Supplementary Information

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S1 Definitions and illustrations for selected ice edge metrics

Most definitions were provided in the main text. Here we present some supplementary information which mainly is concerned with metrics that were not included in the recommended set in Sect. 6.3 in the main text. Also, two figures are included to provide readers with illustrative information regarding the definition and interpretation of some metrics.

S1.1 Separation based IIEE displacement metrics

Provided that the model initialization of the sea ice fraction is close to the observed ice edge fraction at that time, IIEE areas can be expected to emerge as the model ice edge drifts away from the observed edge with an increasing forecast lead time. This evolution is expected to frequently give rise to elongated IIEE areas, and we here adopt the maximum distance inside an IIEE area as the scaling length.

An illustrative example for IIEE and derived metrics is provided in Fig. S1. Here, gray shaded grids represents grids in IIEE area ia, while white grids are outside of the IIEE domain. The scaling length l^{ia}_{max} is indicated by the dashed line. Note that when computing the scaling length we have chosen not to include IIEE grids with only a single IIEE grid neighbour (given by light gray shading in the figure).

Since the definitions of a^{ia} and l^{ia}_{max} take adjacent dry nodes into account, we adopt the hatted notation as introduced in Sect. 2.1 in the main text. The resulting displacement for this area is given as

$$\hat{d}^{ia}_{IIEE} = a^{ia}/l^{ia}_{max} \tag{S1}$$

Note that in theory, a node may be adjacent to two IIEE areas. In such cases, we divide the node's area equally between the two relevant IIEE areas.

A solitary IIEE node is formally treated as a separate IIEE area, with scaling length set to the (average) resolution. Furthermore, let A_0 be the area of the grids where the two ice edges overlap. Letting N_A be the number of IIEE areas, we introduce a set of four corresponding displacement metrics here.

1. The root-mean-squared displacement:

$$\widehat{D^{IIEE}_{RMS}} = \left[\frac{\sum_{ia=1}^{N_A} a^{ia}\left(\hat{d}^{ia}_{IIEE}\right)^2}{A_0 + \sum_{ia=1}^{N_A} a^{ia}}\right]^{1/2} \tag{S2}$$

2. The average displacement:

$$\widehat{D^{IIEE}_{AVG}} = \frac{\sum_{ia=1}^{N_A} a^{ia} \hat{d}^{ia}_{IIEE}}{A_0 + \sum_{ia=1}^{N_A} a^{ia}} \tag{S3}$$

3. The displacement bias:

$$\widehat{\Delta^{IIEE}} = \frac{\sum_{ia^+=1}^{N_A^+} a^{ia^+} \hat{d}^{ia^+}_{IIEE}}{A_0/2 + \sum_{ia^+=1}^{N_A^+} a^{ia^+}} - \frac{\sum_{ia^-=1}^{N_A^-} a^{ia^-} \hat{d}^{ia^-}_{IIEE}}{A_0/2 + \sum_{ia^-=1}^{N_A^-} a^{ia^-}} \tag{S4}$$

4. The maximum displacement:

$$\widehat{D^{IIEE}_{MAX}} = \max(\hat{d}^{ia}) \tag{S5}$$

In order to shed some light on the relation between the D^{IIEE} metric and $\widehat{D^{IIEE}}$ we consider an idealized case where two products' ice edges are y symmetric to each other, and form IIEE in the shape of two rectangles, connected by a line where the edges overlap. Now, take the width (in the x-direction) of the rectangles to be w_1 and w_2 grids respectively, while the length of the mutual edge in between is w_0 grids. The height of the two rectangles are set to h_1 and h_2 grids, respectively. Then, for D^{IIEE}_{AVG} we have

$$\begin{aligned} A^{IIEE} &= w_1 \cdot h_1 + w_2 \cdot h_2, \\ L &= h_1 + w_1 + w_o + h_2 + w_2 \end{aligned} \tag{S6}$$

where L is the ice edge length for both products. Consequently,

$$D^{IIEE}_{AVG} = \frac{w_1 \cdot h_1 + w_2 \cdot h_2}{h_1 + w_1 + w_0 + h_2 + w_2} \tag{S7}$$

To determine $\widehat{D^{IIEE}}$ we first find that

$$\begin{aligned} \hat{d}^{(1,2)}_{IIEE} &= w_{(1,2)} \cdot h_{(1,2)} / l^{(1,2)}_{max}, \\ l^{(1,2)}_{max} &= (w^2_{(1,2)} + h^2_{(1,2)})^{0.5} \end{aligned} \tag{S8}$$

