

Numerical modeling of surface wave development under the action of wind

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Abstract

The numerical modeling of two-dimensional surface wave development under the action of wind is performed. The model is based on three-dimensional equations of potential motion with free surface written in a surface-following non-orthogonal curvilinear coordinate system where depth is counted from moving surface. Three-dimensional Poisson equation for velocity potential is solved iteratively. Fourier transform method, the second-order accuracy approximation of vertical derivatives on a stretched vertical grid and the fourth-order Runge-Kutta time stepping are used. Both the input energy to waves and dissipation of wave energy are calculated on the basis of the earlier developed and validated algorithms. A one-processor version of the model for PC allows us to simulate an evolution of wave field with thousands of degrees of freedom over thousands of wave periods. A long-time evolution of two-dimensional wave structure is illustrated by the spectra of wave surface and input and output of energy.

1. Introduction

Phase resolving modeling of sea waves is the mathematical modeling of surface waves including explicit simulations of surface elevation and velocity field evolution. Compared with the *spectral wave modeling*, the phase resolving modeling is more general since it reproduces a real visible physical process and is based on the well-formulated full equations. The phase resolving models usually operate with a large number of freedom degrees. In general, this method is more complicated and requires more computational resources. The simplest way of such modeling is the calculation of a wave field evolution based on the linear equations. Such approach allows reproducing the main effects of the linear wave transformation due to superposition of wave modes, reflections, refractions etc. This approach is useful for many technical applications while it cannot reproduce a nonlinear nature of waves and the transformation of wave field due to nonlinearity. Another example of a relatively simple object is a case of the shallow-water waves. The nonlinearity can be taken into account in the more sophisticated models derived from the fundamental fluid mechanics equations with some simplifications. The most popular approach is based on nonlinear Schrödinger equation of different orders (see Dysthe, 1979) obtained by expansion of the *surface wave* displacement. This approach is also used for solving the problem of *freak waves*. The main advantage of a simplified approach is that it allows reducing a three-dimensional (3-D) problem to a two-dimensional one (or 2-D problem to 1-D problem). However, it is not always clear which of the non-realistic effects are eliminated or included in the model after simplifications. This is why the most general approach being developed over the past years is based on the initial two-dimensional or three-dimensional equations (still potential). All the tasks based on these

46 equations can be divided into two groups: the periodic and non-periodic problems. An
47 assumption of periodicity considerably simplifies construction of numerical models though such
48 formulation can be applied to the cases when the condition of periodicity is acceptable, for
49 example, when domain is considered as a small part of a large uniform area. For limited domains
50 with no periodicity the problem becomes more complicated since the Fourier presentation cannot
51 be used directly.

52 From the point of view of physics, the problem of phase resolving modeling can be
53 divided into two groups: the adiabatic and non-adiabatic modeling. A simple adiabatic model
54 assumes that the process develops with no input or output of energy. Being not completely free
55 of limitations, such formulation allows investigating the wave motion on the basis of true initial
56 equations. Including the effects of input energy and its dissipation is always connected with the
57 assumptions that generally contradict an assumption of potentiality, i.e., new terms added to the
58 equations should be referred to as pure phenomenological. This is why treatment of a non-
59 adiabatic approach is often based on quite different constructions.

60 All the phase resolving models use the methods of computational mathematics and inherit
61 all their advantages and disadvantages, i.e., on one side, the possibility of a detailed description
62 of the processes, on the other side, a bunch of the specific problems connected with the
63 computational stability, space and time resolution. Mathematical modeling produces tremendous
64 volumes of information the processing of which can be more complicated than the modeling
65 itself.

66 The phase resolving wave modeling takes a lot of computer time since it normally uses a
67 surface-following coordinate system, which considerably complicates the equations. The most
68 time-consuming part of the model is an elliptic equation for velocity potential usually solved
69 with iterations. Luckily, for a two-dimensional problem this trouble is completely eliminated by
70 use of the conformal coordinates reducing the problem to a one dimensional system of equations
71 which can be solved with high accuracy (Chalikov and Sheinin, 1998). For a three-dimensional
72 problem, the reduction to a two-dimensional form is evidently impossible; hence, the solution of
73 a 3D elliptical equation for velocity potential becomes an essential part of the entire problem.
74 This equation is quite similar to the equation for pressure in a non-potential problem. It follows
75 that the 3-D Euler equations, being more complicated, still can be solved over acceptable
76 computer time.

77 There is a large volume of papers devoted to the numerical methods developed for
78 investigation of wave processes over the past decades. It includes a Finite Difference Method
79 (Engsig-Karup et al., 2009, 2012), , a Finite Volume Method (Causon et al., 2010), a Finite
80 Element Method (Ma and Yan, 2010; Greaves, 2010), a Boundary (Integral) Element Method
81 (Grue and Fructus, 2010), Spectral Methods (Ducroset et al., 2007, 2012, 2016;
82 Touboul and Kharif, 2010; Bonnefoy et al., 2010). These include a Smoothed Particle
83 Hydrodynamics method (Dalrymple et al., 2010), a Large Eddy Simulation Method (LES) (Issa
84 et al., 2010; Lubin and Caltagirone J.-P. (2010), a Moving Particle Semi-implicit method (Kim et
85 al., 2014), a Constrained Interpolation Profile method (Zhao, 2016), a Method of Fundamental
86 Solutions (Young et al., 2010) and a Meshless Local Petrov–Galerkin method (Ma, 2010). Fully
87 nonlinear model should be applied to many problems.. Most of the models were designed for
88 engineering applications such as overturning waves, broken waves, waves generated by
89 landslides, freak waves, solitary waves, tsunamis, violent sloshing waves, interaction of extreme
90 waves with beaches, interaction of steep waves with the fixed structures or with different floating

91 structures. The references given above make less than one percent of the publications on those
92 topics

93 Two-dimensional approach (like conformal method) considers a strongly idealized wave
94 field, since even monochromatic waves in the presence of lateral disturbances quickly obtain a
95 two-dimensional structure. The difficulty arising is not a direct result of increase of dimension.
96 The fundamental complication is that the problem cannot be reduced to a two-dimensional
97 problem, and even for the case of a double-periodic wave field the problem of solution of
98 Laplace-like equation for velocity potential arises. The majority of the models designed for
99 investigation of three-dimensional wave dynamics are based on simplified equations such as the
100 second order perturbation methods in which higher order terms are ignored. Overall, it is unclear
101 which effects are missing in such simplified models.

102 The most sophisticated method is based on full three dimensional equations and surface
103 integral formulations (Beale, 2001; Xue et al., 2001; Grilli et al., 2001; Clamond and Grue, 2001;
104 Clamond et al, 2005, 2006; Fructus et al., 2005; Guyenne et al., 2006; Fochesato et al., 2006).
105 Fully nonlinear, three-dimensional water waves, which extends an approach was suggested by
106 Craig and Sulem (1993) originally given in the two-dimensional setting. The model is based
107 upon Hamiltonian formulation (Zakharov, 1968) which allows reducing a problem of surface
108 variables computation by introducing Dirichlet–Neumann operator which is expressed in terms
109 of its Taylor series expansion in homogeneous powers of surface elevation. Each term in this
110 Taylor series can be obtained from the recursion formula and efficiently computed using fast
111 Fourier transform.

112 The main advantage of the boundary integral equation methods (BIEM) is that they are
113 accurate and can describe highly nonlinear waves. A method of solution of Laplace equation is
114 based on use of Green’s function, which allows us to reduce a 3-D water-wave problem to a 2-D
115 boundary integral problem. The surface integral method is well suited for simulation of the wave
116 effects connected with very large steepness, specifically, for investigation of a freak wave
117 generation. These methods can be applied both to periodic and non-periodic flows. The methods
118 do not impose any limitations on wave steepness, so they can be used for simulation of the
119 waves that even approach breaking (Grilli et al., 2001) when the surface obtains a non-single
120 value shape. The method allows us to take into account bottom topography (Grue and Fructus,
121 2010) and investigate an interaction of waves with the fixed structures or with the freely-
122 responding floating structures (Liu et al., 2016, Gou et al., 2010).

123 However, the BIEM method seems to be quite complicated and time-consuming being
124 applied to a long-term evolution of a multi-mode wave field in large domains. The simulation of
125 relatively simple wave fields illustrates an application of the method, and it is unlikely that the
126 method can be applied to the simulation of a long-term evolution of a large-scale multi-mode
127 wave field with a broad spectrum. Implementation of a multi-pole technique for a general
128 problem of the sea wave simulation (Fochesato et al., 2006) can solve the problem but obviously
129 leads to the considerable algorithmic difficulties.

130 Currently, the most popular in oceanography approach is a HOS (High Order Scheme)
131 model developed by Dommermuth and Yue (1987); West et al. (1987). The HOS is based on a
132 paper by Zakharov (1968), where a convenient form of the dynamic and kinematic surface
133 conditions was suggested. Equations used by Zakharov were not intended for modeling, but
134 rather for investigation of stability of finite amplitude waves. In fact, a system of coordinates
135 where depth is counted from the surface, was used, but the Laplace equation for velocity

