Response letter

We thank the Editor and the Reviewers for the careful consideration of our work. In the revised paper, we have addressed all the comments formulated by the Reviewers by replying (in black) to their remarks (in blue). The lines numbers in this rebuttal refer to the revised version of the manuscript.

Responses to comments by Reviewer #1

In this article an analytical solution of the tidal equations is presented to study the interaction of different tidal constituents in the Spanish Guadiana and Guadalquivir estuaries. The solution is based on a method developed by Godin (1999) and Dronkers (1964) for dealing with the non-linear friction term. Different tidal constituents derived from long-term tidal records along both estuaries are compared with amplitudes and phases of these constituents given by the analytical model. Observations and model results are in fair agreement. The article is well written and well organised.

Our reply: We thank the Reviewer for his overall positive assessment of our work.

There is probably an error in figure 9; the damping numbers (as defined in table 1) do not match the x-dependence of the amplitudes of figure 8. This is repeated in the corresponding discussion (lines 320-327). When comparing figures 6 and 8, the damping of M2 tide in the Guadalquivir appears a bit stronger than in the Guadiana, but not an order of magnitude stronger.

Our reply: We thank the Reviewer for this comment. Indeed, we mixed up the unit for the tidal amplitudes imposed at the estuary mouth. The corrected Figure is displayed below (see Figure R1).



Figure R1. Longitudinal variations of tidal damping/amplification number δ_A (a) and wave celerity number λ_A (b) for different tidal constituents along the Guadalquivir estuary.

In the revised paper, we have modified the paragraph as follows:

"Figure 9 shows the longitudinal variations of tidal damping/amplification and wave celerity for the Guadalquivir estuary, which are similar to those in the Guadiana estuary. In general, we observe that the dominant M_2 tide experiences less friction than other secondary semidiurnal tidal constituents although it travels at more or less the same speed in the seaward reach (x=0-35 km). Unlike the Guadiana estuary, the damping experienced by the secondary semidiurnal tides is less than those of diurnal constituents near the estuary mouth (around x=0-7 km; Figure 9a), while the wave celerity is consistently larger in the seaward reach (x=0-38 km; Figure 9b). Similar to the Guadiana estuary, we observe that the tidal damping for the secondary semidiurnal tides is stronger than those of diurnal constituents in the central parts of the estuary (around x=7-52 km), whereas their amplifications are larger in the landward part of the estuary although their wave speeds are less." (see lines 333-341)

The paper can be further improved by adding some clarifications concerning the following points:

1. River discharge is not mentioned at all in the paper. The influence is probably minor in the major part of the estuary, but river discharge could play a role near the sill at the upper end of the estuary, where the tidal velocities go to zero.

Our reply: In the revised paper, we have explicitly mentioned that the model does not account for the influence of river discharge on tidal wave propagation. To be more specific, in abstract part, we emphasized that *"The proposed method could be applicable to other alluvial estuaries with small tidal amplitude to depth ratio and negligible river discharge."* (see lines 13-14)

Meanwhile, in section 2.1 we have explicitly mentioned that "In order to obtain an analytical solution, we assume a negligible river discharge and that the tidal amplitude is small with respect to the mean depth and follow Toffolon and Savenije (2011) to derive the linearized solution of the system of Eqs. (3) and (4)." (see lines 95-96)

In addition, in section 2.2, we have explicitly mentioned that in the Guadiana estuary "the data were collected during an extended (months-long) period of drought with negligible river discharge (e.g., always < 20 m³/s over the preceding 5 months)." (see lines 149-151), while in the Guadalquivir estuary "the results apply to the low river discharge conditions (< 40 m³/s) that usually predominate at the estuary." (see lines 168-169)

2. Close to the sill the tide has the appearance of a standing wave; this gives an almost infinite tidal wave celerity. Tidal wave celerity does not make much sense in this region.

Our reply: In the revised paper, we have explicitly mentioned that: "*It is important to note that a standing wave pattern with celerity approaching infinity is produced near the sill due to the superimposition of the incident and reflected waves (see also Garel and Cai, 2018)."* (see lines 322-324)

3. The Chebyshev coefficients are the coefficients of the expansion of cos(nx) in powers of cos(x).

Our reply: We thank the Reviewer for this comment. In the revised paper, we have clarified that "*The Chebyshev coefficients* $\alpha = 16/(15\pi)$ and $\beta = 32/(15\pi)$ were determined by the expansion of cos(nx) (n=1,2,...) in powers of cos(x) (Godin, 1991, 1999)". (see lines 176-178)

4. It should be mentioned that formula Eq. 12 gives a reasonable approximation only if the diurnal tides are much smaller than the semidiurnal tides.

Our reply: In the revised paper, we have explicitly mentioned this point: "*It is worth noting that Eq. (12) is a reasonable approximation only if the amplitude of secondary constituent is much smaller than that of the dominant one*". (see lines 205-206)

5. The diurnal tides are much less damped than the semidiurnal tides. Apparently, the effects of frictional damping and channel convergence cancel approximately. This might be discussed more clearly in the paper.

Our reply: In the revised paper, we have included a new paragraph to clarify the difference of tidal damping between diurnal and semidiurnal tides.

"In order to clarify the behavior of different tidal constituents, we present Figure 10 [see Figure R2 below] showing the longitudinal variations of estuary shape number γ (representing the channel convergence) and friction number χ_n (representing the bottom friction), two major factors determining the tidal hydrodynamics, in both estuaries. Note that the variable estuary shape number γ observed in the Guadalquivir estuary is due to the adoption of a variable storage width ratio r_s in the analytical model. On the one hand, the estuary shape numbers for diurnal tides are approximately twice larger than those for semidiurnal tides (Figures 10a, d) due to the tidal frequency differences (see definition of γ in Table 1). On the other hand, the effective friction experienced by the diurnal tides is much larger than those of the semidiurnal tides due to the mutual interaction between different tidal constituents (Figure 10b, e, see also Table 3). However, the propagation of different tidal constituents mainly depends on the imbalance between channel convergence and friction, except for those reaches where wave reflection matters (generally close to the head). In particular, in the seaward reach the tidal damping for each tidal constituent can be approximately estimated by $\delta_A = \gamma/2 - \chi_n \mu \cos(\phi)/(2\lambda_A)$ (see equation (20) by Cai et al., 2012). While the channel convergence effect (represented by $\gamma/2$) is much stronger for diurnal tides than for semidiurnal tides, the frictional effect (represented by $\chi_n \mu \cos(\phi)/(2\lambda_A)$) is only slightly larger (Figure 10c, f). Hence, diurnal tides experience a relatively lower damping in the

seaward reach (Figures 7a and 9a). For instance, in the case of the Guadalquivir estuary, diurnal tides are more damped than semidiurnal tides near the estuary mouth (x=0-7 km). For the second (landward) half of the estuary, the lower amplification experienced by diurnal tides is mainly due to the wave reflection from the closed end (see Garel and Cai, 2018)." (see lines 349-369)



Figure R2. Longitudinal variations of estuary shape number γ (a, d), friction number χ_n (b, e) and $\chi_n \mu \cos(\phi)/(2\lambda_A)$ (c, f) in the Guadiana estuary (a, b, c) and Guadalquivir estuary (d, e, f).

The sensitivity of the results to the non-linear frictional interaction between the tidal constituents, being the central theme of the paper, should be discussed more explicitly. Figures 6 and 8 show the combined results of friction, channel convergence and tidal wave reflection. A figure might be added, for example, in which results with and without this frictional interaction are compared.

