 a_{N_2}$
Type 2 vs. Type 3	$\frac{a_{N_2}}{a_{M_2}} = 0.21$	$y = 1$	1	
Type 3 vs. Type 4	$\frac{a_{S_2}}{a_{M_2}} = 0.05$	$x = 4.024$	1.15	

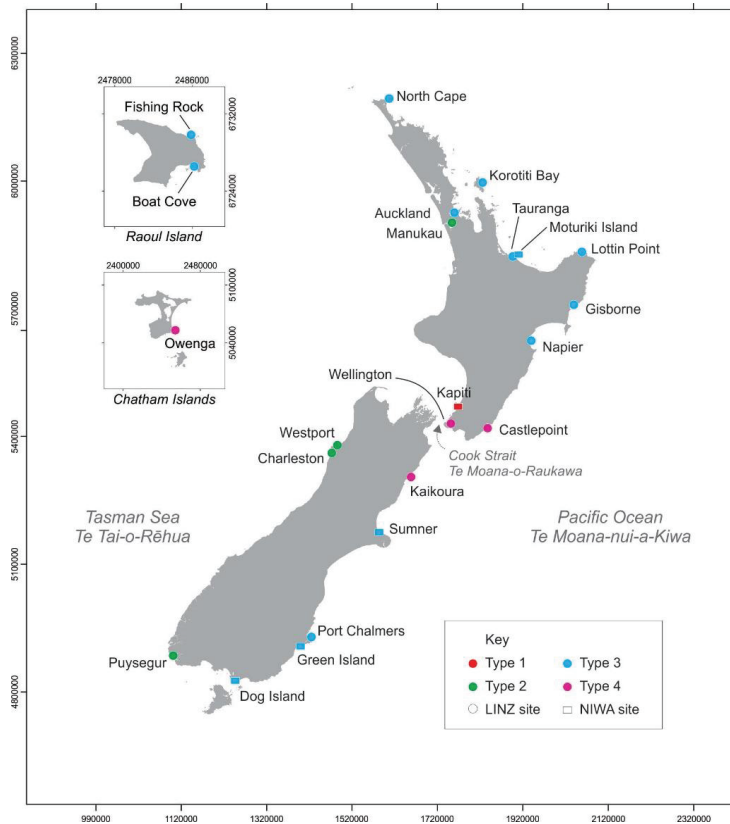
336 *Note.* $x = \frac{a_{N_2}}{a_{S_2}}$ and $y = \frac{a_{S_2}}{a_{N_2}}$. MSR and MTR denote the ratio between the maximum spring tide range to the subsequent spring tide
 337 range and the ratio between the ‘annual’ maximum tidal range to the subsequent tidal range, respectively.