136 potential was taken in its traditional form. However, the Zakharov's followers have accepted this
137 idea literally. They used two coordinate systems: a curvilinear surface-fitting system for surface
138 conditions and the Cartesian system for calculation of a surface vertical velocity. The analytic
139 solution for velocity potential in the Cartesian coordinate system is known. It is based on the
140 Fourier coefficients on a fixed level, while the true variables are the Fourier coefficients for the
141 potential on a free surface. Here a problem of transition from one coordinate system to another
142 arises. This problem is solved by expansion of the surface potential into the Taylor series in the
143 vicinity of the surface. An accuracy of this method depends on that of representation of an
144 exponential function with a finite number of the Taylor series. For the small-amplitude waves
145 and for a narrow wave spectrum, such accuracy is evidently satisfactory. However, for the case
146 of a broad wave spectrum that contains many wave modes, the order of the Taylor series should
147 be high. The problem is now that the waves with high wave numbers are superposed over the
148 surface of larger waves. Since the amplitudes of a surface potential attenuate exponentially, an
149 amplitude of a small wave at a positive elevation increases, and on the contrary, it can approach
150 zero at negative elevations. It is clear that such setting of HOS model cannot reproduce high-
151 frequency waves, which actually reduces the nonlinearity of the model. This is why such model
152 can be integrated for long periods using no high frequency smoothing. Besides, an accuracy of
153 calculation of vertical velocity on the surface depends on full elevation at each point. Hence, the
154 accuracy is not uniform along a wave profile. A substantial increase of the Taylor expansion
155 order can definitely result in the numerical instability due to occasional amplification of modes
156 with high wave numbers. The authors of a surface integral method shared a similar point of view
157 (Clamond et al., 2005). We should note, however, that comparison of HOS method based on the
158 West et al., (1987) approach using a method of the surface integral for an idealized wave field
159 (Clamond et al., 2006) shows quite acceptable results. It was shown in the previous paper that a
160 method suggested by Dommermuth et al., (1987) demonstrates poorer divergence of the
161 expansion for vertical velocity than for method by West et al., (1987). The HOS model has been
162 widely used (for example, Tanaka, 2001; Toffoli et al., 2010; Touboul and Kharif, 2010) and it
163 has shown its ability to efficiently simulate the wave evolution (propagation, nonlinear wave-
164 wave interactions, etc.) in a large-scale domain (Ducrozet et al., 2007, 2012). It is obvious that
165 the HOS model can be used for many practical purposes. Recently, Ecole Centrale Nantes,
166 LHEEA Laboratory (CNRC) announced that the non-linear wave models based on High-Order
167 Spectral (HOS) are published as an open source (<https://github.com/LHEEA/HOS-ocean/wiki>).

168 Opposite to the HOS method based on the analytical solution of Laplace equation in the
169 Cartesian coordinates, a group of models is based on direct solution of the equation for velocity
170 potential in the curvilinear coordinates (Engsig-Karup et al, 2009, 2012; Chalikov et al., 2014).
171 The main advantage of the surface-following coordinate system is that a variable surface is
172 mapped onto the fixed plane. Since wave motion is very conservative, the high accuracy
173 numerical schemes should be used for a good description of nonlinearity and spectrum
174 transformation. This most universal approach is being developed at the Technical University of
175 Denmark (see Engsig-Karup, 2009). Actually, the models *ModelWave3D* developed at TUD are
176 targeted at solution of a variety of problems including such problems as modeling of wave
177 interaction with submerged objects as well as the simulation of wave regime in the basins with
178 real shape and topography.

179 The model is based on the equations of potential flow with a free surface. An effect of
180 variable bathymetry is taken into account by using a so-called σ -coordinate,

181 (straightening out the bottom and surface). At vertical surfaces a normal derivative of the
182 velocity potential is equal to zero. A flexible-order approximation for spatial derivatives is used.
183 The most time-consuming part of this mode is a 3-D equation for the velocity potential. The
184 strategy of the model development is directed at exploiting architectural features of modern
185 GPUs for the mixed precision computations. This approach is tested using a recently developed
186 generic library for fast prototyping of PDE (Partial Differential Equations) solvers. The new
187 wave tool is applicable for solving and analyzing of a variety of large-scale wave problems in the
188 coastal and offshore engineering. A description of the project and references can be found at site
189 (<http://www2.compute.dtu.dk/~apek/OceanWave3D/>).

190 Comparison of *ModelWave3D* with HOS model was presented by Ducroz et al., (2012).
191 It was shown that both model demonstrate high accuracy, but HOS model shows better
192 performance. Note, that comparison of speed of models in this case is not indicative since
193 *ModelWave3D* was designed for investigation of complicated processes, taking into account the
194 real shape of basin, variable depth and even presence of engineering constructions. All these
195 features obviously are not included in HOS model.

196 Development of waves under the action of wind is a process that is difficult to simulate
197 since surface waves are very conservative and change their energy for hundreds and thousands of
198 periods. This is why the most popular method is spectral modeling. Waves as physical objects in
199 this approach are actually absent, since evolution of spectral distribution of wave energy is
200 simulated. The description of input and dissipation in this approach is not connected directly
201 with the formulation of the problem, but it is rather adopted from other branches of wave theory
202 where waves are the objects of investigation. However, the spectral approach turned out to be the
203 only method capable of describing the space and time evolution of wave field in the ocean. The
204 phase resolving models (or 'direct' models) designed for reproducing waves themselves cannot
205 compete with spectral models since a typical size of domain in such models does not exceed
206 several kilometers. Such domain includes just several thousands of large waves. Nevertheless,
207 direct wave modeling plays an ever-increasing role in geophysical fluid dynamics, because it
208 gives the possibility to investigate the processes which cannot be reproduced with spectral
209 models. One of such problems is that of extreme wave generation. (Chalikov, 2009; Chalikov,
210 and Babanin, 2016a). Direct modeling is also a perfect instrument for development of
211 parameterization of physical processes for spectral wave models. Besides, such models can be
212 used for direct simulation of wave regimes of small water basins, for example, port harbors.
213 Other approaches of direct modeling are discussed in (Chalikov et al., 2014; Chalikov, 2016)

214 Until recently, direct modeling was used for reproduction of quasi-stationary wave
215 regime when wave spectrum essentially did not change. An unique example of direct numerical
216 modeling of surface wave evolution is given in Chalikov and Babanin (2014) where development
217 of wave field was calculated with use of a two-dimensional model based on full potential
218 equations written in the conformal coordinates. The model included algorithms for
219 parameterization of input and dissipation of energy (a description of similar algorithms is given
220 below). The model successfully reproduced an evolution of wave spectrum under the action of
221 wind. However, strictly one-dimensional (unidirectional) waves are not realistic; hence, the full
222 problem of wave evolution should be formulated on the basis of three-dimensional equations. An
223 example of such modeling is given in the current paper.

224 **2. Equations**

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227

Let us introduce a *non-stationary surface-following non-orthogonal* coordinate system:

228

$$\xi = x, \quad \vartheta = y, \quad \zeta = z - \eta(\xi, \vartheta, \tau), \quad \tau = t, \quad (1)$$

229

where $\eta(x, y, t) = \eta(\xi, \vartheta, \tau)$ is a moving periodic wave surface given by the Fourier series

230

$$\eta(\xi, \vartheta, \tau) = \sum_{-M_x < k < M_x} \sum_{-M_y < l < M_y} h_{k,l}(\tau) \Theta_{k,l}, \quad (2)$$

231

where k and l are components of wave number vector \mathbf{k} , $h_{k,l}(\tau)$ are Fourier amplitudes for

232

elevations $\eta(\xi, \vartheta, \tau)$, M_x and M_y are the numbers of modes in directions ξ and ϑ , respectively,

233

while $\Theta_{k,l}$ are Fourier expansion basis functions, represented as matrix:

234

$$\Theta_{kl} = \begin{cases} \cos(k\xi + l\vartheta) & -M_x \leq k \leq M_x, -M_y < l < 0 \\ \cos(k\xi) & -M_x \leq k \leq 0, l = 0 \\ \sin(k\xi) & 0 \leq k \leq M_y, l = 0 \\ \sin(k\xi + l\vartheta) & -M_x \leq k \leq M_x, 0 < l \leq M_y \end{cases} \quad (3)$$

235

The 3-D equations of potential waves in the system of coordinates (1) at $\zeta \leq 0$ take the

236

following form:

237

$$\eta_\tau = -\eta_\xi \varphi_\xi - \eta_\vartheta \varphi_\vartheta + (1 + \eta_\xi^2 + \eta_\vartheta^2) \Phi_\zeta, \quad (4)$$

238

$$\varphi_\tau = -\frac{1}{2} (\varphi_\xi^2 + \varphi_\vartheta^2 - (1 + \eta_\xi^2 + \eta_\vartheta^2) \Phi_\zeta^2) - \eta - p, \quad (5)$$

239

$$\Phi_{\xi\xi} + \Phi_{\vartheta\vartheta} + \Phi_{\zeta\zeta} = \Upsilon(\Phi), \quad (6)$$

240

where Υ is the operator:

241

$$\Upsilon(\Phi) = 2\eta_\xi(\Phi)_{\xi\xi} + 2\eta_\vartheta(\Phi)_{\vartheta\vartheta} + (\eta_{\xi\xi} + \eta_{\vartheta\vartheta})(\Phi)_\zeta - (\eta_\xi^2 + \eta_\vartheta^2)(\Phi)_{\zeta\zeta}, \quad (7)$$

242

capital fonts Φ are used for domain $\zeta < 0$ while the lower case φ refers to $\zeta = 0$. Term p in (5)

243

described the pressure on surface $\zeta = 0$.

244

It is suggested in (Chalikov et al., 2014) that it is convenient to represent velocity

245

potential φ as a sum of two components, i.e., an analytical ('linear') component

246

$\bar{\Phi}$, ($\bar{\varphi} = \bar{\Phi}(\xi, \vartheta, 0)$) and an arbitrary ('non-linear') component \tilde{F} , ($\tilde{\varphi} = \tilde{F}(x, y, 0)$):

247

$$j = \bar{j} + \tilde{j}, \quad F = \bar{F} + \tilde{F}. \quad (8)$$

248

The analytical component $\bar{\Phi}$ satisfies Laplace equation:

249

$$\bar{\Phi}_{\xi\xi} + \bar{\Phi}_{\vartheta\vartheta} + \bar{\Phi}_{\zeta\zeta} = 0, \quad (9)$$

250

with known solution:

251

$$\bar{\Phi}(\xi, \vartheta, \zeta, \tau) = \sum_{k,l} \bar{\varphi}_{k,l}(\tau) \exp(|k|\zeta) \Theta_{k,l}, \quad (10)$$

252

(where $|k| = (k^2 + l^2)^{1/2}$, $\bar{\varphi}_{k,l}$ are Fourier coefficients of surface analytical potential $\bar{\varphi}$ at $\zeta = 0$).

