Comment on "Tidal variability in the Hong Kong region" by Devlin et al.

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Changes in oceanic tides—over periods from secular to intra-annual—have been discovered in many ports and are the subject of numerous investigations. Searching for correlations between changes in tidal coefficients and various other environmental variables such as mean sea level is an obvious first step toward understanding the mechanisms behind these changes. Such correlation exercises, however, require that the underlying tidal analysis is carefully done and that the resulting tidal time series (say, of tidal amplitudes) is a legitimate expression of variability in the ocean. That is, care must be taken to ensure that the time series is truly reflecting a change in the tide and is not merely an artifact of tidal analysis. If a tidal time series has evidence of periodicities at 18.6, 8.8, or 4.4 years, which are all well-understood modulations existing in the tidal potential, then it is surely a strong warning that artifacts are present. Computing correlations with such corrupted time series would not be very useful.

In the submitted paper by Devlin et al. (2018) one finds several suspect time series with evident oscillations; see their Figures 9 and 10. My purpose in writing is to describe how these oscillations arise. Ultimately they stem from inadequacies in the tidal analysis, which are sometimes not easily recognized or understood.

The constituents under discussion are N_2, $2N_2$, M_3, and MO_3. The first two are standard semidiurnal constituents arising from the primary degree-2 terms of the tidal potential. The third is a terdiurnal constituent arising from a degree-3 term. The last is a nonlinear compound tide arising from interactions between the linear constituents M_2 and O_1. Devlin et al. (2018) estimate amplitudes and phases of these constituents every year from a half-century of hourly data and show the resulting amplitudes in their Figures 9 and 10. All show clear periodicities or modulations. These arise from tidal spectral lines that are inseparable in one-year analyses.

Here I examine only the time series from Hong Kong, comprising two segments from tide gauges at North Point (1962–1985) and Quarry Bay (1986–2016), locations separated by approximately 1.7 km. Tidal analysis of the entire (24 years) North Point hourly time series yields, for the frequency bands of interest

Table 1: Selected tidal constants for North Point, Hong Kong.

Cluster	Constituent	Doodson number	Amplitude (mm)	Phase (deg)
$2N_2$	$2MK_2$	235.555	3.2	294°
	$2N_2'$	235.655	1.3	231°
	$2N_2$	235.755	12.0	339°
N_2	N_2'	245.555	0.9	255°
	N_2	245.655	83.2	356°
MO_3	MO_3	345.555	7.8	325°
	F_3	345.655	4.2	281°
M_3	M_3	355.555	14.3	307°
	NK_3	355.655	2.5	353°

here, harmonic constants listed in Table 1. The table is split into four clusters of tidal spectral lines. The lines within each cluster form, nominally, a tidal constituent, since the first three digits of their Doodson numbers are identical. Their frequencies are thus all separated by less than 1 cpy. In an analysis of a single year of data, these lines are inseparable. Thus, a time series of annual constituent estimates must yield a modulated time series of the form displayed by Devlin et al. I have reproduced the results of Devlin et al. by solving for annual estimates from the combined North Point and Quarry Bay hourly time series; these are shown as solid circles in Figure 1. The estimates agree fairly well with those given by Devlin et al., except I find a smaller modulation in N_2.

It is interesting to understand the source of some of the tidal lines listed in Table 1. In both N_2 and $2N_2$ there are lines that arise from the third-degree terms of the tidal potential (Cartwright & Tayler, 1971, p. 69), which I denote with a prime symbol. Their Doodson numbers differ from those of the standard degree-2 constituents by one unit in the fourth digit. Since the fourth digit corresponds to the coefficient of the mean longitude of the moon's perigee (period 8.8 years), these degree-2 and degree-3 lines differ in frequency by 1 cycle in 8.8 years. Nominally one therefore needs at least 9 years of observations to separate the degree-2 and degree-3 constituents. In addition, $2N_2$ is perturbed by the compound tide $2MK_2$, thus bringing in a 4.4-year modulation.