Our reply: We thank the Reviewer for the useful suggestion. In the revised paper, we have included a new paragraph to illustrate the importance of mutual interaction between different tidal constituents:

"The importance of mutual interaction between different tidal constituents is illustrated with the iteratively refined model implemented at both case studies (Figures 7 and 9). For comparison, Figure 11 [see Figure R3 below] shows the analytically computed damping/amplification number δ_A and celerity number λ_A without considering mutual interaction (by setting $f_n=1$ in the model). In this case, the damping experienced by both secondary diurnal and semidiurnal tides are apparently underestimated due to the unrealistic friction adopted in the model (Figure 11a, c, see also Figures 7a and 9a, respectively). Similarly, the computed wave celerity for secondary tidal

constituents are apparently overestimated due to the underestimated bottom friction (Figure 11b, d, see also Figures 7b and 9b, respectively). To correctly reproduce the main features of different tidal waves, it is required to use the iteratively refined model proposed in this study." (see lines 370-379)



Figure R3. Longitudinal variations of damping/amplification number δ_A (a, c) and celerity number λ_A (b, d) in the Guadiana estuary (a, b) and Guadalquivir estuary (c, d) in the absence of mutual interaction between different tidal constituents.

Responses to comments by Reviewer #2

This paper deals with numerical modelling of several tidal constituents propagating in an estuary. This is an important problem: estuary models tend to deal with a single constituent at a time (to keep the run length down). However, the friction felt by that constituent will depend on the size and nature of the other tidal constituents in the estuary. The paper is thorough: the problem is first dealt with in an analytical way, numerical solutions are obtained and compared to observations in two estuaries in the Iberian peninsula. Agreement is good.

Our reply: We thank the Reviewer for his overall positive assessment of our work.

I'm not a numerical modeller but I know that the effect of frictional interaction between different tidal constituents has been well studied (the important papers on the subject are referenced here). I would appreciate being told exactly what is new about this paper. Is it the first time that estuaries with an exponential shape have been considered in this way?

Our reply: In the revised paper, we have explicitly mentioned that "*Previous studies explored the effect of frictional interaction between different tidal constituents by quantifying a friction correction factor only (e.g., Dronkers, 1964; Le Provost, 1973; Pingree, 1983; Fang, 1987; Godin, 1999; Inoue and Garrett, 2007). In this study, for the first time, the mutual interactions between tidal constituents in the frictional term were explored using a conceptual analytical model. Specifically, a friction correction factor for each constituent was defined by expanding the quadratic velocity using a Chebyshev polynomials approach." (see lines 60-65)*

The advantage of such conceptual model lies in the deterministic description of the mutual frictional interaction among tidal constituents, which avoids the need of an independent calibration of the friction parameter for the single constituent. The proposed method can be used as a prognostic tool to study the propagation of different tidal constituents in convergent estuaries where the cross-sectional area can be described by an exponential function.

Also, I would be interested to know if the problem could be approached just by matching model results to observations to get the best fit (as I suspect many modellers would do) without worrying too much about the theory.

Our reply: Exactly! Similar to our previous analytical studies for a single tidal constituent (e.g., Toffolon and Savenije, 2011; Cai et al., 2016), the implementation of the new model accounting for the nonlinear interactions between tidal constituents also requires a few dimensionless input parameters representing the external tidal forcing and estuary geometry, which are independent of the tidal hydrodynamics along the estuary. Hence, the problem does solve by matching the model results to observations.

The paper is well written, but it is long and technical. I don't suggest doing anything about it now, but I would encourage the authors to go for a more concise style in the future. Having said that, I found myself wondering why the estuaries behave as they do. WHy does the tidal amplitude first reduce before increasing towards the tidal limit. I think I undesrtand that, but it would be interesting to read the authors opinion in the discussion section.

Our reply: We thank the Reviewer for the useful suggestion. In the revised paper, we have explicitly mentioned the underlying mechanism of tidal hydrodynamics (i.e., damping/amplification along the channel) in these two estuaries.

In the revised paper, we have included a new paragraph to clarify this point:

"In particular, the tidal damping along the first half of these two estuaries is mainly due to the damping of the dominant M_2 wave owning to the fact that the impact of bottom friction dominates over the channel convergence. Along the upper reach, enhanced morphological convergence and reflection effects (that reduce the overall friction experienced by the propagating wave) result in the overall amplification of the tidal wave. For more details of the tidal hydrodynamics in these

two estuaries, readers can refer to Garel and Cai (2018) for the Guadiana estuary and Diez-Minguito et al. (2012) for the Guadalquivir estuary." (see lines 343-348)

There were some small points I noticed which could benefit from correction:

line 83 the storage width Bs is not defined here as far as I can see, although it is defined in the figure. At this stage I am confused about whether the model considers just a rectangular channel (with constant width) or whether the width is allowed to change with the tide.

Our reply: In the revised paper, we have explicitly defined the storage width B_S as "width of the channel at averaged high water level" (see line 87). In this study, we assume a rectangular cross-section with a constant width since the variation of width $\Delta \overline{B}$ with time is usually negligible (i.e., $\Delta \overline{B}/B \ll 1$). On the other hand, the overall influence of storage area is represented by the storage width ratio, defined as the ratio of the storage width B_S (width of the channel at averaged high water level) to the tidally averaged width \overline{B} (i.e., $r_S = B_S/\overline{B}$).

line 115 Why would there be different celerities for elevations and velocities?

Our reply: It was shown by Savenije et al. (2008) that for an infinitely long channel the wave celerities for elevation and velocity are almost the same due to the combined impacts of bottom friction and channel convergence. However, for a semi-closed channel the wave celerities for elevation and velocity would deviate due to the additional impact of reflected wave at the closed end (e.g., Toffolon and Savenije, 2011). Such a celerity difference was recently investigated and detailed by Garel and Cai (2018) for the case in the Guadiana estuary.

equations 10 and 11 it looks line - signs occur where there should be = signs (although that may be a trick of PDF).

Our reply: This is probably due to the PDF viewer, as there is not such typing error on our version.

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Frictional interactions between tidal constituents in tide-dominated estuaries

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Abstract. When different tidal constituents propagate along an estuary, they interact because of the presence of nonlinear terms in the hydrodynamic equations. In particular, due to the quadratic velocity in the friction term, the effective friction experienced by both the predominant and the minor tidal constituents is enhanced. We explore the underlying mechanism with a simple conceptual

- 5 model by utilizing Chebyshev polynomials, enabling the effect of the velocities of the tidal constituents to be summed in the friction term and, hence, the linearized hydrodynamic equations to be solved analytically in a closed form. An analytical model is adopted for each single tidal constituent with a correction factor to adjust the linearized friction term, accounting for the mutual interactions between the different tidal constituents by means of an iterative procedure. The proposed method
- 10 is applied to the Guadiana (southern Portugal-Spain border) and the Guadalquivir (Spain) estuaries for different tidal constituents (M₂, S₂, N₂, O₁, K₁) imposed independently at the estuary mouth. The analytical results appear to agree very well with the observed tidal amplitudes and phases of the different tidal constituents. The proposed method could be applicable to other alluvial estuaries with small tidal amplitude to depth ratio and negligible river discharge.