253

The solution satisfies boundary conditions:

254

$$\begin{aligned} \zeta = 0: \quad & \bar{\Phi} = \bar{\varphi} \\ \zeta \rightarrow -\infty: \quad & \tilde{\Phi}_\zeta \rightarrow 0 \end{aligned} \quad (11)$$

255

The nonlinear component satisfies an equation:

256

$$\tilde{\Phi}_{\xi\xi} + \tilde{\Phi}_{\vartheta\vartheta} + \tilde{\Phi}_{\zeta\zeta} = \Upsilon(\tilde{\Phi}) + \Upsilon(\bar{\Phi}), \quad (12)$$

257 Eq. (12) is solved with the boundary conditions:

$$\begin{aligned}
 258 \quad \zeta = 0: \quad \tilde{\Phi} &= 0 \\
 \zeta \rightarrow -\infty: \quad \tilde{\Phi}_\zeta &\rightarrow 0
 \end{aligned}
 \tag{13}$$

259 The derivatives of linear component $\bar{\Phi}$ in (7) are calculated analytically. The scheme
 260 combines 2-D Fourier transform method in the ‘horizontal surfaces’ and a second-order finite-
 261 difference approximation on a stretched staggered grid defined by relation $\Delta\zeta_{j+1} = \chi\Delta\zeta_j$ ($\Delta\zeta$ is
 262 a vertical step, while $j=1$ at the surface). The stretched grid provides increase of accuracy of
 263 approximation for the exponentially decaying modes. The values of stretching coefficient χ lie
 264 within the interval 1.01-1.20. A finite-difference second-order approximation of vertical
 265 operators in Eq. (12) on a non-uniform vertical grid is quite straightforward. Equation (12) is
 266 solved as Poisson equations with iterations over the right-hand side. At each time step the
 267 iterations start with right-side calculated at previous time step. The initial elevation was
 268 generated as superposition of linear waves corresponding to JONSWAP spectrum (Hasselmann
 269 et al., 19734) with random phases. The initial Fourier amplitudes for surface potential were
 270 calculated by formulas of linear wave theory. A detailed description of the scheme and its
 271 validation is given in Chalikov et al., (2014) and Chalikov (2016).

272 Equations (4) – (6) are written in a non-dimensional form by using the following scales:
 273 length L where $2\pi L$ is (dimensional) period in the horizontal direction; time $L^{1/2}g^{-1/2}$ and velocity
 274 potential $L^{3/2}g^{1/2}$ (g is acceleration of gravity). The pressure is normalized by water density, so
 275 that the pressure scale is Lg . Equations (4) – (6) are self-similar to the transformation with
 276 respect to L . The dimensional size of domain $2\pi L$, so scaled size is 2π . All the results
 277 presented in this paper are nondimensional. Note that the number of Fourier modes can be
 278 different in x and y directions. In this case is assumed that two length scales L_x and L_y are used.
 279 The nondimensional length of domain in y -direction remains equal 2π and factor $r = L_x / L_y$ is
 280 introduced in definition of differentiation in Fourier space.

281

282 **3. Energy input and dissipation**

283

284 Input energy to waves describes a pressure term p in a dynamic boundary condition (5).
 285 The tangent stress on the surface cannot be taken into account in potential formulation.
 286 Dissipation cannot be also described with use of potential equations, but for realistic description
 287 of wave dynamics, dissipation of wave energy should be taken into account, i.e., we should
 288 include in equations (4) and (5) additional terms which, strictly speaking, contradict the
 289 assumption of potentiality.

290 **3.1 Energy input from wind**

291 According to the linear theory (Miles, 1957), the Fourier components of surface pressure
 292 p are connected with those of surface elevation through the following expression:

$$293 \quad p_{k,l} + ip_{-k,-l} = \frac{\rho_a}{\rho_w} (\beta_{k,l} + i\beta_{-k,-l}) (h_{k,l} + ih_{-k,-l}), \tag{14}$$

294 where $h_{k,l}, h_{-k,-l}, \beta_{k,l}, \beta_{-k,-l}$, are real and imaginary parts of elevation η and the so-called β -
 295 function (i.e., Fourier coefficients at COS and SIN, respectively); ρ_a / ρ_w is a ratio of air and

296 water densities. Eq. (14) is a standard presentation of pressure above multi-mode surface. It
 297 means that every wave mode with amplitude $(h_{k,l}^2 + h_{-k,-l}^2)^{1/2}$ initiates the pressure mode with
 298 amplitude $(p_{k,l}^2 + p_{-k,-l}^2)^{1/2}$ shifted by phase of wave mode by angle $\alpha = \text{atan} \frac{\beta_{-k,-l}}{\beta_{k,l}}$. Both
 299 coefficients in (14) are function of ratio of wind velocity at half of mode length height $\lambda_{k,l} / 2$ to
 300 virtual phase velocity. Hence, for derivation of shape of beta-function it is necessary to
 301 simultaneously measure wave surface elevation and non-static pressure on the surface.
 302 Experimental measurement of surface pressure is a very difficult problem since the
 303 measurements should be done very close to a moving surface, preferably, with a surface-
 304 following sensor. Such measurements are done quite seldom, especially, in the field. The
 305 measurements were carried out for the first time by a team of authors both in laboratory and field
 306 (Snyder et al, 1981; Hsiao and Shemdin, 1983; Hasselmann and Bösenberg, 1991; Donelan et al.,
 307 2005, 2006). The data obtained in this way allowed constructing an imaginary part of beta-
 308 function used in some versions of wave forecasting models (Rogers et al., 2012). Such
 309 measurements and their processing are quite complicated since wave-produced pressure
 310 fluctuations are masked by turbulent pressure fluctuations. The second way of beta-function
 311 evaluation is based on the results of numerical investigations of statistical structure of the
 312 boundary layer above waves with use of Reynolds equations and an appropriate closure scheme.
 313 In general, this method works so well that many problems in the technical fluid mechanics are
 314 often solved using numerical models, not experimentally (Gent and Taylor, 1976; Riley et al.,
 315 1982; Al-Zanaidi and Hui, 1984).. This method was being developed beginning from (Chalikov,
 316 1978, 1986), followed by (Chalikov and Makin, 1991; Chalikov and Belevich, 1992; Chalikov,
 317 1995). The results were implemented in WAVEWATCH model, i.e., a third-generation wave
 318 forecast model (Tolman and Chalikov, 1996) and thoroughly validated against the experimental
 319 data in the course of developing WAVEWATCH-III (Tolman et al., 2014). This method was
 320 later improved on the basis of more advanced coupled modeling of waves and boundary layer
 321 (Chalikov and Rainchik, 2010), while the beta-function used in WAVEWATCH-III was
 322 corrected and extended up to high frequencies. Direct calculation of energy input to waves
 323 requires both real and imaginary parts of the beta-function. The total energy input to waves
 324 depends on imaginary part of β -function, while the moments of higher order depend both on
 325 imaginary and real parts of β . This is why full approximation constructed in (Chalikov and
 326 Rainchik, 2010) was used in the current work. Note that in the range of relatively low
 327 frequencies the nw method is very close to the scheme implemented in WAVEWATCH-III.

328 It is a traditional suggestion that both coefficients are the functions of virtual
 329 nondimensional frequency $\Omega = \omega_{k,l} U \cos \psi = U / c_{k,l} \cos \psi$ (where $\omega_{k,l}$ and U are the
 330 nondimensional radian frequency and wind speed, respectively; $c_{k,l}$ is a phase speed of the k^{th}
 331 mode; ψ is an angle between wind and wave mode directions). Most of the schemes for
 332 calculations of β -function consider a relatively narrow interval of nondimensional frequencies
 333 Ω . In the current work, the range of frequencies covers an interval $(0 < \Omega < 10)$, and
 334 occasionally the values of $\Omega > 10$ can appear. This is another reason why the function derived in
 335 (Chalikov and Rainchik, 2010) through coupled simulations of waves and boundary layer, is
 336 used here. Wave model is based on potential equations for a flow with free surface, extended

337 with an algorithm for breaking dissipation (see below description of the breaking dissipation
 338 parameterization). Wave boundary layer (WBL) model is based on Reynolds equations closed
 339 with $K - \varepsilon$ scheme; solutions for air and water are matched through the interface. The β -
 340 function was used for evaluation of accuracy of the surface pressure p calculations. A shape of
 341 β -function connecting surface elevations and surface pressure, is studied up to high
 342 nondimensional wave frequencies both in positive and negative (i.e., for wind opposite to waves)
 343 domains. The data on β -function exhibit wide scatter, but since the volume of data was quite
 344 large (47 long-term numerical runs allowed us to generate about 1,400,000 values of β), the
 345 shape of β -function was defined with satisfactory accuracy up to very high nondimensional
 346 frequencies ($-50 < \Omega < 50$). As a result, the data on β -function in such a broad range, allow us
 347 to calculate wave drag up to very high frequencies and to explicitly divide the fluxes of energy
 348 and momentum transferred by the pressure and molecular viscosity. This method is free of
 349 arbitrary assumptions on the drag coefficient C_d ; and, on the other hand, such calculations allow
 350 investigating the nature of wave drag (see Ting et al., 2012).

351 The most reliable data on β -function are concentrated in interval $-10 < \Omega < 10$ (negative
 352 values of Ω correspond wave modes running against wind). real and imaginary parts of beta
 353 function are shown in Fig. (1). It is an corrected version of approximation given in Chalikov and
 354 Rainchik (2010), where data at negative Ω were interpreted erroneously. In current calculations
 355 the modes running against wind are absent.

356

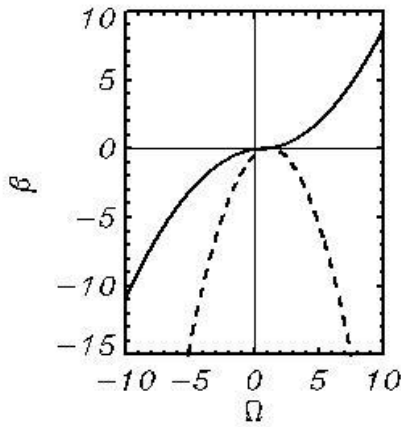


Figure 1. Real (dashed curve) and imaginary (solid curve) parts of β -function.