[On a technical note, it is important to realize that forming tidal admittances,

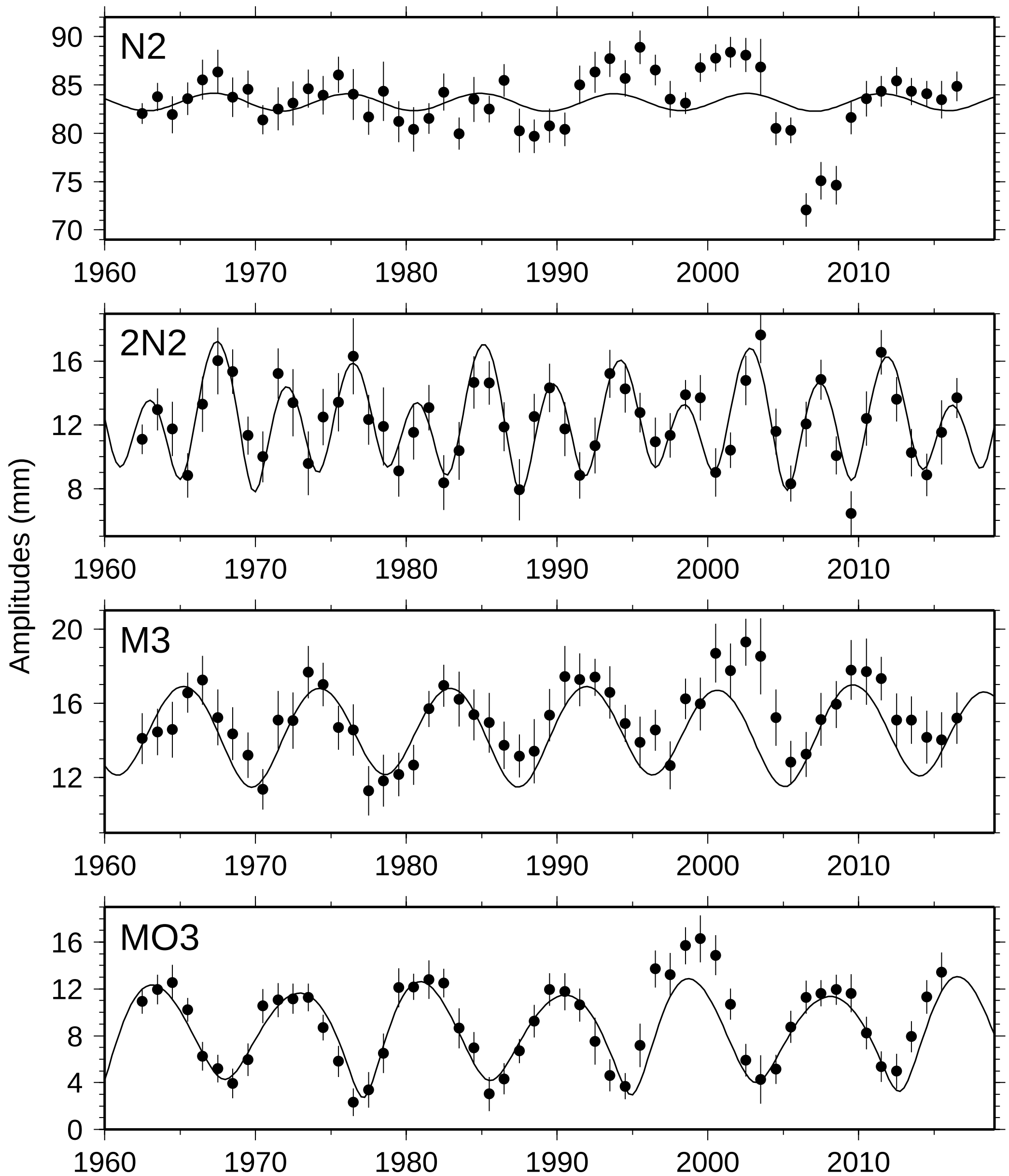

Figure 1: Four tidal constituents at Hong Kong (North Point and Quarry Bay). Solid circles: annual amplitude estimates based on analyses of hourly data, with error bars based on the spectral energy of the tidal residuals integrated over a small window surrounding each constituent. Solid lines: predicted amplitude modulations based on the harmonic constants of Table 1, which accounts for the existance of clustered tidal spectral lines that are inseparable in yearly analyses.