15 1 Introduction

Numerous studies have been conducted in recent decades to model tidal wave propagation along an estuary since an understanding of tidal dynamics is essential for exploring the influence of humaninduced (such as dredging for navigational channels) or natural (such as global sea level rises) interventions on estuarine environments (Schuttelaars et al., 2013; Winterwerp et al., 2013). Analytical

- 20 models are invaluable tools and have been developed to study the basic physics of tidal dynamics in estuaries; for instance, to examine the sensitivity of tidal properties (e.g., tidal damping or wave speed) to change in terms of external forcing (e.g., spring–neap variations of amplitude) and geometry (e.g., depth or channel length). However, most analytical solutions developed to date, which make use of the linearized Saint-Venant equations, can only deal with one predominant tidal con-
- 25 stituent (e.g., M_2), which prevents consideration of the nonlinear interactions between different tidal constituents. The underlying problem is that the friction term in the momentum equation follows a quadratic friction law, which causes a nonlinear behavior causing tidal asymmetry as tide propagates upstream. If the friction law were linear, one would expect that the effective frictional effect for different tidal constituents (e.g., M_2 and S_2) could be computed independently (Pingree, 1983).
- To explore the interaction between different constituents of the tidal flow, the quadratic velocity u|u| (where *u* is the velocity) is usually approximated by a truncated series expansion, such as a Fourier expansion (Proudman, 1953; Dronkers, 1964; Le Provost, 1973; Pingree, 1983; Fang, 1987; Inoue and Garrett, 2007). If the tidal current is composed of one dominant constituent and a much smaller second constituent, it has been shown by many researchers (Jeffreys, 1970; Heaps, 1978;
- 35 Prandle, 1997) that the weaker constituent is acted on by up to 50% more friction than that of the dominant constituent. However, this requires the assumption of a very small value of the ratio of the magnitudes of the weaker and dominant constituents, which indicates that this is only a first-order estimation. Later, some researchers have extended the analysis to improve the accuracy of estimates and to allow for more than two constituents (Pingree, 1983; Fang, 1987; Inoue and Garrett, 2007).
- 40 Pingree (1983) investigated the interaction between M_2 and S_2 tides, resulting in a second-order correction of the effective friction coefficient acting on the predominant M_2 tide and a fourth-order value for the weaker S_2 constituent of the tide. Fang (1987) derived exact expressions of the coefficients of the Fourier expansion of u|u| for two tidal constituents but did not provide exact solutions for the case of three or more constituents. Later, Inoue and Garrett (2007) used a novel approach
- 45 to determine the Fourier coefficients of u|u|, which allows the magnitude of the effective friction coefficient to be determined for many tidal constituents. For the general two-dimensional tidal wave propagation, the expansion of quadratic bottom friction using a Fourier series was first proposed by Le Provost (1973) and subsequently applied to spectral models for regional tidal currents (Le Provost et al., 1981; Le Provost and Fornerino, 1985; Molines et al., 1989). Building on the previous work by
- 50 Le Provost (1973), the importance of quadratic bottom friction in tidal propagation and damping was discussed by Kabbaj and Le Provost (1980) and reviews of friction term in models were presented by Le Provost (1991).

In contrast, as noted by other researchers (Doodson, 1924; Dronkers, 1964; Godin, 1991, 1999), the quadratic velocity u|u| is, mathematically, an odd function, and it is possible to approximate it

55 by using a two- or three-term expression, such as $\alpha u + \beta u^3$ or $\alpha u + \beta u^3 + \xi u^5$, where α , β , and ξ are suitable numerical constants. The linear term αu represents the linear superposition of different

constituents, while the nonlinear interaction is attributed to a cubic term βu^3 and a fifth-order term ξu^5 . It is to be noted that such a method has the advantage of keeping the hydrodynamic equations resolvable in a closed form (Godin, 1991, 1999).

- 60 In this paper, a conceptual analytical model is presented to understand the propagation of Previous studies explored the effect of frictional interaction between different tidal constituents that one might wish to treat independently. The key lies in the treatment of the quadratic velocity in the friction term . by quantifying a friction correction factor only (e.g., Dronkers, 1964; Le Provost, 1973; Pingree, 1983; Fang, 1987; Godin, 1999; Inoue and Garrett, 2007)
- 65 . In this study, for the first time, the mutual interactions between tidal constituents in the frictional term were explored using a conceptual analytical model. Specifically, a friction correction factor for each constituent was defined by expanding the quadratic velocity using a Chebyshev polynomials approach. The model has subsequently been applied to the Guadiana and the Guadalquivir estuaries in southern Iberian Peninsula, for which case the mutual interaction between the predominant M_2
- tidal constituent and other tidal constituents (e.g., S_2 , N_2 , O_1 , K_1) is explored.

2 Materials and methods

2.1 Hydrodynamic model

We are considering a semi-closed estuary that is forced by one predominant tidal constituent (e.g., M₂) with the tidal frequency ω = 2π/T, where T is the tidal period. As the tidal wave propagates
75 into the estuary, it has a wave celerity of water level c_A, a wave celerity of velocity c_V, an amplitude of tidal elevation η, a tidal velocity amplitude v, a phase of water level φ_A, and a phase of velocity φ_V. The length of the estuary is indicated by L_e.

The geometry of a semi-closed estuary is shown in Figure 1, where x is the longitudinal coordinate, which is positive in the landward direction, and z is the free surface elevation. The tidally
averaged cross-sectional area A and width B are assumed to be exponentially convergent in the landward direction, which can be described by

$$A = A_0 \exp(-x/a),\tag{1}$$

$$\overline{B} = \overline{B_0} \exp(-x/b), \tag{2}$$

- 85 where $\overline{A_0}$ and $\overline{B_0}$ are the respective values at the estuary mouth (where x=0), and a and b are the convergence lengths of cross-sectional area and width, respectively. We also assume a rectangular cross-section, from which it follows that the tidally averaged depth is given by $\overline{h} = \overline{A}/\overline{B}$. The possible influence of storage area is described by the storage width ratio r_S , defined as the ratio of the storage width B_S (width of the channel at averaged high water level) to the tidally averaged
- 90 width $\overline{\underline{B}}$ (i.e., $r_S = B_S/\overline{B}$).

With the above assumptions, the one-dimensional continuity equation reads

$$r_S \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + h \frac{\partial u}{\partial x} + \frac{hu}{\overline{B}} \frac{\mathrm{d}\overline{B}}{\mathrm{d}x} = 0, \qquad (3)$$

where t is the time and h the instantaneous depth. Assuming negligible density effects, the onedimensional momentum equations can be cast as follows

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$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial z}{\partial x} + \frac{g \, u |u|}{K^2 h^{4/3}} = 0,$$
 (4)

where g is the acceleration due to gravity and K is the Manning-Strickler friction coefficient.

In order to obtain an analytical solution, we assume a negligible river discharge and that the tidal amplitude is small with respect to the mean depth and follow Toffolon and Savenije (2011) to derive the linearized solution of the system of Eqs. (3) and (4). However, different from the standard linear solutions, we will retain the mutual interaction among different harmonics originated by the nonlinear frictional term, which contains two sources of nonlinearity: the quadratic velocity u|u| and the variable depth at the denominator. While we neglect the latter factor, consistent with the assumption of small tidal amplitude, we will exploit Chebyshev polynomials to represent the

harmonic interaction in the quadratic velocity (see Section 3.1). For sake of clarity, we report here 105 the linearized version of the momentum equation

$$\frac{\partial u}{\partial t} + g \frac{\partial z}{\partial x} + \kappa \, u |u| = 0, \tag{5}$$

and the friction coefficient

$$\kappa = \frac{g}{K^2 \overline{h}^{4/3}} \,. \tag{6}$$

Toffolon and Savenije (2011) demonstrated that the tidal hydrodynamics in a semi-closed estuary are controlled by a few dimensionless parameters that depend on geometry and external forcing (for detailed information about analytical solutions for tidal hydrodynamics, readers can refer to Appendix A). These parameters are defined in Table 1 and can be interpreted as follows.

The independent dimensionless parameters are: ζ_0 is the dimensionless tidal amplitude (the subscript 0 indicating the seaward boundary condition); γ is the estuary shape number (representing 115 the effect of cross-sectional area convergence); χ_0 is the friction number (describing the role of the frictional dissipation); L_e^* is the dimensionless estuary length. The dimensional quantities used in the definition of the dimensionless parameters are: η_0 is the tidal amplitude at the seaward boundary; $c_0 = \sqrt{g\overline{h}/r_S}$ is the frictionless wave celerity in a prismatic channel; $L_0 = c_0/\omega$ is the tidal length scale related to the frictionless tidal wave length by a factor 2π .