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369 Function β can be approximated by formulas:

370

$$371 \quad \beta_{k,l} = \begin{cases} \beta_0 + a_0(\Omega - \Omega_0) + a_1(\Omega - \Omega_0)^2 & \Omega_0 < \Omega \\ \beta_0 + a_0(\Omega - \Omega_0) - a_1(\Omega - \Omega_0)^2 & \Omega < \Omega_0 \end{cases}, \quad (15)$$

$$372 \quad \beta_{-k,-l} = \begin{cases} \beta_1 + a_3(\Omega - \Omega_2) & \Omega < \Omega_2 \\ a_2(\Omega - \Omega_1)^2 & \Omega_2 < \Omega < \Omega_3, \\ \beta_1 - a_3(\Omega - \Omega_3) & \Omega_3 < \Omega \end{cases}, \quad (16)$$

373 where the coefficients are:

$$374 \quad \Omega_0 = 0.02277, \Omega_1 = 1.20, \Omega_2 = -18.8, \Omega_3 = 21.2, a_0 = 0.02277, a_1 = 0.09476, a_2 = -0.3718, \\ a_3 = 14.80, b_0 = -0.02, b_1 = -148.0.$$

375 It was indicated above that an initial wave field is assigned as superposition of linear
376 modes of which amplitudes are calculated with JONSWAP spectrum with initial peak wave
377 number $k_p^0 = 100$. The initial value $U / c_p^0 = 6$ was chosen, i.e., a ratio of the nondimensional
378 wind speed at height of half of initial peak wave length $\lambda_0 / 2 = 2\pi / 100$ and the phase speed
379 $c_p^0 = (k_p^0)^{-1/2}$ is equal to 6. Such a high ratio corresponds to initial stages of wave development.
380 Wind velocity $6c_p^0$ remains constant during all time of integration. The values of Ω for other
381 wave numbers are calculated by assuming that wind profile is logarithmic:

$$382 \quad \Omega_{k,l} = \frac{U}{c_{k,l}} \ln \frac{\lambda_{k,l}}{2z_0} \left(\ln \frac{\lambda_0}{2z_{00}} \right)^{-1} \cos \psi_{k,l}, \quad (17)$$

383 where z_{00} is effective nondimensional roughness for the initial wind profile, while z_0 is the
384 actual roughness parameter that depends on the energy in a high-frequency part of spectrum and
385 on the wind profile. We call it ‘effective’, since very close to the surface the wind profile is not
386 logarithmic (Chalikov, 1995; Tolman and Chalikov, 1996; Chalikov and Rainchik, 2010). The
387 value of this parameter depends on the wind velocity and energy in a high-wave number interval
388 of wave spectrum, as well as on the length scale of the problem. All these effects are possible to
389 include by matching the wave model with a one dimensional WBL model (Ting et al., 2012).
390 Here, a simplified scheme for the roughness parameter is chosen. It is well known that the
391 roughness parameter (as well as a drag coefficient) increases with decrease of the inverse wave
392 age. In our case wind speed is fixed, and dependence for the nondimensional roughness
393 parameter is constructed on the basis of the results obtained in (Chalikov and Rainchik, 2010):

$$394 \quad z_0 = 15z_{00}\Omega, \quad (18)$$

395 where $z_{00} = 10^{-3}$ is the initial value of the roughness parameter. Eq. (18) approximates
396 dependence of the effective roughness at the stage of wave development. Note that the results are
397 not sensitive to variation of the roughness parameter within reasonable limits.

398

399 **3.2 High wave number energy dissipation**

400

401 A nonlinear flux of energy directed to the small wave numbers produces downshifting of
402 spectrum, while an opposite flux forms a shape of spectral tail. The second process can produce
403 accumulation of energy near ‘cut’ wave number. Both processes become more intensive with
404 increase of energy input. Growth of amplitudes at high wave numbers is followed by the growth
405 of local steepness and numerical instability. This well-known phenomenon in numerical fluid
406 mechanics is eliminated by use of a highly selective filter simulating nonlinear viscosity. To
407 support stability, additional terms are included into the right hand sides of equations (4) and (5):

$$408 \quad \frac{\partial \eta_{k,l}}{\partial \tau} = E_{k,l} - \mu_{k,l} \eta_{k,l}, \quad (19)$$

$$409 \quad \frac{\partial \varphi_{k,l}}{\partial \tau} = F_{k,l} - \mu_{k,l} \varphi_{k,l} \quad (20)$$

410

411 ($E_{k,l}$ and $F_{k,l}$ are Fourier amplitudes of the right-hand sides of equations (4) and (5) while factor
412 $\mu_{k,l}$ is calculated using a formula:

$$413 \quad \mu_{k,l} = \begin{cases} 0 & |k| < k_d \\ c_m k_0 \left(\frac{|k| - k_d}{(k_0 - k_d)} \right)^2 & k_d \leq |k| \leq k_0 \\ c_m k_0 & |k| > k_0 \end{cases} \quad (21)$$

414 where k and l are components of wave number $|k|$, while coefficients k_d and k_0 are defined by the
415 expressions:

$$416 \quad k_d = d_m^2 M_x M_y \left(\left(l |k|^{-1} d_m M_x \right)^2 + \left(k |k|^{-1} d_m M_y \right)^2 \right)^{-1/2} \quad (22)$$

$$417 \quad k_0 = M_x M_y \left(\left(l |k|^{-1} M_x \right)^2 + \left(k |k|^{-1} M_y \right)^2 \right)^{-1/2} \quad (23)$$

418 where $c_m = 0.1$, $d_m = 0.75$. Expressions (18) - (20) can be interpreted in a straightforward way:
419 the value of $\mu_{k,l}$ is equal to zero inside the ellipse with semi-axes $d_m M_x$ and $d_m M_y$; then it grows
420 linearly with $|k|$ up to the value c_m and is equal to c_m outside the outer ellipse. This method of
421 filtration that we call ‘tail dissipation’ was developed and validated with a conformal model by
422 Chalikov and Sheinin (1998). The sensitivity of the results to the parameters in (21) - (23) is not
423 high. The aim of the algorithm is support of smoothness and monotonicity of wave spectrum
424 within a high wave number range. Since the algorithm affects the amplitudes of small modes, it
425 actually does not reduce the total energy, though it efficiently prevents development of
426 numerical instability. Note that any long-term calculations cannot be performed without ‘tail
427 dissipation’ eliminating development of the numerical instability at high wave numbers.

428

429 **3.3 Dissipation due to wave breaking**

430

431 The main process of wave dissipation is wave breaking. This process is taken into
432 account in all spectral wave forecasting models similar to WAVEWATCH (see Tolman and
433 Chalikov, 1996). Since there are no waves in spectral models, no local criteria of wave breaking
434 can be formulated. This is why breaking dissipation is represented in spectral models in a
435 distorted form. A real breaking occurs in relatively narrow areas of physical space; however,
436 spectral image of such breaking is stretched over the entire wave spectrum, while in reality the
437 breaking decreases height and energy of dominant waves. This contradiction occurs because
438 waves in spectral models are assumed as linear ones, while in fact the breaking occurs in
439 physical space with nonlinear sharp wave, usually composed of several modes. However,
440 progress has been gradually made in spectral wave modeling over the past decade. One important
441 outcome is that the wave breaking term in the state-of-art wave models now accounts for the threshold-
442 behavior of dominant wave breaking, that is, waves won't break unless their steepness exceeds a threshold
443 (Alves and Banner, 2003; Babanin et al., 2010).

444 The mechanics of wave breaking at developed wave spectrum differs from that in a wave
445 field represented by few modes, normally considered in many theoretical and laboratory
446 investigations (e.g., Alberello et al., 2018). Since the breaking in laboratory conditions is initiated

447 by special assigning of amplitudes and phases, it cannot be similar to the breaking in natural
448 conditions. To some degree, the wave breaking is similar to development of extreme wave that
449 appears suddenly with no pronounced prehistory (Chalikov and Babanin, 2016a, 2016b). There
450 are no signs of modulational instability in both phenomena, which suggests a process of taking
451 energy from other modes. The evolution leading to breaking or ‘freaking’ seems just opposite:
452 full energy of main wave remains nearly constant while the columnar energy is focusing around
453 the crest of this wave which becomes sharper and unstable. Probably, even more frequent cases
454 of wave breaking and extreme wave appearance can be explained by local superposition of
455 several modes.

456 The instability of interface leading to breaking is an important and poorly developed
457 problem of fluid mechanics. In general, this essentially nonlinear process should be investigated
458 for a two-phase flow. Such approach was demonstrated, for example, by Iafrati (2001).
459 However, the progress in solving this highly complicated problem is not too fast.

460 The problem of breaking parameterization includes two points: (1) establishing of a
461 criterion of breaking onset and (2) developing of an algorithm of breaking parameterization. The
462 problem of breaking is discussed in details in Babanin (2011). Chalikov and Babanin (2012)
463 performed numerical investigation of the processes leading to breaking. It was found that a clear
464 predictor of breaking, formulated in dynamical and geometrical terms, probably does not exist.
465 The most evident criterion of breaking is the breaking itself, i.e., the process when some part of
466 upper portion of sharp wave crest is falling down. This process is usually followed by separation
467 of detached volume of liquid into water and air phases. Unfortunately, there is no possibility to
468 describe this process within the scope of potential theory.

469 Some investigators suggest using the physical velocity approaching the rate of surface
470 movement in the same direction as a criterion of breaking onset. This is incorrect, since the
471 kinematic boundary condition suggests that these quantities are exactly equal to each other. It is
472 quite clear that the onset of breaking can be characterized by appearance of non-single-value
473 piece of surface. This stage can be investigated with two-dimensional model which due to a high
474 flexibility of the conformal coordinates allows us to reproduce a surface with the inclination in
475 the Cartesian coordinates larger than 90 degrees. (In the conformal coordinates the dependence
476 of elevation on curvilinear coordinate is always single-value). The duration of this stage is
477 extremely short, the calculations being always interrupted by the numerical instability with sharp
478 violation of conservations laws (constant integral invariants, i.e., full energy and volume) and
479 strong distortion of the local structure of flow. Numerous numerical experiments with conformal
480 model showed that after appearance of non-single value, the model never returns to stability.
481 However, introducing of appearance of the non-single-surface as a criterion of breaking
482 instability even in conformal model is impossible, since a behavior of model at a critical point is
483 unpredictable, and the run is most likely to be terminated, no matter what kind of
484 parameterization of breaking is introduced. It means that even in a precise conformal model,
485 stabilization of solution should be initiated prior to breaking.