Furthermore, $A_0 = w_0 \cdot 1$, and introducing these quantities into Eq. S3 we find

$$\widehat{D^{IIEE}_{AVG}} = \frac{w_1^2 \cdot h_1^2/(w_1^2 + h_1^2)^{0.5} + w_2^2 \cdot h_2^2/(w_2^2 + h_2^2)^{0.5}}{w_0 \cdot 1 + w_1 \cdot h_1 + w_2 \cdot h_2} \tag{S9}$$

Now consider some selected cases:

Case 1 Identical squares, *i.e.*, $w_1 = w_2 = h_1 = h_2 = w$; $w_0 = \nu w$. Then,

$$\frac{\widehat{D^{IIEE}_{AVG}}}{D^{IIEE}_{AVG}} = \frac{1 + \nu/4}{1 + \nu/(2w)} \sqrt{2} \geq \sqrt{2} \tag{S10}$$

To take an example, assume that the squares have sides with 20 grids. Then, if $\nu = 1/4$ (the squares are 5 grids apart) the fraction in Eq. S10 is approximately 1.5. If $\nu = 4$ (a separation of 80 grids) the fraction has a value of about 3.

Case 2 Different sized squares, *i.e.*, $w_1 = h_1 = w;\ \ w_2 = h_2 = \alpha w;\ \ w_0 = \nu w$. Then,

$$\frac{\widehat{D^{IIEE}_{AVG}}}{D^{IIEE}_{AVG}} = \frac{1+\alpha^3}{1+\alpha^2}\ \frac{1+\alpha+\nu/2}{1+\alpha^2+\nu/w}\sqrt{2} \tag{S11}$$

Consider the case $\alpha = 1/4$, and set $w = 20$ grids. Then, the fraction in Eq. S11 becomes about 1.7 and 2.5 when we set $\nu = 1/4$ and $\nu = 4$, respectively.

Case 3 Identical rectangles, *i.e.*, $w_1 = w_2 = w;\ \ h_1 = h_2 = \delta w;\ \ w_0 = \nu w$. Then,

$$\frac{\widehat{D^{IIEE}_{AVG}}}{D^{IIEE}_{AVG}} = \frac{1}{\sqrt{1+\delta^{-2}}}\ \frac{1+\delta^{-1}(1+\nu/2)}{1+\delta^{-1}\nu/(2w)} \tag{S12}$$

In the model results, the IIEE areas are usually elongated in the direction parallell to the main direction of the ice edge, *i.e.*, $\delta < 1$. When we investigate the case $\delta = 1/4$ and again set $w = 20$ grids, the fraction in Eq. S12 becomes approximately 1.35 and 2.3 for $\nu = 1/4$ and $\nu = 4$, respectively.

Based on these idealized examples, we will expect that the definition of $\widehat{D^{IIEE}_{AVG}}$ leads to values that are larger than the corresponding values for D^{IIEE}_{AVG}. If the results from the idealized examples are representative in operational applications, the ratio of these quantities will be in the approximate range of 1.5-3.

S1.2 Fractions skill score

An idealized example provided to shed light on FSS metrics is given in Fig. S2. Here, two gridded contour lines are displayed by filled boxes. On the original grid the two lines extend over 9 and 12 grid cells, respectively, including four cells where they overlap. Let us associate the gridded line shown by light gray and black boxes with observations of the sea ice edge, and take the dark gray and black boxes to represent a model result. Then, the ice edge fractions for a neighbourhood size $n = 3$ becomes

$$I_o^{n=3} = \frac{1}{9}\begin{pmatrix} 0 & 0 & 0 \\ 3 & 1 & 3 \\ 0 & 2 & 0 \end{pmatrix} \quad ; \quad I_M^{n=3} = \frac{1}{9}\begin{pmatrix} 2 & 0 & 2 \\ 2 & 1 & 3 \\ 0 & 2 & 0 \end{pmatrix} \tag{S13}$$

and we find that $\mathrm{MSE}^{n=3} = 9/9^3$, $\mathrm{MSE}^{n=3}_{\mathrm{ref}} = 49/9^3$ and the fractions skill score for a neighbourhood size $n = 3$ is $\mathrm{FSS}^{n=3} = 40/49 \approx 0.82$. For the skill score with the original 9×9 grid we have $\mathrm{MSE}^{n=1} = 13/81$, $\mathrm{MSE}^{n=3}_{\mathrm{ref}} = 21/81$, and we find that $\mathrm{FSS}^{n=1} = 8/21 \approx 0.38$.

Moreover, we note from Eq.s 18 and 19 in the main text that the FSS score will not change if we introduce a set of additional grids where neither product has an ice edge, provided that non-events dominate events (*i.e.*, the first term in Eq. 19 is used, here, that the number of nodes without an ice edge is larger than the number of edge nodes). This observation has consequences for two different aspects in the present study.

