S1 Flow over the Scottish shelf from altimetry

In order to get a more complete estimate of the total Atlantic inflow to the Arctic Mediterranean, we extend the monitoring section across the Faroe-Shetland Channel (FSC) so that it crosses the Scottish shelf all the way to the Orkneys and use data from satellite altimetry to estimate the variations in volume transport through this section.

For a completely barotropic flow - where the along-flow velocity $v(x,t)$ only depends on the cross-flow coordinate x and time t - over a flat bottom at depth D, the volume transport between point x_1 and point x_2 is:

$$Q(t) = \int_{x_1}^{x_2} v(x,t) \cdot D \cdot dx = \int_{x_1}^{x_2} \frac{g}{f} \cdot \frac{dh(x,t)}{dx} \cdot D \cdot dx = \frac{g \cdot D}{f} \cdot [h(x_2,t) - h(x_1,t)] \qquad (S1)$$

where $h(x,t)$ is sea level height and we have assumed geostrophic balance. The assumption of a flat bottom is seldom justified, but the same relationship holds for a varying bottom depth if the velocity is spatially homogeneous and D is the average bottom depth along the section.

If reliable values for absolute sea level height are known, then this relationship can be used to calculate absolute values for volume transport. Even without this requirement, we can, however, calculate the temporal variations in volume transport from the temporal variations in sea level height, the "Sea Level Anomalies" (SLA). If the average volume transport can be estimated in some other way, then a time series of absolute volume transport can be generated by combining these two terms.

In the region between the Faroe Islands and Scotland, the horizontal variation in absolute sea level (mean dynamic topography) seems to be too smooth. We therefore use satellite altimetry only to look at the variations in volume transport. The SLA data were downloaded (http://www.aviso.altimetry.fr) from the global gridded ($0.25° \times 0.25°$) AVISO+ data set. In this data set, the two points e_1 and e_2 in Figure S1 straddle the FSC and the difference in SLA values: $\Delta h_{12}(t) = e_2(t)-e_1(t)$ has been used to generate the time series for FS-inflow in our data set based on the analysis by Berx et al. (2013).

In Figure S1, we have added three more points, e_3, e_4, and e_5, along a section that crosses the Scottish shelf in continuation of the section across the channel (Table S1). As long as the flow is barotropic as well as geostrophic, the SLA-difference between e_2 and the other three points on the shelf ought to represent the variations of the current on the shelf. From Figure S2, the seasonal transport variation on the shelf is similar, although not identical, in timing to that of the FS-inflow. The curves for e_4-e_2 and e_5-e_2 in the figure are very similar, indicating little net flow between e_4 and e_5. Since e_5, is almost in the North Sea, we therefore choose the difference $\Delta h_{24}(t) = e_4(t)-e_2(t)$ to represent the current over the Scottish shelf. Using Eq. (S1) with an average bottom depth $D = 100$ m together with this series and the information in the main document on the average transport, we have generated monthly values for ES-inflow.

In summary, our use of the Aviso+ altimetry data is based on assumptions of barotropy and homogeneity, which are not likely to be fulfilled especially during summer when parts of the section may be stratified. The variations in volume transport that they indicate are, however, probably of the correct order of

magnitude. Thus, the altimetry data indicate a seasonal variation with maximum (northeastward) volume transport in January and an amplitude on the order of 0.2 Sv.

S2 Iterative determination of seasonal and long-term variations

The time series considered in this study may be seen as super-positions of slowly varying signals + seasonal signals + random variations. We use an iterative decomposition method to separate seasonal and long-term variations. The seasonal variation generally has a roughly sinusoidal shape, and a simple analysis may be made by regressing the time series on a sinusoidal seasonal variation, where the phase lag is varied to give maximum correlation. The longterm variation may then be calculated as a running mean of de-seasoned values. We have chosen to average over a 3-year period for the running mean. This choice is somewhat arbitary, but driven by data availability. From the determined long-term variation, a new estimate of seasonal variation can then be calculated.

This procedure is repeated iteratively and rapidly converges so that we obtain both a seasonal signal that is less contaminated by long-term variations and a time series of a 3-year-running mean, which is the average of all the de-seasoned values within each 3-year period. Since our time series have monthly resolution, there will ideally be 36 values to be averaged per 3-year mean, but gaps in the time series reduce this number in practice. In addition to the average, we also calculate the standard error of each 3-year mean value. As a simple method to correct for serial correlation, the number of months (sample size) is replaced by the "equivalent sample size" (von Storch, 1999) calculated from the autocorrelation function of the de-seasoned time series. With the traditional assumption of normal distribution, the standard error is then converted into a rough estimate of the 95 % confidence limit through multiplication by 1.96.