120 The main dependent dimensionless parameters are also presented in Table 1, including: ζ is the actual tidal amplitude; χ is the actual friction number; μ is the velocity number (the ratio of the actual velocity amplitude to the frictionless value in a prismatic channel); λ_A and λ_V are, respectively, the celerity for elevation and velocity (the ratio between the frictionless wave celerity in a prismatic

channel and actual wave celerity); δ_A and δ_V are, respectively, the amplification number for elevation

125 and velocity (describing the rate of increase, δ_A (or δ_V) > 0, or decrease, δ_A (or δ_V) < 0, of the wave amplitudes along the estuary axis); $\phi = \phi_V - \phi_A$ is the phase difference between the phases of velocity and elevation.

It is important to remark that several nonlinear terms are present both in the continuity and in the momentum equations (Parker, 1991), which are responsible, for instance, of the internal generation

130 of overtides (e.g., M_4). In this approximated approach, we disregard them and focus exclusively on the mutual interaction among the external tidal constituents mediated by the quadratic velocity dependence in the frictional term. In fact, it crucially affects the propagation of the tidal waves associated with the different constituents that are already present in the tidal forcing at the estuary mouth.

135 2.2 Study areas

Both the Guadiana and the Guadalquivir estuaries are located in the southwest part of the Iberian Peninsula. These systems are good candidates for the application of a 1D hydrodynamic model of tidal propagation. Both estuaries are featured with a simple geometry, consisting of a single, narrow and moderately deep channel with relatively smooth bathymetric variations. Moreover, their

140 tidal prism exceeds their average freshwater inputs by several orders of magnitude due to strong regulation by dams. Under these largely predominant low river discharge conditions, both estuaries are well-mixed, and the water circulation is mainly driven by tides.

The Guadiana estuary, at the southern border between Spain and Portugal, connects the Guadiana River to the Gulf of Cadiz. Tidal water level oscillations are observed along the channel until a weir

145 located 78 km upstream of the river mouth (Garel et al., 2009). Both the cross-sectional area and the channel width are convergent and can be described by an exponential function, with convergence lengths of *a*=31 km and *b*=38 km, respectively (Figure 2). The flow depth is generally between 4 m and 8 m, with a mean depth of about 5.5 m (Garel, 2017).

The tidal dynamics in the Guadiana estuary are derived from records obtained using eight pressure transducers deployed for a period of 2 months (31 July to 25 September 2015) approximately every 10 km along the estuary (from the mouth to ~ 70 km upstream). The data were collected during an extended (months-long) period of drought with negligible river discharge (e.g., always < 20 m³/s over the preceding 5 months). For each station, the amplitude and phase of elevation of the tidal constituents were obtained from standard harmonic analysis of the observed pressure records using

the "t-tide" Matlab toolbox (Pawlowicz et al., 2002). The harmonic results are displayed in Table 2. Near the mouth, the largest diurnal (K_1), semi-diurnal (M_2) and quarter-diurnal (M_4) frequencies are similar to those previously reported at the same location based on pressure records taken over ~ 9 months (see Garel and Ferreira, 2013). In particular, the value $(\eta_{K_1} + \eta_{O_1})/(\eta_{M_2} + \eta_{S_2})$ is less than 0.1 at the sea boundary, which indicates that the tide is dominantly semi-diurnal.

- 160 The Guadalquivir estuary is located in southern Spain, at ~ 100 km to the east of the Guadiana River mouth. The estuary has a length of 103 km starting from the mouth at Sanlucar de Barrameda to the Alcala del Rio dam. The geometry of the Guadalquivir estuary can be approximated by exponential functions with convergence length of a=60 km for the cross-sectional area and b=66 km for the width (see Diez-Minguito et al., 2012). The flow depth is more or less constant (7.1 m).
- 165 Tidal dynamics along the Gualdalquivir estuary was analysed by Diez-Minguito et al. (2012) based on harmonic analyses of field measurements collected from June to December 2008. The amplitude and phase of tidal constituents near the mouth are highly similar to those at the entrance of the Guadiana estuary (Table 2), producing a semi-diurnal and mesotidal signal with a mean spring tidal range of 3.5 m. In this paper, the tidal observations of the Guadalquivir estuary are directly
- 170 taken from Diez-Minguito et al. (2012). The results apply to the low river discharge conditions ($< 40 \text{ m}^3/\text{s}$) that usually predominate at the estuary.

3 Conceptual model

3.1 Representation of quadratic velocity u|u| using Chebyshev polynomials approach

The Chebyshev polynomials can be used to approximate the quadratic dependence of the friction
term on the velocity, u|u|. Adopting a two-term approximation, it is known that (Godin, 1991, 1999)

$$u|u| = \hat{v}^2 \left[\alpha \left(\frac{u}{\hat{v}}\right) + \beta \left(\frac{u}{\hat{v}}\right)^3 \right],\tag{7}$$

where \hat{v} is the sum of the amplitudes of all the harmonic constituents. The Chebyshev coefficients were determined as $\alpha = 16/(15\pi)$, and $\beta = 32/(15\pi)$ were determined by the expansion of $\cos(nx)$.

180 (n=1,2,...) in powers of $\cos(x)$ (Godin, 1991, 1999). It is important to note that, unlike series developments (e.g., Fourier expansion), the Chebyshev coefficients α and β vary with the number of terms that are used in the development. Godin (1991) already showed that a two-term approximation (such as Eq. 7) is adequate to satisfactorily account for the friction.

For a single harmonic

$$185 \quad u = v_1 \cos(\omega_1 t), \tag{8}$$

where v_1 is the velocity amplitude and ω_1 its frequency, Eq. (7) can be expressed by exploiting standard trigonometric relations as

$$u|u| \cong v_1^2 \left[\frac{8}{3\pi} \cos(\omega_1 t) + \frac{8}{15\pi} \cos(3\omega_1 t) \right].$$
 (9)

Focusing only on the original harmonic constituent leads to

190
$$u|u| \cong \frac{8}{3\pi} v_1^2 \cos(\omega_1 t),$$
 (10)

which coincides exactly with Lorentz's classical linearization (Lorentz, 1926) or a Fourier expansion of u|u| (Proudman, 1953).

Considering a second tidal constituent, the velocity is given by

$$u = v_1 \cos(\omega_1 t) + v_2 \cos(\omega_2 t) = \hat{v} \left[\varepsilon_1 \cos(\omega_1 t) + \varepsilon_2 \cos(\omega_2 t) \right], \tag{11}$$

195 where v_2 and ω_2 are the amplitude and frequency of the second constituent, $\varepsilon_1 = v_1/\hat{v}$ and $\varepsilon_2 = v_2/\hat{v}$ are the ratios of the amplitudes to that of the maximum possible velocity $\hat{v} = v_1 + v_2$. Note that the possible phase lag between the two constituents is neglected assuming a suitable time shift (Inoue and Garrett, 2007). In this case, the truncated Chebyshev polynomials approximation of u|u| (focusing on two original tidal constituents) is expressed as (see also Godin, 1999)

200
$$u|u| \cong \frac{8}{3\pi} \widehat{v}^2 \left[F_1 \varepsilon_1 \cos(\omega_1 t) + F_2 \varepsilon_2 \cos(\omega_2 t) \right],$$
 (12)

with

$$F_{1} = \frac{3\pi}{8} \left[\alpha + \beta \left(\frac{3}{4} \varepsilon_{1}^{2} + \frac{3}{2} \varepsilon_{2}^{2} \right) \right] = \frac{1}{5} (2 + 3\varepsilon_{1}^{2} + 6\varepsilon_{2}^{2}) = \frac{1}{5} \left(8 + 9\varepsilon_{1}^{2} - 12\varepsilon_{1} \right), \tag{13}$$

$$F_{2} = \frac{3\pi}{8} \left[\alpha + \beta \left(\frac{3}{4} \varepsilon_{2}^{2} + \frac{3}{2} \varepsilon_{1}^{2} \right) \right] = \frac{1}{5} (2 + 3\varepsilon_{2}^{2} + 6\varepsilon_{1}^{2}) = \frac{1}{5} \left(5 + 9\varepsilon_{1}^{2} - 6\varepsilon_{1} \right), \tag{14}$$

- 205 where F_1 and F_2 represent the effective friction coefficients caused by the nonlinear interactions between tidal constituents. The last equality in Eqs. (13) and (14) is due to the fact that $\varepsilon_1 + \varepsilon_2 = 1$. It is worth noting that Eq. (12) is a reasonable approximation only if the amplitude of secondary constituent is much smaller than that of the dominant one.
- For illustration, approximations using Eqs. (7) and (12) for a typical tidal current with $\varepsilon_1 = 3/4$ 210 and $\varepsilon_2 = 1/4$ are displayed in Figure 3 for the case of two tidal constituents. It can be seen that the Chebyshev polynomials approximation (Eq. 7) matches the nonlinear quadratic velocity well, while Eq. (12), retaining only the original frequencies (ω_1 and ω_2), is still able to approximately capture the first-order trend of the quadratic term.