486 Consideration of exact criterion for breaking onset for the models using transformation of
487 the coordinate type of (1) is useless, since the numerical instability in such models arises not
488 because of the breaking approaching but because of appearance of large local steepness. Multiple
489 experiments with direct 3-D wave model show that appearance of local steepness
490 $\max\left(\frac{\partial\eta}{\partial x}, \frac{\partial\eta}{\partial y}\right)$ exceeding ≈ 2 (that corresponds to a slope of about 60 degrees) is always

491 followed by numerical instability but instability can happen far before reaching this value.
 492 Decrease of time step does not make any effect. As seen, a surface with such slope is very far
 493 from being a vertical ‘wall’, when real breaking starts. However, an algorithm for breaking
 494 parameterization must prevent a numerical instability. The situation is similar to the numerical
 495 modeling of turbulence (LES technique), where the local highly selective viscosity is used to
 496 prevent appearance of too large local gradients of velocity. The description of breaking in direct
 497 wave modeling should satisfy the following conditions. (1) It should prevent the onset of
 498 instability at each point of half million of grid points over more than 100 thousand of time
 499 steps.(2) It should describe in a more or less realistic way the loss of kinetic and potential
 500 energies with preservation of balance between them. (3) It should preserve the volume. It was
 501 suggested in (Chalikov, 2005) that an acceptable scheme can be based on the local highly
 502 selective diffusion operator with special diffusion coefficient. Several schemes of such type were
 503 validated, and finally the following scheme was chosen:

$$504 \quad \eta_\tau = E_\eta + J^{-1} \left(\frac{\partial}{\partial \xi} B_\xi \frac{\partial \eta}{\partial \xi} + \frac{\partial}{\partial \vartheta} B_\vartheta \frac{\partial \eta}{\partial \vartheta} \right), \quad (24)$$

$$505 \quad \varphi_\tau = F_\varphi + J^{-1} \left(\frac{\partial}{\partial \xi} B_\xi \frac{\partial \varphi}{\partial \xi} + \frac{\partial}{\partial \vartheta} B_\vartheta \frac{\partial \varphi}{\partial \vartheta} \right), \quad (25)$$

506 where F_η and F_φ are the right-hand sides of equations (4) and (5) including the terms introduced
 507 in terms of Fourier coefficients by (19) – (23), B_ξ and B_ϑ are diffusion coefficients. It was
 508 suggested in the first versions of the scheme that diffusion coefficient depends on a local slope,
 509 however, such scheme did not prove to be very reliable since it did not prevent all of the events
 510 of numerical instability. A scheme based on the calculation of the local curvilinearity $\eta_{\xi\xi}$ and $\eta_{\vartheta\vartheta}$
 511 turned out to be a lot more robust. The calculations of 75 different runs were performed with full
 512 3-D model in (Chalikov et al., 2014) over period of $t = 350$ (70,000 time steps). The total number
 513 of values used for the calculations of dependence in Fig. 2 (thick curve) is about 6 billion. The
 514 normal probability calculated with the same dispersion is shown by thin curve.

515

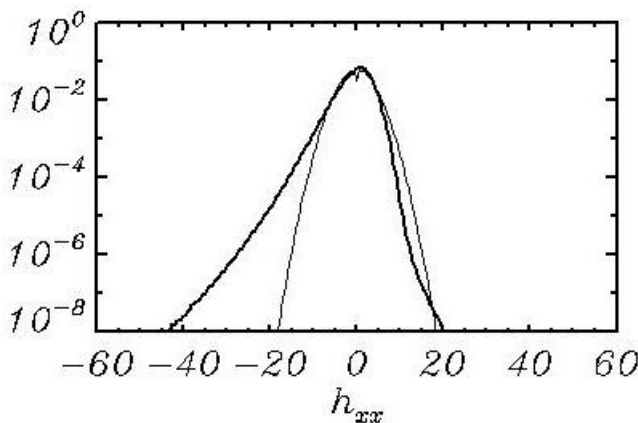


Figure 2. Probability of curvilinearity $\eta_{\xi\xi}$. Thick curve calculated with full 3-D model; thin curve is a probability calculated over ensemble of linear modes with the same spectrum.

527

528 It is seen that the probability of large negative values of curvilinearity is by orders larger than the
 529 probability calculated over ensemble of linear modes with spectra generated by nonlinear
 530 model.

531 The curvilinearity turned out to be very sensitive to the shape of surface. This is why it

532 was chosen as a criterion of breaking approach. Coefficients B_ξ and B_g depend nonlinearly on
 533 the curvilinearity

$$534 \quad B_\xi = \begin{cases} \Delta\xi C_B \eta_{\xi\xi}^2 & \eta_{\xi\xi} < \eta_{\xi\xi}^{cr} \\ 0 & \eta_{\xi\xi} \geq \eta_{\xi\xi}^{cr} \end{cases} \quad (26)$$

$$535 \quad B_g = \begin{cases} \Delta\zeta C_B \eta_{g,g}^2 & \eta_{g,g} < \eta_{g,g}^{cr} \\ 0 & \eta_{g,g} \geq \eta_{g,g}^{cr} \end{cases} \quad (27)$$

536

537 where $\Delta\xi$ and $\Delta\zeta$ are horizontal steps in x and y direction in grid space, and coefficients are
 538 $C_B = 2.0$, $\eta_{\xi\xi}^{cr} = \eta_{g,g}^{cr} = -50$. Algorithm (24) - (27) does not change the volume and decreases the
 539 local potential and kinetic energy. It is assumed that the lost momentum and energy are
 540 transferred to current and turbulence (see Chalikov and Belevich, 1992). Besides, the energy also
 541 goes to other wave modes. The choice of parameters in (24) - (27) is based on simple
 542 considerations: local piece of surface can closely approach the critical curvilinearity but not
 543 exceed it. The values of the coefficients are picked with reserve to provide stability of long runs.

544 We do not think that the suggested breaking parameterization is a final solution of the
 545 problem. Other schemes will be tried in the next version of the model. However, the results
 546 presented below show that the scheme is reliable and provides a realistic energy dissipation rate.

547

548 **4. Calculations and results**

549

550 The elevation and surface velocity potential fields are approximated in the current
 551 calculations by $M_x = 256$ and $M_y = 128$ modes in directions x and y . The corresponding grid
 552 includes $N_x \times N_y = (1024 \times 512)$ knots. The vertical derivatives are approximated at vertical
 553 stretched grid $d\zeta_{j+1} = \chi d\zeta_j$, ($j = 1, 2, 3, \dots, L_w$) where $\nu = 1.2$ and $L_w = 10$. The small number of
 554 levels used for solution of the equation for nonlinear component of the velocity potential is
 555 possible because just a surface vertical derivative for the velocity potential $\partial\Phi / \partial\zeta (\zeta = 0)$ is
 556 required. The velocity potential mainly consists of an analytical component $\bar{\varphi}$, while a nonlinear
 557 component provides but small correction. To reach an accuracy of solution $\varepsilon = 10^{-6}$ for equation
 558 (11), no more than two iterations were usually sufficient.

559 The parameters chosen were used for solution of the problem of wave field evolution
 560 over acceptable time (of the order of 10 days). The initial conditions were assigned on the basis
 561 of empirical spectrum JONSWAP (Hasselmann et al., 1973) with a maximum placed at wave
 562 number $k_p = 100$ with angle spreading $(\cosh \psi)^{256}$. Details of initial conditions are of no
 563 importance because an initial energy level is quite low.

564 The total energy of wave motion $E = E_p + E_k$ (E_p - is potential energy, while E_k is
 565 kinetic energy) is calculated with the following formulas:

566

$$567 \quad E_p = 0.25 \overline{\eta^2}, \quad E_k = 0.5 \overline{(\varphi_x^2 + \varphi_y^2 + \varphi_z^2)}, \quad (28)$$

568

569 where single bar denotes averaging over the ξ and \mathcal{G} coordinates, while double bar denotes
 570 averaging over entire volume. The derivatives in (25) are calculated according to transformation
 571 (1). An equation of integral energy $E = E_p + E_k$ evolution can be represented in the following
 572 form:

$$573 \quad \frac{dE}{dt} = \bar{\bar{I}} + \bar{\bar{D}}_b + \bar{\bar{D}}_t + \bar{\bar{N}}, \quad (29)$$

574 where $\bar{\bar{I}}$ is the integral input of energy from wind (Eqs. (14) – (18)); $\bar{\bar{D}}_b$ is a rate of energy
 575 dissipation due to the wave breaking (Eqs. (24) – (27)); $\bar{\bar{D}}_t$ is a rate of energy dissipation due to
 576 filtration of high-wave number modes (‘tail dissipation’, Eqs. (19) – (23)); $\bar{\bar{N}}$ is an integral effect
 577 of the nonlinear interactions described by the right-hand side of the equations when surface
 578 pressure p is equal to zero. The differential form for calculation of the energy transformation can
 579 be, in principle, derived from Eqs. (4) – (6), but here a more convenient and simple method was
 580 applied. Different rates of integral energy transformations can be calculated with help of
 581 fictitious time steps (i.e., apart from the basic calculations). For example, the value of $\bar{\bar{I}}$ is
 582 calculated by the following relation:

$$583 \quad \bar{\bar{I}} = \frac{1}{\Delta t} \left(\bar{\bar{E}}^{t+\Delta t} - \bar{\bar{E}}^t \right), \quad (30)$$

584
 585 where $\bar{\bar{E}}^{t+\Delta t}$ is the integral energy of wave field obtained after one time step with the right side of
 586 equation (6) containing only the surface pressure calculated with Eqs. (14) – (18). For
 587 calculation of the dissipation rate due to filtration, the right-hand side of the equations contains
 588 just the terms introduced in Eqs. (19) - (23), while for calculation of the effects of breaking, only
 589 the terms introduced in (24) – (27) are in use.