First, when modeling the ocean, dry nodes are usually not considered to be part of the computational domain, and are assigned a special value in numerical results. When integrating over a neighbourhood as in Eq. 22 in the main text one option would be to discard the grids that are dry in the original representation. We will then be left with a result which has a non-constant neighbourhood size of n^2 where dry nodes are not present, and $< n^2$ for neighbourhoods where dry nodes are present. Here, we choose to avoid the problem of non-constant neighbourhood sizes by adopting $I_o = I_m = 0$ for dry grids.

Second, the grid for *n=3* indicated by thick lines in Fig. S2 is only one of nine possible configurations. Since the FSS results are not affected by additional grids where neither product has an ice edge, we can expand the original domain by adding a padding region of *n-1* grids. In the case of *n=3* all configurations are attained by shifting the neighbourhood by 0, 1 and 2 original grids in both directions. The average FSS score from all of the configurations will be used, since the alternative is a set of results that will depended on an arbitrary configuration subset choice.

S2 Map of GODAE regions

The map of GODAE regions in the Arctic Ocean and adjacent seas, which was referred to near the end of the main text, is available as Fig. S3.

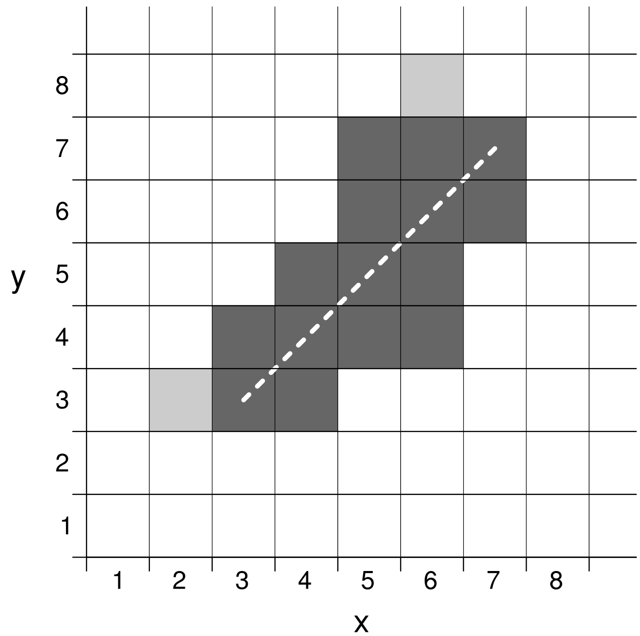

Figure S1. Illustration for scaling length of continuous IIEE areas. Here, the IIEE area is shown as gray shaded grids, which in this example is a 17 grid area. When determining the scaling length, IIEE area grids with only one IIEE area grid neighbour are disregarded (light gray shading). The scaling length is then set to the largest distance between the centers of the remaining IIEE area grids. This distance is indicated by the white dashed line. The displacement given by Eq. S1 in the metrics defined in Sect. S1.1 of this continuous IIEE area is then the area (17 grids) divided by its scaling length.

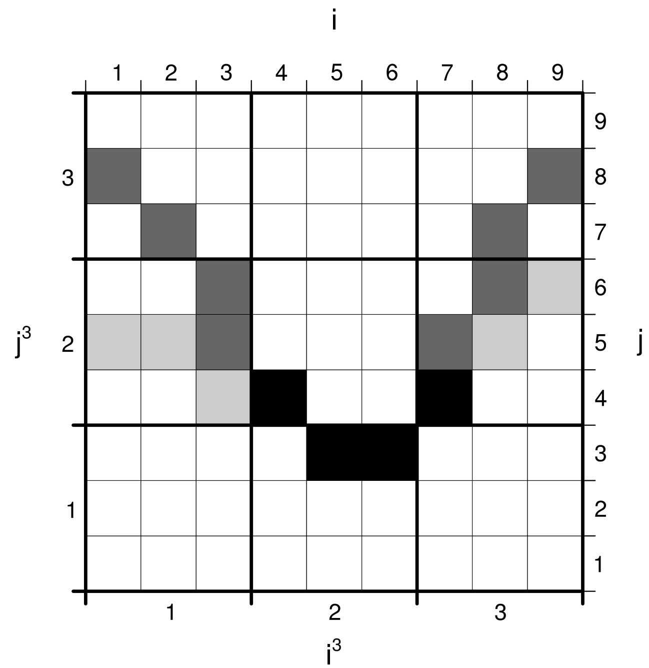

Figure S2. Illustration for computation of fractions skill score for gridded contour lines. One of the gridded lines is shown as light gray boxes, whereas the other is shown as dark gray. Grids where the two lines overlap are black. The original grid is displayed by thin grid lines with x-axis indices at the top and y-axis indices to the right. Thick grid lines correspond to the grid with $n = 3$, with x- and y-axis indices at the bottom and to the left, respectively. See the text for details.