It can be seen from Eqs. (13) and (14) that when $\varepsilon_2 \ll 1$ (hence, $\varepsilon_1 \simeq 1$ for the dominant tidal 215 constituent), $F_1 \simeq 1$, $F_2 \simeq 1.6$, thus the weaker constituent experiences proportionately 60% more friction than the dominant constituent, which is slightly larger than the classical result of 50% more friction for the weaker tidal constituent. Figure 4 shows the solutions of effective friction coefficients F_1 and F_2 as a function of ε_1 for the case of two constituents. As expected, we see a symmetric response of these coefficients in the function of ε_1 since $\varepsilon_1 + \varepsilon_2 = 1$. Specifically, we note that the

220

the dominant constituent is twice larger than the weaker constituent.

Similarly, we are able to extend the same approach to the case of a generic number n of astronomical tidal constituents (e.g., K₁, O₁, M₂, S₂, N₂)

effective friction coefficient F_1 reaches a minimum when $\varepsilon_1=2/3$, when the velocity amplitude of

$$u = \sum_{i=1}^{n} v_1 \cos(\omega_i t) = \widehat{v} \sum_{i=1}^{n} \varepsilon_i \cos(\omega_i t), \qquad (15)$$

in which the subscript i represents the i-th tidal constituent. Considering only the original tidal constituents, the quadratic velocity can be approximated as

$$u|u| \cong \frac{8}{3\pi} \widehat{v}^2 \sum_{i=1}^n F_i \varepsilon_i \cos(\omega_i t), \tag{16}$$

and the general expression for the effective friction coefficients of j-th tidal constituents is given by

$$F_j = \frac{3\pi}{8} \left\{ \alpha + \beta \left[\sum_{i=1, i \neq j}^n \frac{3}{2} \varepsilon_i^2 - \frac{3}{4} \varepsilon_j^2 \right] \right\} = \frac{1}{5} \left(2 + 3\varepsilon_j^2 + \sum_{i=1, i \neq j}^n 6\varepsilon_i^2 \right).$$
(17)

230 We provide the complete coefficients for the cases of one to three constituents in Appendix B.

3.2 Effective friction in the momentum equation

For a single tidal constituent $u = v_1 \cos(\omega_1 t)$, the quadratic velocity term u|u| is often approximated by adopting Lorentz's linearization equation (Eq. 10) and thus the friction term in Eq. (5) becomes

$$\kappa u|u| = \left(\kappa \frac{8}{3\pi} \upsilon_1\right) u = r u, \tag{18}$$

which is the "standard" case for a monochromatic wave, i.e. when we only deal with a predominant tidal constituent (e.g., M_2).

For illustration of the method, we consider a tidal current that is composed of one dominant constituent (e.g., M_2 with velocity u_1) and a weaker constituent (e.g., S_2 with velocity u_2), which is a simple but important example in estuaries, i.e., $u = u_1 + u_2$. In this case, combination of Eq. (5) and the Chebyshev polynomials expansion of u|u| (Eq. 12) yields

$$\frac{\partial u_1}{\partial t} + \frac{\partial u_2}{\partial t} + g \frac{\partial z_1}{\partial x} + g \frac{\partial z_2}{\partial x} + \kappa \frac{8}{3\pi} \widehat{v}(F_1 u_1 + F_2 u_2) = 0, \qquad (19)$$

where z_1 is the free surface elevation for the dominant constituent and z_2 for the secondary constituent. Exploiting the linearity of Eq. (19), we can solve the two problems independently. As a result, we see that the actual friction term that is felt in Eq. (19) is different from that would be felt by the single constituent alone (Eq. 18).

Introducing a general form of the linearized momentum equation for the generic *i*-th constituent

$$\frac{\partial u_i}{\partial t} + g \frac{\partial z_i}{\partial x} + f_i r_i u_i = 0,$$
⁽²⁰⁾

with

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$$r_i = \kappa \frac{8}{3\pi} v_i \,, \tag{21}$$

250 as in the standard case, we see that the effective friction term contains a correction factor

$$f_i = \frac{F_i}{\varepsilon_i},\tag{22}$$

through the coefficient F_i . Since the ratio ε_i can be quite small for a weaker constituent, the friction actually felt can be significantly stronger.

4 Results

255 4.1 Hydrodynamic modeling incorporating the friction correction factor

If there are many tidal constituents, then the friction experienced by one is affected by the others. As suggested by our conceptual model, the mutual effects can be incorporated by using the friction correction factor f_n defined in Eq. (22) if the other (weaker) constituents are treated in the same way as the predominant constituent. As a result, the friction number χ_n for each tidal constituent can be modified as

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 $\chi_n = f_n \chi, \tag{23}$

where χ is the friction number (see definition in Table 1) experienced if only a single tidal constituent is considered.

We note that the modified friction number χ_n in Eq. (23) contains the friction coefficient K. In
many applications, K is calibrated separately for each tidal constituent to account for the different friction exerted due to the combined tide, either changing K directly or through calibration of the different correction friction factors f_n (see, e.g., Cai et al., 2015, 2016). The current study aims at avoiding the need to adjust K individually, so that only a single value of K can be calibrated, which is based on the physical consideration that friction mostly depends on bottom roughness, and the 270 other factors (tide interaction) are to be correctly modelled.

4.2 Procedure to study the propagation of the different constituents

With a hydrodynamic model for a single constituent (see Appendix A), an iterative procedure can be designed to study the propagation of the different constituents by calibrating a single value of the Manning-Strickler friction parameter K. The flow chart illustrating the computation process is pre-

- 275 sented in Figure 5. Initially, we assume the friction correction factor $f_i=1$ for each tidal constituent, and compute the first tentative values of velocity amplitude v_i along the channel using the hydrodynamic model. This allows defining \hat{v} and, hence, ε_i . Taking into account the frictional interaction between tidal constituents, the revised f_i is calculated using Eqs. (17) and (22). Subsequently, using the updated f_i , the new velocity amplitude v_i along the channel can be computed using the hydro-
- 280 dynamic model. This process is repeated until the result is stable. In this paper, two examples of Matlab scripts are provided together with the observed tidal data in the Guadiana and Guadalquivir estuaries (see Supporting Information).

It is worth stressing that the single constituents are not calibrated independently, as was done in previous analyses (e.g., Cai et al., 2015). Conversely, only a single friction parameter, K, is calibrated or estimated based on the physical knowledge of the system (bed roughness). This feature

285 calibrated or estimated based on the physical knowledge of the system (bed roughness). This feature represents a major advantage of the proposed method because the frictional interaction is modelled in mechanistic terms using Eq. (22).

4.3 Application to the Guadiana and Guadalquivir estuaries

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In this study, the analytical model for a semi-closed estuary presented in Section 2.1 was applied to the Guadiana and Guadalquivir estuaries to reproduce the correct tidal behavior for different tidal constituents. The analytical results were compared with observed tidal amplitude η and associated phase of elevation ϕ_A .