References

von Storch H. (1999) Misuses of Statistical Analysis in Climate Research. In: von Storch H., Navarra A. (eds) Analysis of Climate Variability. Springer, Berlin, Heidelberg

Table S1. Location and altimetric characteristics of the five altimetry grid points selected. Avg and Std indicate average and standard deviation, respectively, of the SLA-values at each grid point.

Point	Latitude	Longitude	Avg (cm)	Std (cm)
e1	61.125°N	6.125°W	3.0	5.2
e2	60.125°N	3.875°W	3.2	5.9
e3	59.875°N	3.375°W	3.3	7.4
e4	59.625°N	2.875°W	3.4	9.0
e5	59.375°N	2.375°W	3.3	9.3
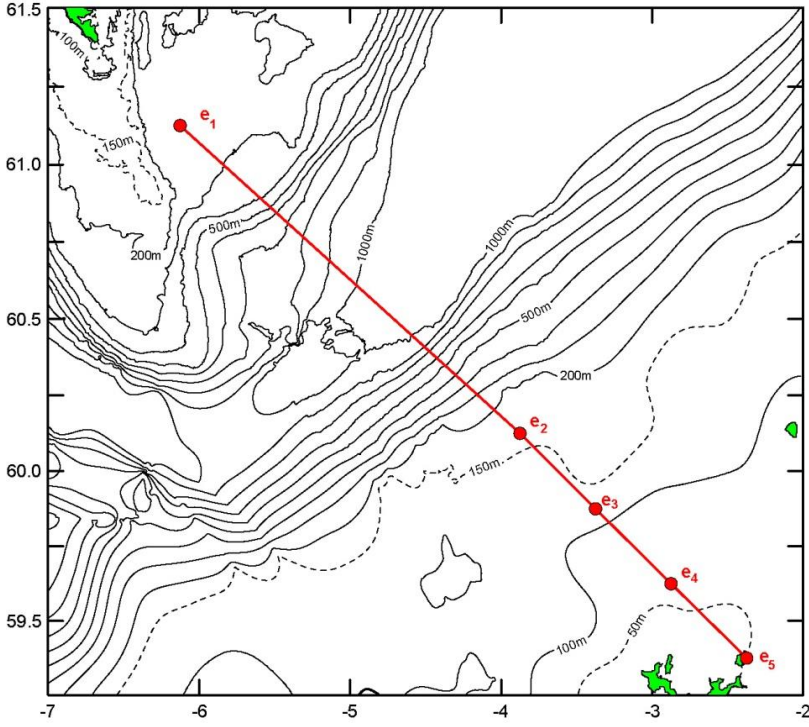
Figure S1. Bottom topography of the region with the five selected altimetry grid points shown by red circles.
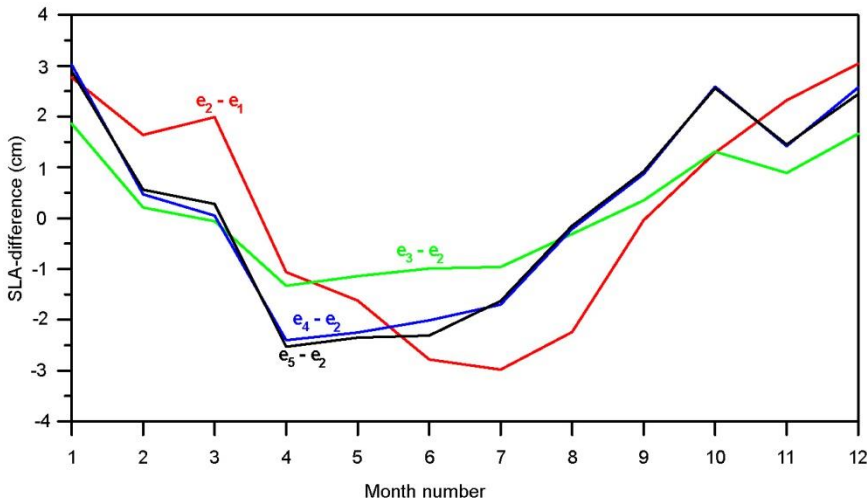
Figure S2. Monthly averages of the difference in SLA-values between pairs of altimetry points. The red curve is across the FSC ($e_2 - e_1$). The other curves are between e_2 and the other three points on the Scottish shelf. The period is January 1, 1993 to December 31, 2016.