591 An evolution of the characteristics calculated by formula (30) is shown in Fig. 3.

592

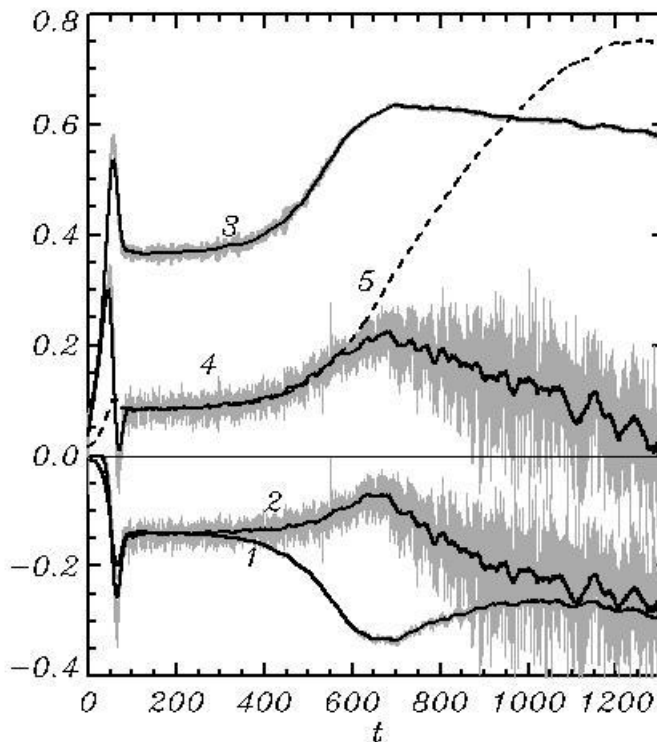


Figure 3. Evolution of integral characteristics of solution, rate of evolution of integral energy multiplied by 10^7) due to: 1 – tail dissipation D_t (Eqs. 19-23); 2 – breaking dissipation D_b (Eqs. 24-27); 3 – input of energy from wind I (Eqs. 14-18); 4 – balance of energy $I + D_t + D_b$. Curve 5 shows the evolution of wave energy $10^5 E$. Vertical bars of grey color show the instantaneous values; thick curve shows the smoothed behavior.

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Sharp variations of all characteristics at $t < 50$ can be probably explained by adjustment of linear initial fields to nonlinearity. Up to the end of integration, the sum of all energy transition terms (tail dissipation $\overline{\overline{D_t}}$, breaking dissipation $\overline{\overline{D_b}}$ and energy input \overline{I}) is approaching zero (curve 4), and the energy growth E (Curve 5) stops. Then the energy tends to decrease, but we are not sure about the nature of this effect. Such behavior can be explained by a fluctuating character of mutual adjustment of input and dissipation or simply by worsening of the approximation because of the downshifting process. Note that opposite to a more or less monotonic behavior of tail dissipation (Curve 1), the breaking dissipation is highly intermittent, which is consistent with common views on the nature of wave breaking.

The data on evolution of wave spectrum are shown in Fig. 4.

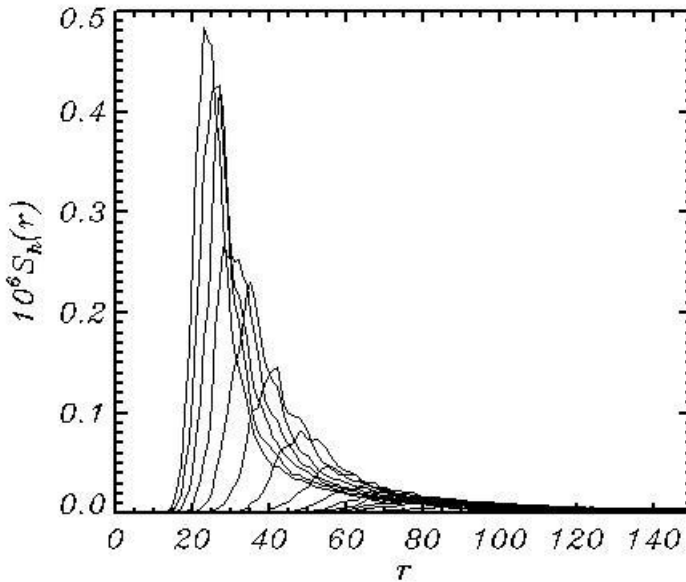


Figure 4. The wave spectra $S_h(r)$ integrated over angle ψ in the polar coordinates and averaged over consequent intervals of length about 100 units of nondimensional time t . The spectra are growing and shifting from right to left.

640 The 2-D wave spectrum $S(k,l)$ ($0 \leq k \leq M_x, -M_y \leq l \leq M_y$) averaged over 13 time
641 intervals of length equal to $\Delta t \approx 100$, was transferred to the polar coordinates $S_p(\psi, r)$
642 ($-\pi/2 \leq \psi \leq \pi/2, 0 \leq r \leq M_x$) and then averaged over angle ψ to obtain 1-D spectrum $S_h(r)$:

$$S_h(r) = \sum S_p(\psi, r) r \Delta\psi. \quad (31)$$

644 An angle $\psi = 0$ coincides with the direction of wind U , $\Delta\psi = \pi/180$.

645

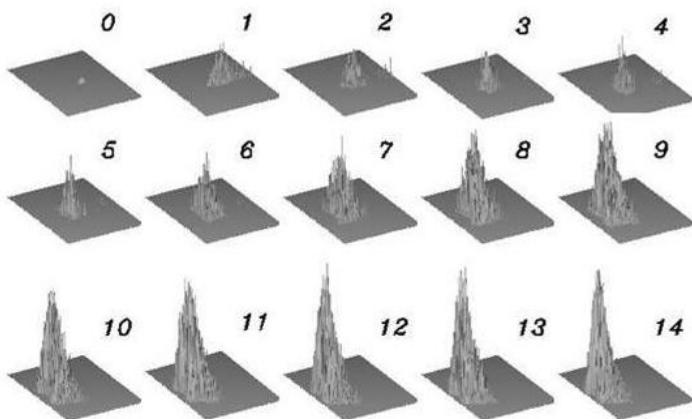


Figure 5. Sequence of 3-D images of $\lg_{10}(S(k,l))$ where each panel corresponds to single curve in Fig. 3. The left side refers to wave number $l(-M_y \leq l \leq M_y)$ and front side – to $k(0 \leq k \leq M)$. The numbers indicate

653 end of time interval expressed in hundreds of nondimensional time units.

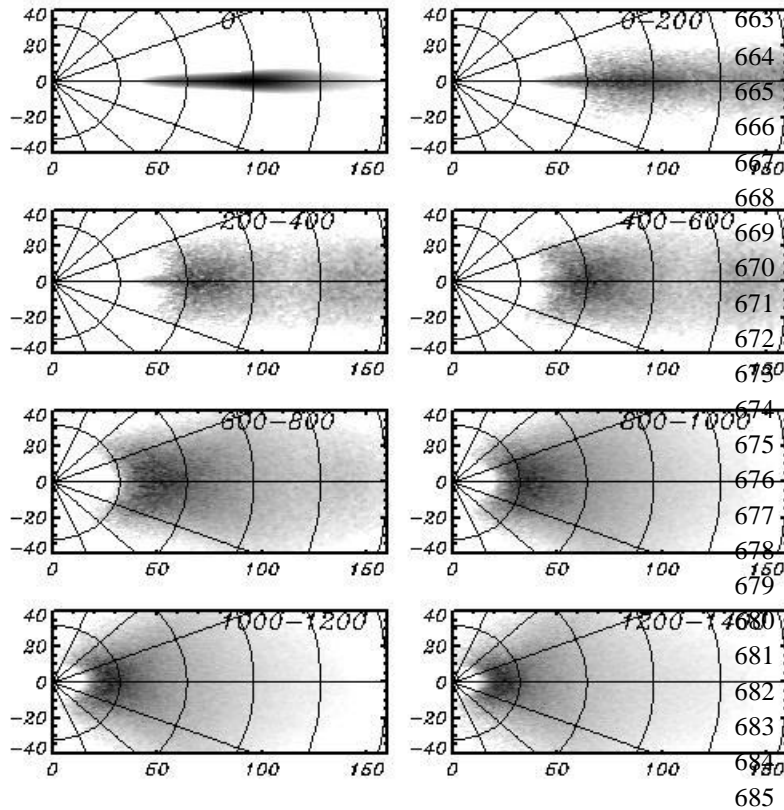
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655

656

657 As seen, each spectrum consists of separated peaks and holes¹. This phenomenon was first
 658 observed and discussed by Chalikov et al. (2014). The repeated calculations with different
 659 resolution showed that such structure of 2-D spectrum is typical. It cannot be explained by fixed
 660 combination of interacting modes, since in different runs (with the same initial conditions but
 661 different set of phases for the modes) peaks are located in different locations in Fourier space.

662



686 **Figure 6.** Sequence of 2-D images of $\lg_{10}(S(k,l))$ averaged over consequent seven periods length $\Delta t = 200$.

687 Numbers indicate the period of averaging (first panel marked 0, refers to initial conditions). Horizontal and vertical
 688 axes correspond to wave numbers k and l correspondingly

689

690 Another presentation is given in Fig 6 where the $\log_{10}(S(\psi, r))$, averaged over the successive
 691 seven period length $\Delta t = 200$, is given. The first panel with a mark 0 refers to initial conditions.
 692 Disturbances within the range $(125 < k < 150)$ reflect initial adjustment of the input and
 693 dissipation at high wave number slope of spectrum. The pictures characterize well the
 694 downshifting and angle spreading of spectrum due to nonlinear interactions.

695 Evolution of the integrated over angles ψ wave spectrum $S_h(r)$ can be described with
 696 the equation

697

¹The wave spectrum looks rather like La Sagrada Familia (Gaudi) in Barcelona than the St. Mary
 Axe ('Pickle') in London.