338



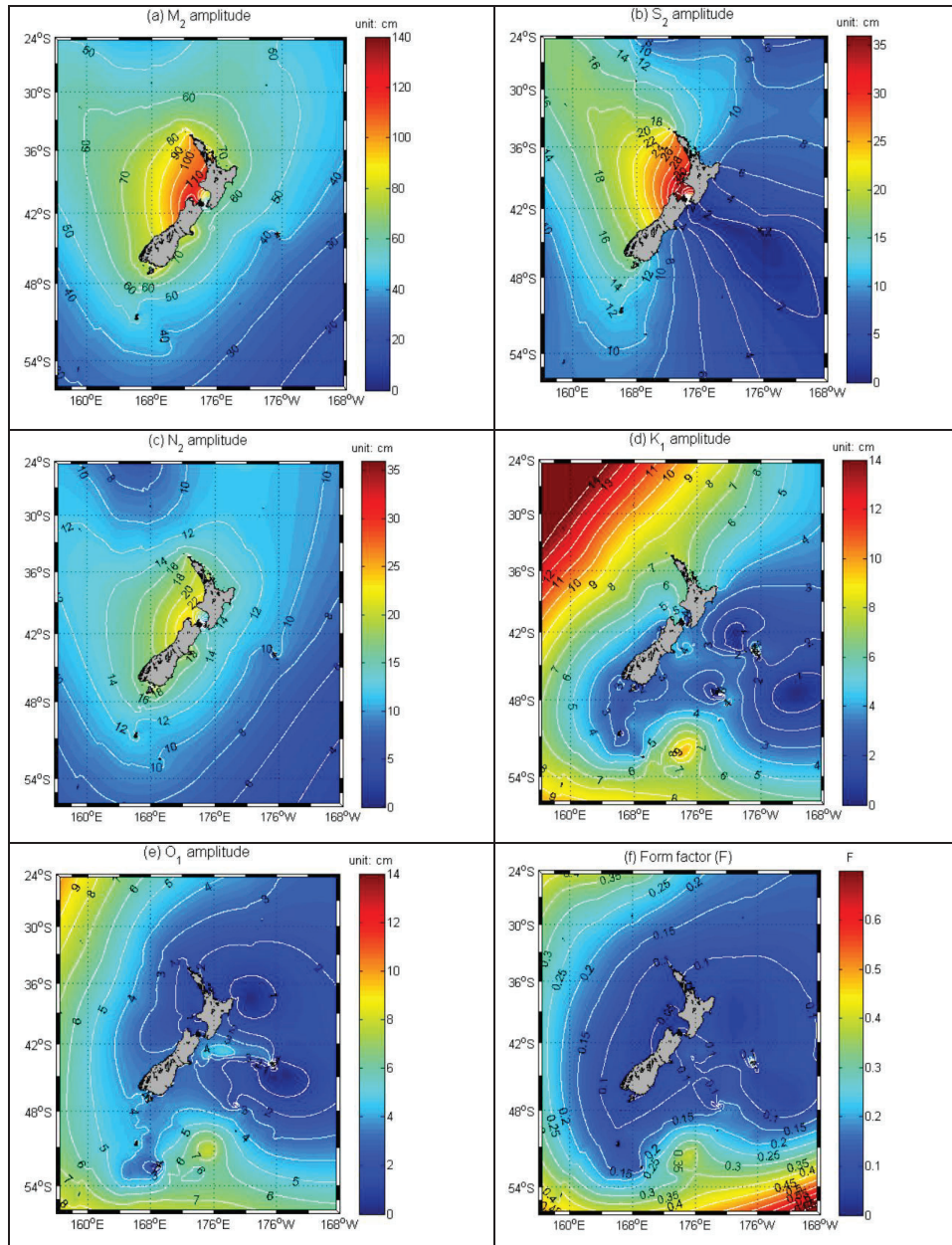
342 **Figure 1.** (a) Global distribution of daily form factor (F) values, indicating daily tidal regime types ($F < 0.25$: semi-diurnal; $F > 0.25$ to
343 $F < 1.5$ mixed-mainly semi-diurnal; $F > 1.5$ to $F < 3$: mixed-mainly diurnal; and $F > 3$: diurnal, according to the classification of van der
344 Stok 1897, and Courtier 1938); (b) the world's semi-diurnal tidal areas ($F < 0.25$) divided into those where spring-neap (green) versus
345 perigean-apogean (blue) signals are the main influence on the monthly tidal envelope; and (c) semi-diurnal tidal regimes (in red)
346 where the S_2/M_2 constituent amplitude ratio is < 0.04 and thus spring-neap tidal signals are very weak so that perigean-apogean
347 signals are prominent, as derived from FES2014 tidal harmonic constants.

348



349
350 **Figure 2.** Location of 23 Aotearoa New Zealand sea level observation stations investigated in this research: circles indicate LINZ
351 sites, rectangles indicate NIWA sites; each site is colored according to monthly tidal envelope type. Offshore islands are not shown
352 to scale (Raoul & Chatham Islands). The coordinate system is NZGD2000, Transverse Mercator.