The morphology of the Guadiana estuary was represented in the model with a constant depth (5.5 m), an exponentially converging width (length scale, 38 km) and a constant storage ratio of

- 295 1 representative of the limited salt marsh areas (about 20 km², see Garel (2017)). The Manning-Strickler friction coefficient ($K = 42 \text{ m}^{1/3}\text{s}^{-1}$) was determined by calibrating the model outputs (obtained using the iterative procedure presented in section 4.2) with observations. It can be seen from Figure 6 that the computed tidal amplitude and phase of elevation are in good agreement with the observed values for different tidal constituents in the Guadiana estuary. The N_2 amplitude is
- 300 slightly overestimated in the central part of the estuary, which may suggest that the harmonic analysis has some difficulties to resolve this constituent in relation to the length of the considered time series (54 days). In support, the N_2 amplitude (0.16 m) from a longer time series (85 days) collected in 2017 at 58 km from the mouth matches better the model output, while results for other constituents are similar in 2015 and 2017 (Garel, unpublished data). Otherwise, the correspondence is poorest for
- 305 the semi-diurnal constituents at the most upstream station, owing to truncation of the lowest water levels by a sill located at about 65 km from the river mouth (Garel, 2017). Table 3 displays the mean friction correction coefficient f obtained from the iterative procedure to account for the nonlinear interaction between different tidal constituents. In particular, the mean friction correction factors ffor the minor constituents S_2 , N_2 , O_1 , and K_1 are 4.6, 8.1, 41.1, and 49.8, respectively.
- To understand the tidal dynamics between different tidal constituents along the Guadiana estuary, the longitudinal variations of the tidal damping/amplification number δ_A and celerity number λ_A (see their definitions in Table 1) are shown in Figure 7 where similar minor constituents in semidiurnal (S_2, N_2) and diurnal (O_1, K_1) band behave more or less the same. As shown in Figure 7a, the minor constituents S_2, N_2, O_1 , and K_1 experience more friction compared with the predominant
- 315 M₂ tide. Interestingly, we observe a stronger damping (δ_A < 0) of semidiurnal constituents (S₂, N₂) than those of diurnal constituents (O₁, K₁) in the seaward part of the estuary (around x=0-40 km) although the amplitudes of the diurnal constituents are less than those of the semidiurnal ones. In contrast, the amplification (δ_A > 0) of semidiurnal constituents (S₂, N₂) is more apparent than those of diurnal constituents (O₁, K₁) in the landward part of the estuary. For the wave celerity, as
- 320 expected the dominant M_2 tide travels faster (smaller λ_A) than minor tidal constituents. In addition, we observe that the wave celerity of semidiurnal tidal constituents is larger than those of diurnal constituents in the seaward reach (around x=0-30 km), while it is the opposite in the landward reach, which suggests a complex relation between tidal damping/amplification and wave celerity due to the combined impacts of channel convergence, bottom friction and reflected wave. It is important to

325 note that a standing wave pattern with celerity approaching infinity is produced near the sill due to the superimposition of the incident and reflected waves (see also Garel and Cai, 2018).

For the Guadalquivir estuary, the geometry can be approximated as a converging estuary with a width convergence length of b=65.5 km and a constant stream depth of about 7.1 m. A linear reduction of the storage width ratio of 1.5-1 was adopted over the reach 0-103 km. The observed

tidal amplitudes and phases are best reproduced by using the model for $K = 46 \text{ m}^{1/3} \text{s}^{-1}$ (see Figure 8). In general, the observed tidal properties (tidal amplitude and phase) of different constituents are well reproduced. The enhanced frictional coefficient f for minor constituents S_2 , N_2 , O_1 , and K_1 are 5.4, 9.7, 40.7, and 43.7, respectively (Table 3).

Figure 9 shows the longitudinal variations of tidal damping/amplification and wave celerity for

- 335 the Guadalquivir estuary. Similar to, which are similar to those in the Guadiana estuary. In general, we observe that the dominant M_2 -M₂ tide experiences less tidal damping and travels faster than other minor tidal constituents. It can be seen from Figure 9 that the magnitude of tidal damping is approximately one order larger than that in the Guadiana estuary (friction than other secondary semidiurnal tidal constituents although it travels at more or less the same speed in the seaward reach
- 340 (x=0-35 km). Unlike the Guadiana estuary, the damping experienced by the secondary semidiurnal tides is less than those of diurnal constituents near the estuary mouth (around x=0-7 km; Figure 9a)and hence, while the wave celerity is comparatively smaller (larger λ_A , consistently larger in the seaward reach (x=0-38 km; Figure 9b). Unlike Similar to the Guadiana estuary, the damping experienced by the minor we observe that the tidal damping for the secondary semidiurnal tides
- 345 is less stronger than those of diurnal constituents in the seaward reach central parts of the estuary (around x=0.55-7.52 km), while the wave celerity is consistently larger for the whole channel. whereas their amplifications are larger in the landward part of the estuary although their wave speeds are less.
 - In particular, the tidal damping along the first half of these two estuaries is mainly due to the
- 350 damping of the dominant M_2 wave owning to the fact that the impact of bottom friction dominates over the channel convergence. Along the upper reach, enhanced morphological convergence and reflection effects (that reduce the overall friction experienced by the propagating wave) result in the overall amplification of the tidal wave. For more details of the tidal hydrodynamics in these two estuaries, readers can refer to Garel and Cai (2018) for the Guadiana estuary and
- 355 Diez-Minguito et al. (2012) for the Guadalquivir estuary.

In order to clarify the behavior of different tidal constituents, we present Figure 10 showing the longitudinal variations of estuary shape number γ (representing the channel convergence) and friction number χ_n (representing the bottom friction), two major factors determining the tidal hydrodynamics, in both estuaries. Note that the variable estuary shape number γ observed in the

360 Guadalquivir estuary is due to the adoption of a variable storage width ratio r_S in the analytical model. On the one hand, the estuary shape numbers for diurnal tides are approximately twice

larger than those for semidiurnal tides (Figures 10a, d) due to the tidal frequency differences (see definition of γ in Table 1). On the other hand, the effective friction experienced by the diurnal tides is much larger than those of the semidiurnal tides due to the mutual interaction

- 365 between different tidal constituents (Figure 10b, e, see also Table 3). However, the propagation of different tidal constituents mainly depends on the imbalance between channel convergence and friction, except for those reaches where wave reflection matters (generally close to the head). In particular, in the seaward reach the tidal damping for each tidal constituent can be approximately estimated by $\delta_A = \gamma/2 - \chi_n \mu \cos(\phi)/(2\lambda_A)$ (see equation (20) by Cai et al., 2012)
- 370 . While the channel convergence effect (represented by $\gamma/2$) is much stronger for diurnal tides than for semidiurnal tides, the frictional effect (represented by $\chi_n\mu\cos(\phi)/(2\lambda_A)$) is only slightly larger (Figure 10c, f). Hence, diurnal tides experience a relatively lower damping in the seaward reach (Figures 7a and 9a). For instance, in the case of the Guadalquivir estuary, diurnal tides are more damped than semidiurnal tides near the estuary mouth (x=0-7 km). For the second (landward)
- 375 half of the estuary, the lower amplification experienced by diurnal tides is mainly due to the wave reflection from the closed end (see Garel and Cai, 2018).

The importance of mutual interaction between different tidal constituents is illustrated with the iteratively refined model implemented at both case studies (Figures 7 and 9). For comparison, Figure 11 shows the analytically computed damping/amplification number δ_A and celerity number

- 380 λ_A without considering mutual interaction (by setting $f_n=1$ in the model). In this case, the damping experienced by both secondary diurnal and semidiurnal tides are apparently underestimated due to the unrealistic friction adopted in the model (Figure 11a, c, see also Figures 7a and 9a, respectively). Similarly, the computed wave celerity for secondary tidal constituents are apparently overestimated due to the underestimated bottom friction (Figure 11b, d, see also Figures 7b and 9b, respectively).
- 385 To correctly reproduce the main features of different tidal waves, it is required to use the iteratively refined model proposed in this study.