698
$$\frac{dS_h(r)}{dt} = I(r) + D_t(r) + D_b(r) + N(r), \quad (32)$$

699 where $I(r)$, $D_t(r)$, $D_b(r)$ and $N(r)$ are the spectra of the input energy, tail dissipation, breaking
700 dissipation and a rate of nonlinear interactions, all obtained by integration over angles ψ . All of
701 the spectra shown below were obtained by transformation of 2-D spectra into the polar
702 coordinate (ψ, r) and then integrated over angles ψ within the interval $(-\pi/2, \pi/2)$. The
703 spectra can be calculated using an algorithm similar to the algorithm (30) for integral
704 characteristics. For example, the spectrum of energy input $I(k, l)$ is calculated as follows:

705
$$I(k, l) = (S_c^{t+\Delta t}(k, l) - S_c^t(k, l)) / \Delta t, \quad (33)$$

706 where $S_c(k_x, k_y)$ is a spectrum of columnar energy calculated by relation

707
$$S_c(k, l) = \frac{1}{2} \left(h_{k,l}^2 + h_{-k,-l}^2 + \int_{-H}^0 (u_{k,l}^2 + u_{-k,-l}^2 + v_{k,l}^2 + v_{-k,-l}^2 + w_{k,l}^2 + w_{-k,-l}^2) d\zeta \right) \quad (34)$$

708 where grid values of velocity components u, v, w are calculated by relations:

709
$$u = \varphi_\xi + \varphi_\zeta \eta_\xi, \quad v = \varphi_\eta + \varphi_\zeta \eta_\eta, \quad w = \varphi_\zeta, \quad (35)$$

710 and $u_{k,l}$, $v_{k,l}$ and $w_{k,l}$ are their Fourier coefficients.

711 For calculation of $I(k, l)$ the fictitious time steps Δt are made only with a term
712 responsible for the energy input, i.e., surface pressure p . Spectrum $I(k, l)$ was averaged over the
713 periods $\Delta t \approx 100$, then transformed into a polar coordinate system and integrated in Fourier
714 space over angles ψ within the interval $(-\pi/2, \pi/2)$.

715

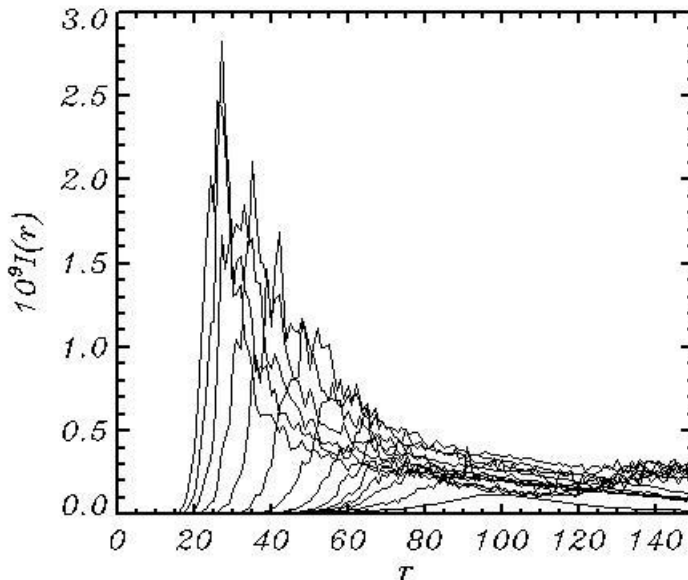


Figure 7. The spectrum of energy input $I(r)$ integrated over angle ψ in the polar coordinates and averaged over consequent intervals of length about 100 units of nondimensional time t .

732 The evolution of input spectra (Fig. 7) is, in general, similar to that of wave spectra shown in
733 Fig. 4. Note that a maximum of spectra is located at the maximum of wave spectra since the
734 input depends mainly on spectral density, while the dependence on frequency is less important.

735 Algorithm (32) – (35) was applied for calculation of the dissipation spectra due to
736 dumping of a high-wave number part of spectrum (tail dissipation) and for calculation of the

737 spectrum of breaking dissipation. In the first case, the fictitious time step was made taking into
 738 account the terms described by Eqs (19) – (23), while in the second case the time step was made
 739 using the terms described by Eqs (24) – (27).

740 The spectra of tail dissipation calculated similar to spectra $I(r)$ are shown in Fig. 8.
 741 Dissipation occurs at the periphery of spectrum, outside the ellipse with semi-axes $d_m M_x$ and
 742 $d_m M_y$ ². This is why such dissipation, averaged over angles, seems to affect a middle part of 1-D
 743 spectrum. The tail dissipation effectively stabilizes the solution.

744 The breaking dissipation averaged over angles is presented in Fig. 8. As seen, the
 745 breaking dissipation has a maximum at spectral peak. It does not mean that in the vicinity of
 746 wave peak the probability of large curvilinearity is quite high. The high rate of breaking
 747 dissipation can be explained by high wave energy in the vicinity of wave peak. The energy lost
 748 through breaking, described by the diffusion mechanism, correlates with the energy of breaking
 749 waves. Opposite to high wave number dissipation which regulates shape of spectral tail, the
 750 breaking dissipation forms the main energy-containing part of spectrum.

751 The diffusion mechanism suggested in (24) - (27) modifies an elevation and surface
 752 stream function in a close vicinity of breaking point. The amplitudes of side perturbation are
 753 small and decrease very quickly over the distance from a breaking point.

754

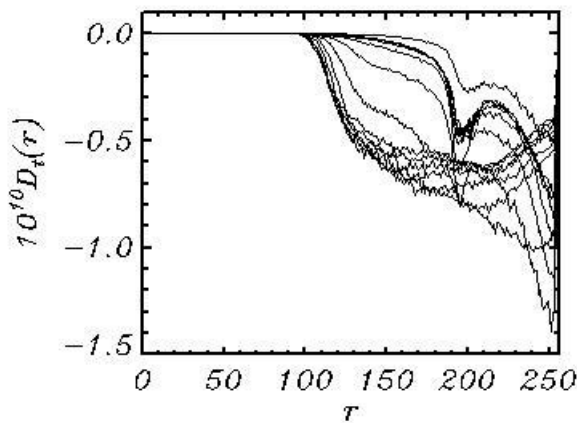


Figure 8. Tail dissipation spectra $D_t(r)$ integrated over angle ψ in the polar coordinates and averaged over consequent intervals of length about 100 units of nondimensional time t .

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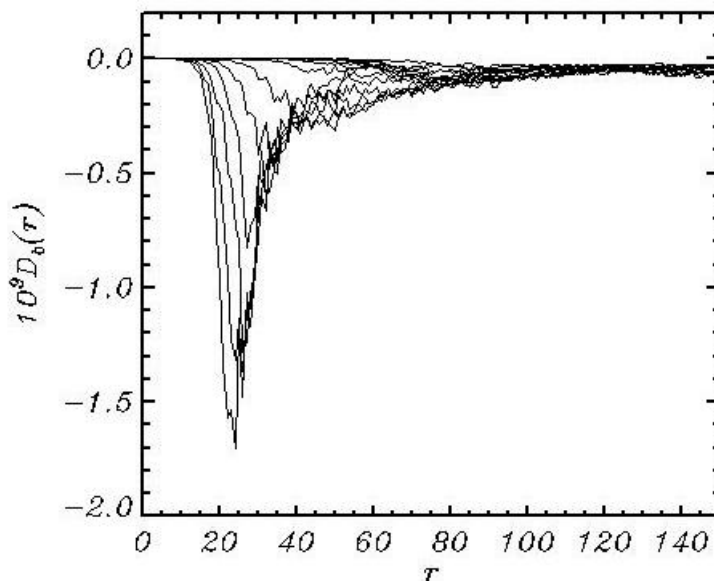


Figure 9. Breaking dissipation spectra $D_b(r)$ integrated over angle ψ in the polar coordinates and averaged over consequent intervals of length about 100 units of nondimensional time t .

²The 2-D Fourier spectral ‘tail’ looks like ‘peacock’ tail.

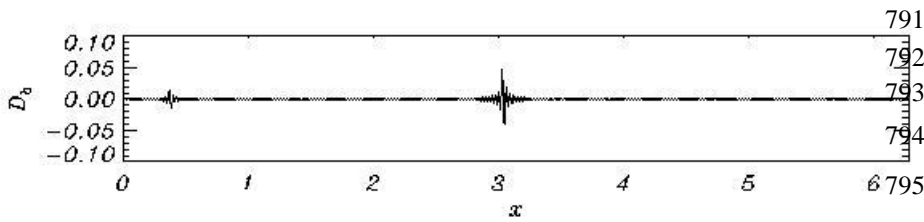
780

781 An example of profile of the energy input due to breaking $D_b(x)$ is given in Fig. 10. As
 782 seen, energy input is fluctuating around the breaking point. A diffusion operator chosen for
 783 breaking parameterization not only decreases total energy but also redistributes the energy
 784 between Fourier modes in Fourier space.

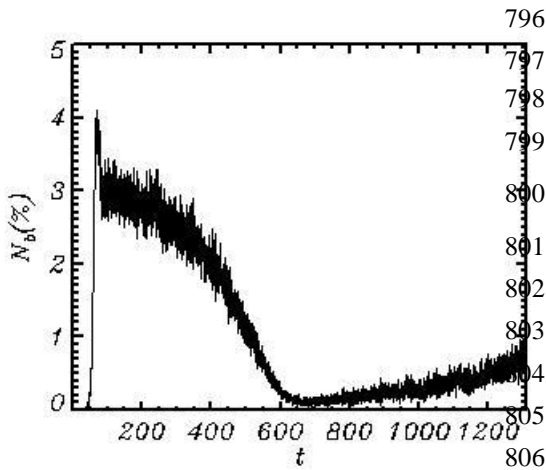
785 In general, for the specific conditions considered in the paper, the breaking is an
 786 occasional process taking place in a small part of domain. The kurtosis of input energy due to the
 787 breaking $D_b(\xi, \vartheta)$, i.e., the value

$$788 \quad Ku = \overline{\overline{D_b^4}} \left(\overline{\overline{D_b^2}} \right)^{-2} - 3 \quad (36)$$

789 is of the order of 10^3 , which corresponds to plain function with occasional separated peaks.
 790



791
792 **Figure 10.** Example of
793 energy input due to
794 breaking $D_b(x)$.



796
797 **Figure 11.** Evolution of number of wave breaking
798 events N_b expressed in percentage of the number
799 of grid points $N_x \times N_y$.
800
801
802
803
804
805

808 The number of breaking points in terms of percentage of the total number of points is given in
 809 Fig. 11. As seen, the number of breaking events is going down to $t = 600$ and then is growing up
 810 to the end of the calculations. The number of breaking events is not directly connected with
 811 intensity of breaking, which is seen when comparing Fig. 11 and curve 2 in Fig.3.