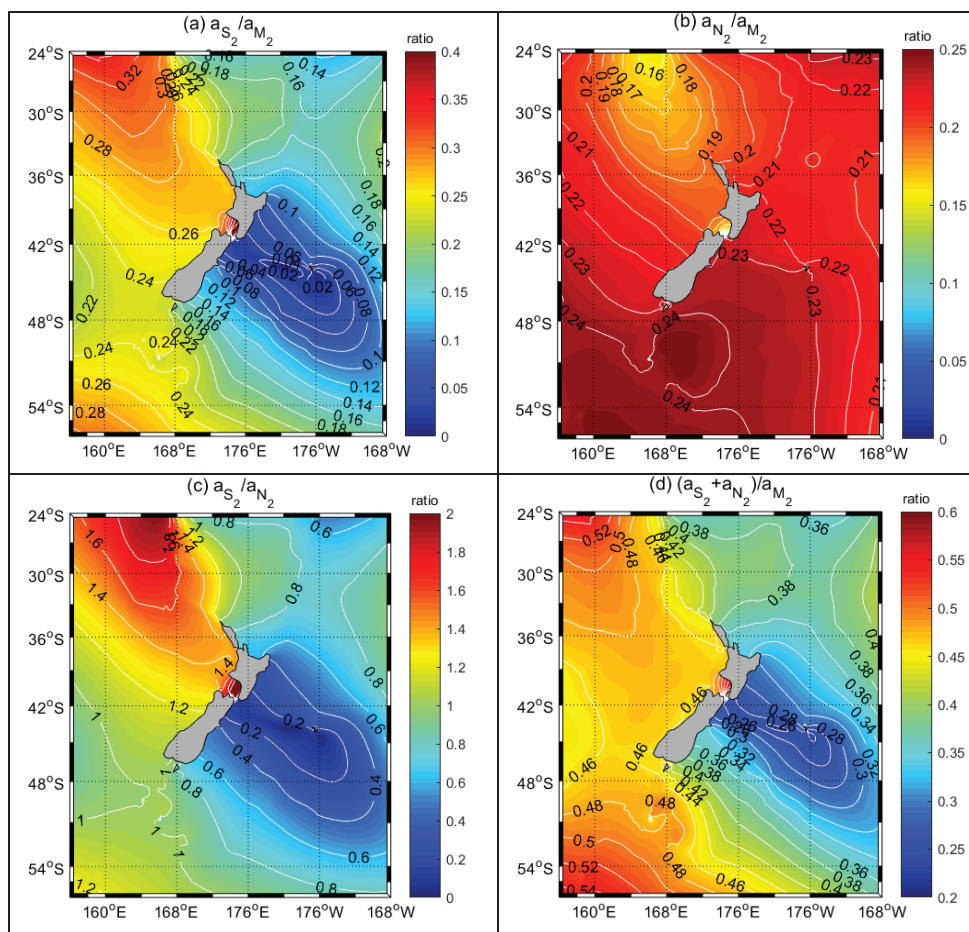
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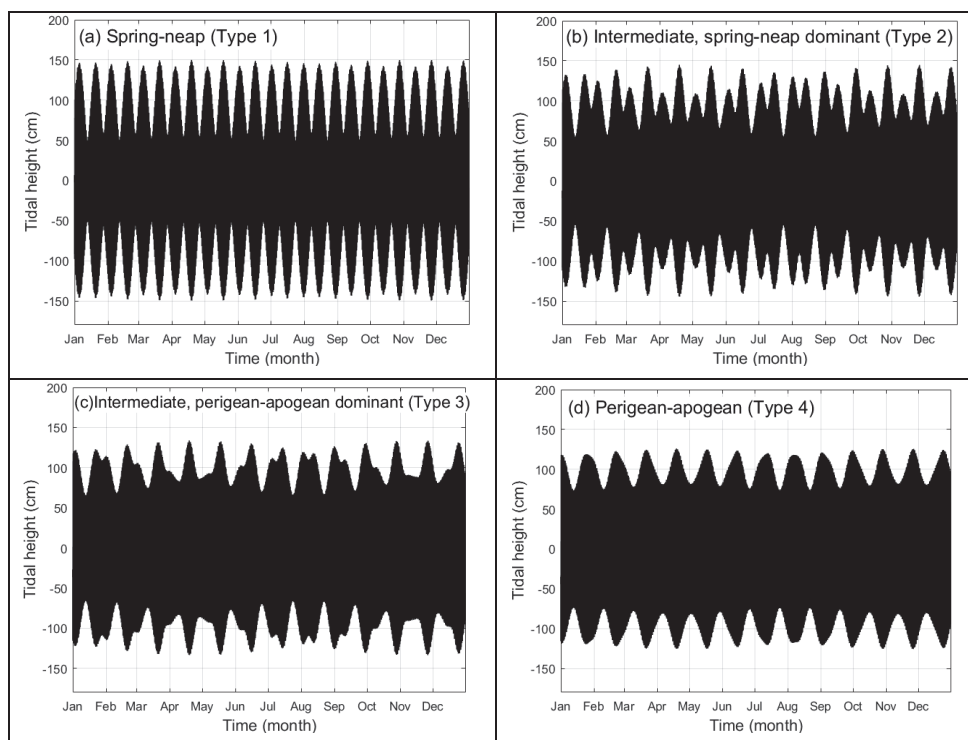
354 **Figure 3. Amplitude contours for the (a) M₂, (b) S₂, (c) N₂, (d) K₁, and (e) O₁ tides around ANZ, and (f) the resultant horizontal**
355 **distribution of F , daily tidal form factor values, as derived and calculated from the FES2014 tide model database at a scale of**
356 **1°/16×1°/16. Note that the amplitude color scales vary between plots a and e.**