5 Conclusions

In this study, we provide insight into the mutual interactions between one predominant (e.g., M_2) and other tidal constituents in estuaries and the role of quadratic friction on tidal wave propagation.

- 390 An analytical method exploiting the Chebyshev polynomials was developed to quantify the effective friction experienced by different tidal constituents. Based on the linearization of the quadratic friction, the conceptual model has been used to explore the nonlinear interaction of different tidal constituents, which enables them to be treated independently by means of an iterative procedure. Thus, an analytical hydrodynamic model for a single tidal constituent can be used to reproduce the
- 395 correct wave behavior for different tidal constituents. In particular, it was shown that a correction of the friction term needs to be used to correctly reproduce the tidal dynamics for minor tidal con-

stituents. The application to the Guadiana and the Guadalquivir estuaries shows that the conceptual model can interpret the nonlinear interaction reasonably well when combined with an analytical model for tidal hydrodynamics.

400 A crucial feature of the proposed approach is the deterministic description of the mutual frictional interaction among tidal constituents, which avoids the need of an independent calibration of the friction parameter for the single constituent. In this respect, further work is required to explore whether a reliable value of the friction coefficient estimated through this method can be parametrized based on observations of the bottom roughness of the estuary.

405 Appendix A

Analytical solutions of tidal hydrodynamics for a single tidal constituent

In this paper, analytical solutions for a semi-closed estuary proposed by Toffolon and Savenije (2011) were used to reproduce the longitudinal tidal dynamics along the estuary axis. The solution makes use of the parameters that are defined in Table 1.

410 The analytical solutions for the tidal wave amplitudes and phases are given by:

$$\eta = \zeta_0 \overline{h_0} |A^*|, \qquad v = r_S \zeta_0 c_0 |V^*|, \tag{A1}$$

$$\tan\left(\phi_{A}\right) = \frac{\Im\left(A^{*}\right)}{\Re\left(A^{*}\right)}, \qquad \tan\left(\phi_{V}\right) = \frac{\Im\left(V^{*}\right)}{\Re\left(V^{*}\right)}, \tag{A2}$$

where \Re and \Im are the real and image parts of the corresponding term, and A^* and V^* are unknown 415 complex functions varying along the dimensionless coordinate $x^* = x/L_0$:

$$A^* = a_1^* \exp\left(w_1^* x^*\right) + a_2^* \exp\left(w_2^* x^*\right),\tag{A3}$$

$$V^* = v_1^* \exp\left(w_1^* x^*\right) + v_2^* \exp\left(w_2^* x^*\right).$$
(A4)

For a tidal channel with a closed end, the analytical solutions for the unknown variables in Eqs. (A3) and (A4) are listed in Table 4, where Λ is a complex variable, defined as

$$\Lambda = \sqrt{\gamma^2/4 - 1 + i\widehat{\chi}}, \qquad \widehat{\chi} = \frac{8}{3\pi}\mu\chi, \tag{A5}$$

where the coefficient $8/(3\pi)$ stems from the adoption of Lorentz's linearization when considering only one single predominant tidal constituent (e.g., M_2).

Since the friction parameter $\hat{\chi}$ depends on the unknown value of μ (or v), an iterative procedure was used to determine the correct wave behavior. In addition, to account for the longitudinal variation of the cross-section (e.g., estuary depth) a multi-reach technique was adopted by subdividing the

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entire estuary into multiple sub-reaches and the solutions obtained by solving a set of linear equations with internal boundary conditions at the junction of the sub-reaches satisfying the continuity condition (see details in Toffolon and Savenije, 2011).

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For given computed values of A^* and V^* , the dependent parameters defined in Table 1 can be computed using the following equations:

$$\mu = |V^*|, \qquad \phi = \phi_V - \phi_A, \tag{A6}$$

$$\delta_A = \Re\left(\frac{1}{A^*}\frac{\mathrm{d}A^*}{\mathrm{d}x^*}\right), \qquad \delta_V = \Re\left(\frac{1}{V^*}\frac{\mathrm{d}V^*}{\mathrm{d}x^*}\right),\tag{A7}$$

435
$$\lambda_A = \left|\Im\left(\frac{1}{A^*}\frac{\mathrm{d}A^*}{\mathrm{d}x^*}\right)\right|, \qquad \lambda_V = \left|\Im\left(\frac{1}{V^*}\frac{\mathrm{d}V^*}{\mathrm{d}x^*}\right)\right|. \tag{A8}$$

Appendix B

Coefficients of the Godin's expansion

The following trigonometric equation

440
$$\cos^3(\omega_1 t) = \frac{3}{4}\cos(\omega_1 t) + \frac{1}{4}\cos(3\omega_1 t),$$
 (B1)

is used to convert the third-order terms of Eq. (7) to the harmonic constituents. For a single harmonic, it follows that

$$u|u| = v_1^2 \left[\left(\alpha + \frac{3}{4}\beta \right) \cos(\omega_1 t) + \frac{1}{4}\beta \cos(3\omega_1 t) \right].$$
(B2)

For two harmonic constituents, the Chebyshev polynomials approximation of u|u| is expressed as

445
$$u|u| = v_1^2 \left\{ \alpha \left[\varepsilon_1 \cos(\omega_1 t) + \varepsilon_2 \cos(\omega_2 t) \right] + \beta \left[\varepsilon_1 \cos(\omega_1 t) + \varepsilon_2 \cos(\omega_2 t) \right]^3 \right\}.$$
 (B3)

In Eq. (B3), the cubic term can be expanded as

$$[\varepsilon_1 \cos(\omega_1 t) + \varepsilon_2 \cos(\omega_2 t)]^3 = \varepsilon_1^3 \cos^3(\omega_1 t) + 3\varepsilon_1 \varepsilon_2^2 \cos(\omega_1 t) \cos^2(\omega_2 t) + 3\varepsilon_2 \varepsilon_1^2 \cos(\omega_2 t) \cos^2(\omega_1 t) + \varepsilon_2^3 \cos^3(\omega_2 t).$$
(B4)

Making use of the trigonometric equations to expand the power of the cosine functions (e.g., cos³(ω₁t) and cos²(ω₁t)) and extracting only the harmonic terms with frequencies ω₁ and ω₂, Eq.
450 (B3) can be reduced to Eq. (12).

14

For the case of many constituents, here we only provide the exact coefficients for n=3:

$$F_1 = \frac{3\pi}{8} \left[\alpha + \beta \left(\frac{3}{4} \varepsilon_1^2 + \frac{3}{2} \varepsilon_2^2 + \frac{3}{2} \varepsilon_3^2 \right) \right] = \frac{1}{5} \left(2 + 3\varepsilon_1^2 + 6\varepsilon_2^2 + 6\varepsilon_3^2 \right), \tag{B5}$$

$$F_{2} = \frac{3\pi}{8} \left[\alpha + \beta \left(\frac{3}{4} \varepsilon_{2}^{2} + \frac{3}{2} \varepsilon_{1}^{2} + \frac{3}{2} \varepsilon_{3}^{2} \right) \right] = \frac{1}{5} \left(2 + 3\varepsilon_{2}^{2} + 6\varepsilon_{1}^{2} + 6\varepsilon_{3}^{2} \right), \tag{B6}$$

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$$F_{3} = \frac{3\pi}{8} \left[\alpha + \beta \left(\frac{3}{4} \varepsilon_{3}^{2} + \frac{3}{2} \varepsilon_{1}^{2} + \frac{3}{2} \varepsilon_{2}^{2} \right) \right] = \frac{1}{5} \left(2 + 3\varepsilon_{3}^{2} + 6\varepsilon_{1}^{2} + 6\varepsilon_{2}^{2} \right).$$
(B7)

Equations (B5) to (B6) reduce to Eqs. (13) and (14) when $\varepsilon_3 = 0$ (i.e., $v_3=0$).