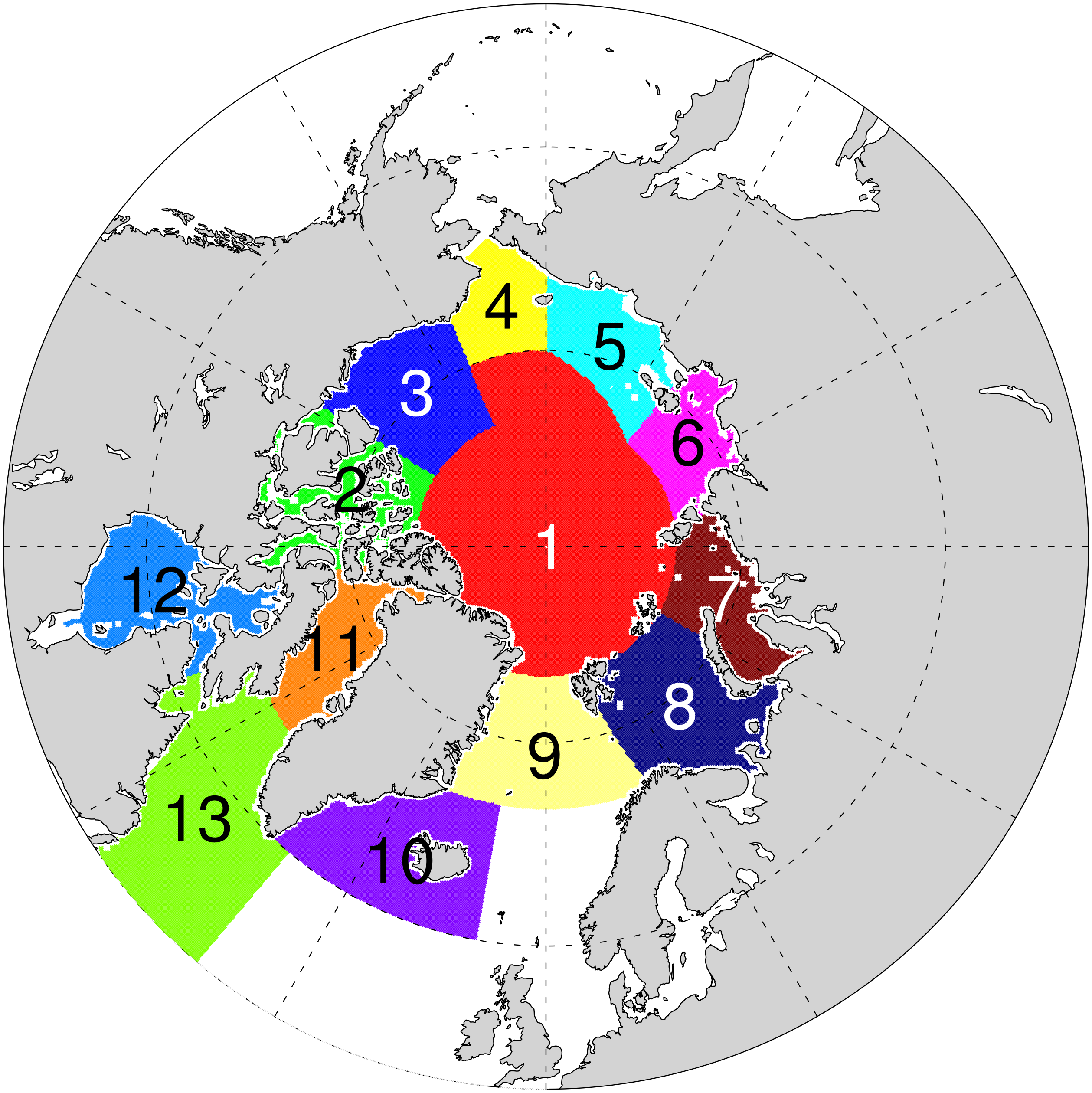

Figure S3. Arctic sub-regions as defined in GODAE OceanView. The numbered regions are (1) Arctic Deep Basin, (2) Queen Elisabeth Islands, (3) Beaufort Sea, (4) Chuckchi Sea, (5) Siberian Sea, (6) Laptev Sea, (7) Kara Sea, (8) Barents Sea, (9) Greenland Basin, (10) Southeast Greenland, (11) Baffin Bay, (12) Hudson Bay, and (13) Labrador Sea.