357



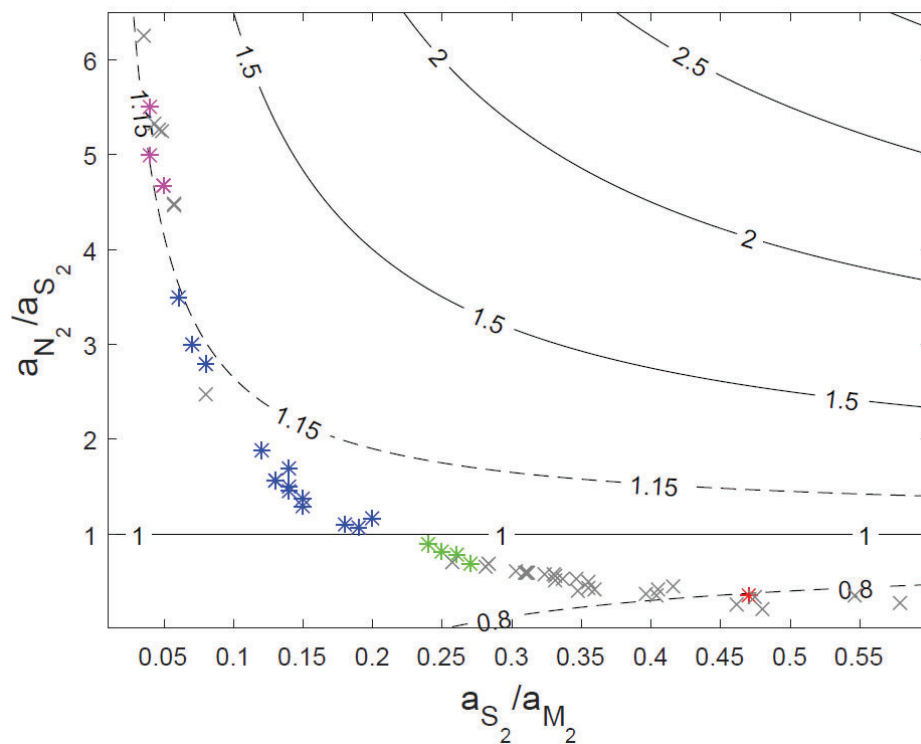
358 Figure 4. Horizontal distributions of tidal constituent amplitude ratios around ANZ for: (a) $\frac{a_{S_2}}{a_{M_2}}$; (b) $\frac{a_{N_2}}{a_{M_2}}$; (c) $\frac{a_{S_2}}{a_{N_2}}$ and (d) $\frac{a_{S_2}+a_{N_2}}{a_{M_2}}$; as
359 calculated using the FES2014 tide model database at a scale of $1^\circ/16 \times 1^\circ/16$. Note that the amplitude color scales vary between plots
360 a and d.