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 Table 1. Definitions of dimensionless parameters.

Independent parameters	Dependent parameters
Tidal amplitude at the mouth	Tidal amplitude
$\zeta_0 = \eta_0 / \overline{h_0}$	$\zeta=\eta/\overline{h}$
Friction number at the mouth	Friction number
$\chi_0 = r_S c_0 \zeta_0 g / \left(K^2 \omega \overline{h_0}^{4/3} \right)$	$\chi = r_S c_0 \zeta g / \left(K^2 \omega \overline{h}^{4/3} \right)$
Estuary shape	Velocity number
$\gamma = c_0/(\omega a)$	$\mu = v/(r_S \zeta c_0) = v \overline{h}/(r_S \eta c_0)$
Estuary length	Damping number for water level
$L_e^* = L_e/L_0$	$\delta_A = c_0 \mathrm{d}\eta / (\eta \omega \mathrm{d}x)$
	Damping number for velocity
	$\delta_V = c_0 \mathrm{d}\upsilon / (\upsilon \omega \mathrm{d}x)$
	Celerity number for water level
	$\lambda_A = c_0/c_A$
	Celerity number for velocity
	$\lambda_V = c_0/c_V$
	Phase difference
	$\phi = \phi_V - \phi_A$

	Amplitude (m)							
<i>x</i> (km)	M_{sf}	O_1	K_1	N_2	M_2	S_2	M_4	M_6
2.4	0.01 (0.03)	0.06 (0.01)	0.07 (0.01)	0.23 (0.01)	0.97 (0.01)	0.37 (0.02)	0.02 (0.00)	0.01 (0.00)
10.7	0.01 (0.07)	0.06 (0.01)	0.07 (0.01)	0.22 (0.01)	0.93 (0.01)	0.34 (0.01)	0.02 (0.01)	0.01 (0.00)
22.8	0.03 (0.04)	0.06 (0.01)	0.07 (0.01)	0.20 (0.02)	0.86 (0.02)	0.29 (0.02)	0.04 (0.01)	0.02 (0.01)
33.9	0.06 (0.05)	0.06 (0.01)	0.07 (0.01)	0.20 (0.02)	0.85 (0.02)	0.27 (0.02)	0.04 (0.01)	0.03 (0.01)
43.6	0.06 (0.06)	0.06 (0.01)	0.07 (0.01)	0.21 (0.02)	0.87 (0.02)	0.27 (0.02)	0.05 (0.01)	0.03 (0.01)
51.4	0.05 (0.05)	0.06 (0.01)	0.07 (0.01)	0.22 (0.02)	0.90 (0.02)	0.28 (0.02)	0.07 (0.01)	0.03 (0.01)
60.1	0.07 (0.06)	0.06 (0.01)	0.07 (0.01)	0.22 (0.02)	0.93 (0.02)	0.30 (0.02)	0.08 (0.01)	0.04 (0.01)
69.6	0.10 (0.06)	0.06 (0.01)	0.06 (0.01)	0.19 (0.03)	0.78 (0.03)	0.24 (0.03)	0.16 (0.03)	0.02 (0.01)
Phase (°)								
2.4	190 (149)	310 (6)	73 (5)	54 (4)	62 (1)	93 (2)	151 (8)	219 (18)
10.7	8 (190)	319 (7)	85 (6)	68 (3)	75 (1)	108 (3)	103 (14)	237 (15)
22.8	38 (66)	331 (9)	103 (7)	87 (4)	93 (1)	130 (3)	131 (12)	294 (16)
33.9	49 (56)	343 (7)	116 (6)	104 (5)	109 (1)	151 (4)	166 (8)	336 (11)
43.6	51 (58)	348 (8)	123 (8)	116 (5)	121 (1)	166 (4)	189 (6)	12 (14)
51.4	48 (48)	352 (9)	128 (8)	123 (6)	128 (1)	175 (5)	203 (5)	43 (19)
60.1	53 (58)	356 (9)	133 (8)	131 (6)	135 (1)	184 (5)	219 (4)	69 (21)
69.6	51 (43)	7 (9)	146 (8)	146 (9)	148 (2)	200 (7)	261 (11)	15 (18)

Table 2. Tidal elevation amplitudes (m) and phases (°) estimates (with 95% confidence intervals in brackets) from harmonic analyses of pressure records along the Guadiana estuary (x: distance from the mouth, km).

Table 3. Mean correction friction factor f for different tidal constituents along the Guadiana and the Guadalquivir estuaries.

Tidal constituents	M_2	S_2	N_2	K_1	O_1
Guadiana	1.1	4.6	8.1	41.1	49.8
Guadalquivir	1.1	5.4	9.7	40.7	43.7

Table 4. Analytical expressions for unknown complex variables for the case of a closed estuary.

a_{1}^{*}, a_{2}^{*}	v_1^*, v_2^*	w_1^*, w_2^*
$a_1^* = \left[1 + \exp\left(\Lambda L_e^*\right) \frac{\Lambda + \gamma/2}{\Lambda - \gamma/2} ight]^{-1}$	$v_1^* = rac{-ia_1^*}{\Lambda - \gamma/2}$	$w_1^*=\gamma/2+\Lambda$
$a_2^* = 1 - a_1^*$	$v_2^* = \frac{i(1-a_1^*)}{\Lambda + \gamma/2}$	$w_2^*=\gamma/2-\Lambda$



Figure 1. Geometry of a semi-closed estuary and basic notation (after Savenije et al. (2008)). HW, high water; LW, low water.



Figure 2. Tidally averaged depth (m, black dots), width (m, blue dots) and cross-sectional area (m², green dots) along the Guadiana estuary. Red lines represent exponential fit curves for the width and cross-sectional area.



Figure 3. Approximation to the quadratic velocity u|u| by the Chebyshev polynomials approach for the case of two tidal constituents (i.e., M_2 and K_1). Here, $u = 0.6 \cos(\omega_1 t) + 0.2 \cos(\omega_2 t)$, where ω_1 and ω_2 represent the tidal frequencies of M_2 and K_1 , respectively.



Figure 4. Computed effective friction coefficients F_1 (a) and F_2 (b) from Eqs. (13) and (14) as a function of ε_1 .



Figure 5. Computation process for tidal properties of different constituents in an estuary.



Figure 6. Tidal constituents (a) M_2 ; (b) S_2 ; (c) N_2 ; (d) K_1 ; (e) O_1 : modelled against observed values of tidal amplitude (m) and phase (°) of elevation along the Guadiana estuary.



Figure 7. Longitudinal variations of tidal damping/amplification number δ_A (a) and wave celerity number λ_A (b) for different tidal constituents along the Guadiana estuary.



Figure 8. Tidal constituents (a) M_2 ; (b) S_2 ; (c) N_2 ; (d) K_1 ; (e) O_1 : modelled against observed values of tidal amplitude (m) and phase (°) of elevation along the Guadalquivir estuary.



Figure 9. Longitudinal variations of tidal damping/amplification number δ_A (a) and wave celerity number λ_A (b) for different tidal constituents along the Guadalquivir estuary.



Figure 10. Longitudinal variations of estuary shape number γ (a, d), friction number χ_{R} (b, e) and $\chi_{R}\mu\cos(\phi)/(2\lambda_{A})$ (c, f) in the Guadiana estuary (a, b, c) and Guadalquivir estuary (d, e, f).



Figure 11. Longitudinal variations of damping/amplification number δ_A (a, c) and celerity number λ_A (b, d) in the Guadiana estuary (a, b) and Guadalquivir estuary (c, d) in the absence of mutual interaction between different tidal constituents.