as was done by Devlin et al., cannot automatically account for the presence of both degree-2 and degree-3 tides, even though this approach is indeed useful for accounting for some close spectral lines (e.g, it can usually account for the lines associated with 18.6-y nodal modulations). Admittances must be computed separately for degree-2 and degree-3 tides. Although their frequencies are nearly identical, their spatial forcing patterns over the global ocean are markedly different, so the ocean's response to these different components is also very different. See, for example, the approach to admittance calculations taken by Munk & Cartwright (1966) and the very different ocean responses for degree-2 and degree-3 diurnal tides observed by Cartwright (1975).]

The amplitude of N_2' is small relative to the dominant N_2 line, so one should expect only small modulations in annual estimates of N_2. In other locations, the degree-3 constituent is larger and the resulting modulations more pronounced. One such example occurs in the Gulf of Maine (Doodson, 1924).

In the terdiurnal band, the linear M_3 tide, which is the principal lunar constituent from the third-degree potential, and the compound constituent NK_3 again differ in frequency by 1 cycle in 8.8 y. The compound NK_3 is often very small, but it is noticeable at this location because the tidal regime is mixed, with comparably strong tides in both diurnal and semidiurnal bands. Similarly, falling near the compound tide MO_3 is a small linear constituent, which is sometimes denoted F_3 in honor of Admiral A. Franco who noticed it in records from Cananeia, Brazil; these again require 9 years to separate.

Given the harmonic constants of Table 1, it is straightforward to predict the modulations that will be observed in annual estimates of these four constituents. Consider that any cluster is of the form

$$h = A_0 \cos(\omega_0 t - \varphi_0) + A_1 \cos(\omega_1 t - \varphi_1) + \dots \tag{1}$$

Set $\delta\omega_1 = \omega_1 - \omega_0 \ll \omega_0$, and similarly $\delta\omega_2 = \omega_2 - \omega_0$, etc., so that

$$\cos \omega_1 t = \cos \omega_0 t \cos \delta\omega_1 t - \sin \omega_0 t \sin \delta\omega_1 t$$

with similar expressions for the sine components. If only the first term in h appears, its in-phase and quadrature components are simply $A_0 \cos \varphi_0$ and $A_0 \sin \varphi_0$, respectively. But when the other terms in h are present, we find after gathering terms that the components are:

$$\begin{aligned} \text{In-phase} &= A_0 \cos \varphi_0 + A_1 \cos \varphi_1 \cos \delta\omega_1 t + A_1 \sin \varphi_1 \sin \delta\omega_1 t + \dots \\ \text{Quadrature} &= A_0 \sin \varphi_0 + A_1 \sin \varphi_1 \cos \delta\omega_1 t - A_1 \cos \varphi_1 \sin \delta\omega_1 t + \dots \end{aligned}$$

Evaluating these terms for the four constituents of interest here, using the coefficients tabulated in Table 1, leads to the solid lines in Figure 1. The agreement with the annual estimates (solid circles) is reasonably good, including the extrapolation into the period of the Quarry Bay series (recall Table 1 is based only on North Point data). Thus is explained the major modulations in the tidal constants shown by Devlin et al.

Any physically-based analysis of changes in these tidal constituents should be based on the differences between the annual estimates (solid circles) and their understood modulations (solid lines). The large offset in N_2 amplitudes around 2007, and earlier highs around 1994 and 2002, are obvious, and these may represent either true changes in tide or merely instrumental problems. But any analysis of cause must begin with a careful tidal analysis as a first step. Otherwise, computations of tidal correlations are of dubious value.

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