361



362 Figure 5. Idealized examples of four different monthly tidal envelopes over one year, calculated using the amplitude value $a_{M_2} =$
 363 100 cm and the amplitude ratio values of: (a) $\frac{a_{S_2}}{a_{M_2}} = 0.46$, $\frac{a_{S_2}}{a_{N_2}} = 11.5$, $\frac{a_{N_2}}{a_{M_2}} = 0.04$; (b) $\frac{a_{S_2}}{a_{M_2}} = 0.27$, $\frac{a_{S_2}}{a_{N_2}} = 1.5$, $\frac{a_{N_2}}{a_{M_2}} = 0.18$; (c)
 364 $\frac{a_{S_2}}{a_{M_2}} = 0.12$, $\frac{a_{S_2}}{a_{N_2}} = 0.5455$, $\frac{a_{N_2}}{a_{M_2}} = 0.22$; and (d) $\frac{a_{S_2}}{a_{M_2}} = 0.04$, $\frac{a_{S_2}}{a_{N_2}} = 0.1818$, $\frac{a_{N_2}}{a_{M_2}} = 0.22$. Note that the F_M^S values of these plots are:
 365 (a) 0.71; (b) 0.93; (c) 1.09; and (d) 1.17.

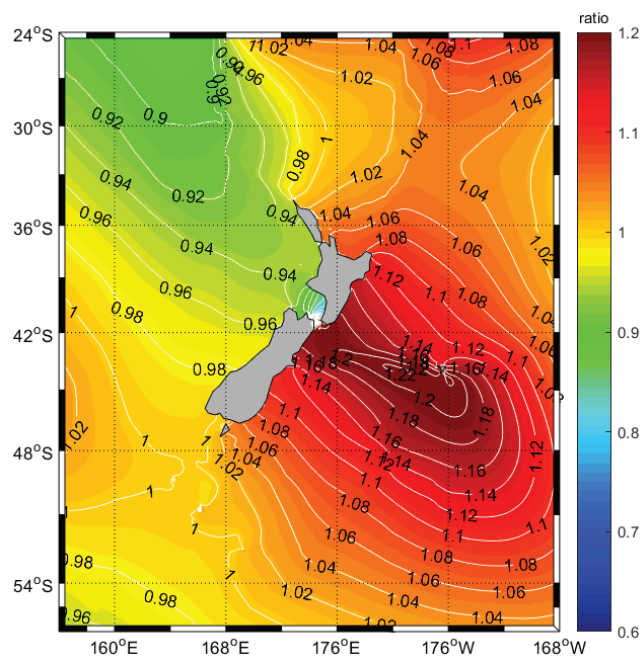
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368 **Figure 6.** Plot of the relationship between the $\frac{a_{N_2}}{a_{S_2}}$ and $\frac{a_{S_2}}{a_{M_2}}$ ratios (y and x axes respectively) versus F_M^S values (shown as plot contours),
369 with data points corresponding to Aotearoa New Zealand waters Type 1 sites (red star); Type 2 sites (green stars); Type 3 sites (blue
370 stars); and Type 4 sites (pink stars), all from Table 2; and tidal data representative of the greater Cook Strait area (grey crosses)
371 from Walters et al. (2010, Tables 1 and 3).

372



373

374 **Figure 7.** Distribution of monthly tidal envelope factor (F_M^S) values in the waters around ANZ, calculated using FES2014 data. See
375 **Table 3 and Figure 5** for corresponding monthly tidal envelope factor classes and envelope patterns.