812 An integral term describing nonlinear interaction $\overline{\overline{N}}$ in Eq. (29) is small (compared with local
 813 values of $N_{k,l}$), but the magnitude of spectrum $N(r)$ is comparable with input $I(r)$ and
 814 dissipation $D_t(r)$ and $D_b(r)$ terms. The presentation of term $N(r)$ in a form shown in Figs. (6)
 815 – (8) is not clear. This is why the spectra $10^8 N(r)$ averaged over interval $\Delta t = 100$ are plotted
 816 separately in Fig. 11 for the last eight intervals (thick curves) together with the wave spectrum
 817 $10^6 S_h(r)$. In general, the shapes of spectrum $N(r)$ agree with the conclusions of the quasi-
 818 linear Hasselmann (1962) theory (Hasselmann et al., 1985). At low wave number slope of
 819 spectrum the nonlinear influx of energy is positive while at the opposite slope it is negative. This

820 process produces shifting of spectrum to the lower wave number (downshifting). Opposite to the
 821 Hasselmann's theory, these results are obtained by solution of full three-dimensional equations.
 822 It would be interesting to compare our results with the calculations of Hasselmann's integral
 823

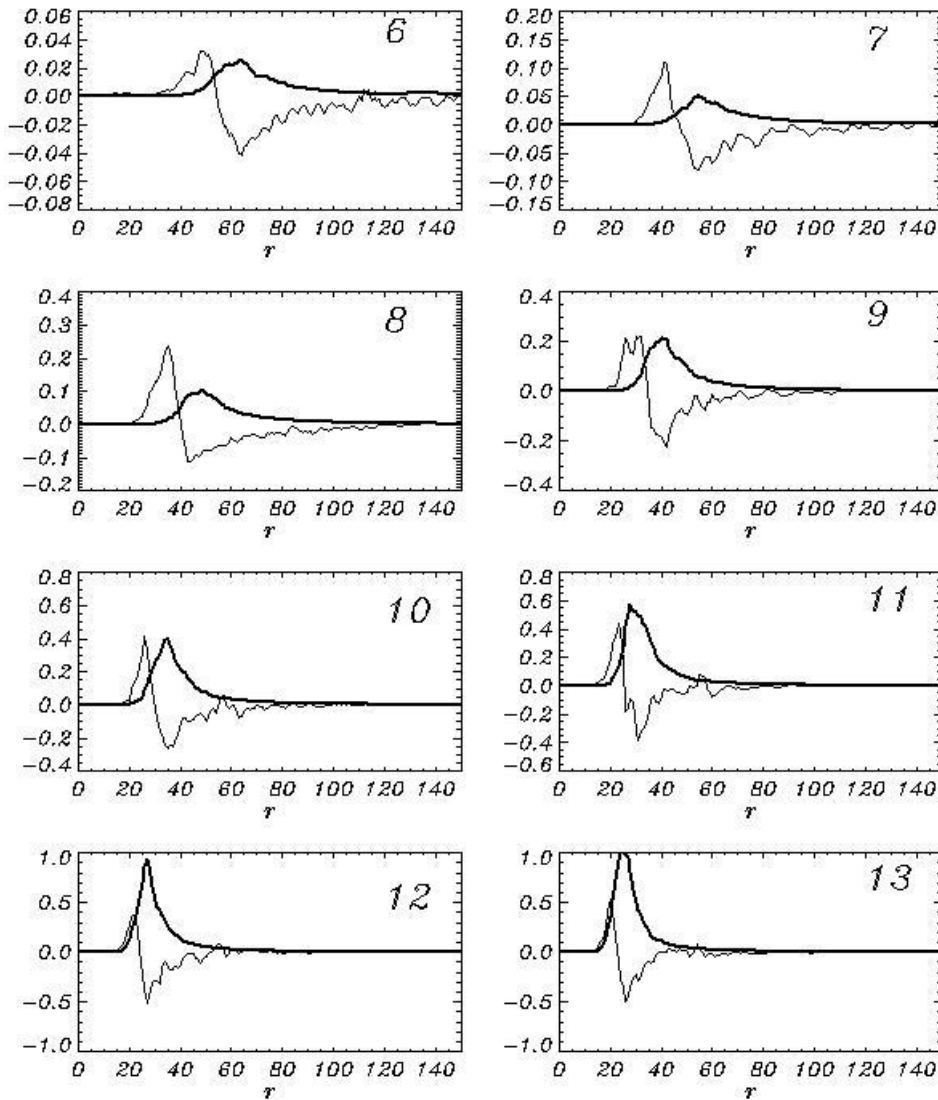


Figure 12. Sequence of wave spectra $S_h(r)$ (thick curves) and nonlinear input term $N(r)$ (thin curves) averaged over consequent eight periods of length $\Delta t = 100$ starting from 6th period.

851 Unfortunately, neither of the existing programs of such type permits doing calculations with such
 852 a high resolution that was used in the current model. Note that nonlinear interactions also
 853 produce widening of spectrum.

854 Obviously, the nonlinearity is quite an important property of surface waves. The
 855 contribution of nonlinearity can be estimated, for example, by comparison of the kinetic energy
 856 of linear component $E_l = 0.5(\overline{\varphi_x^2} + \overline{\varphi_y^2} + \overline{\varphi_z^2})$ and the total kinetic energy E_k (Fig. 13). A ratio
 857 E_l / E_k as a function of time remains very close to 1, which proves that the nonlinear part of
 858 energy makes up just a small percentage of the total energy. It does not mean that the role of
 859 nonlinearity is small; its influence can manifest itself over large time scales.

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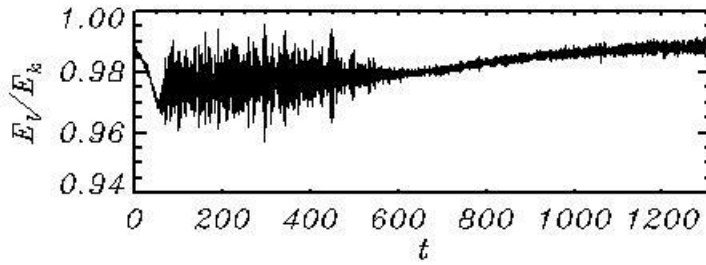


Figure 13. Time evolution of ratio E_l / E_k .

868

The time evolution of integral spectral characteristics is presented in Fig. 14.

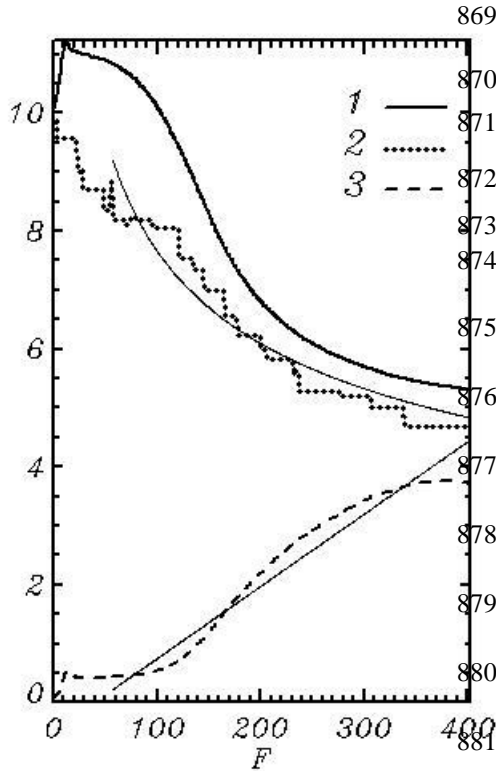


Figure 14. Time evolution of: weighted frequency ω_w (1) (Eq. 34); spectral peak frequency $\omega_p = k_p^{1/2}$ (2); full energy E (3) (Eq. 28). Thin curves are empirical a distance passed by the spectral peak.

882

883 Curve 1 corresponds to the weighted frequency ω_w

$$884 \quad \omega_w = \left(\frac{\int k S k dl}{\int S k dl} \right)^{1/2}, \quad (37)$$

885 where integrals are taken over the entire Fourier-domain. The value ω_w is not sensitive to the
 886 details of spectrum, hence, it well characterizes the position of spectrum and its shifting. Curve2
 887 describes evolution of the spectral maximum. The step shape of curve corresponds to the
 888 fundamental property of downshifting. Opposite to the common views, development of spectrum
 889 occurs not monotonically, but by appearance of a new maximum at lower wave number as well
 890 as by attenuation of the previous maximum. . Interesting that the same phenomenon is also
 891 observed in spectral model (Rogers et al., 2012). Curve 3 describes the change of total energy
 892 $E = E_p + E_k$. As seen all three curves have a tendency for slowing down of evolution rate. Then
 893 the energy tends to decrease, but we are not sure about the nature of this effect. Such behavior
 894 can be explained by a fluctuating character of mutual adjustment of input and dissipation or

895 simply by worsening of the approximation because of the downshifting process. The numerical
 896 experiment reproduces the case when development of wave field occurs under the action of
 897 permanent and uniform wind. This case corresponds to JONSWAP experiment. Despite large
 898 scatter, the data allow us to construct empirical approximations of wave spectrum, as well as to
 899 investigate the evolution of spectrum as a function of fetch F . In particular, it is suggested that
 900 the frequency of spectral peak changes as $F^{-1/3}$, while full energy grows linearly with F . Neither
 901 of the dependences can be exact, since they do not take into account the approaching to a
 902 stationary regime. Besides, the dependence of frequency on fetch is singular at $F = 0$.

903 The value of fetch in periodic problem can be calculated by integration of peak phase
 904 velocity $c_p = |k|^{-1/2}$ over time.

$$905 \quad F = \int_{t_0}^t c_p dt \quad (38)$$

906 The JONSWAP dependencies for the frequency of spectral peak ω_p and full energy E are
 907 shown in Fig 14 by thin curves. Dependence $\omega_p \sim F^{1/3}$ is qualitatively valid. Dependence of the
 908 total energy on fetch does not look like a linear one, but it is worth to note that JONSWAP
 909 dependence is evidently inapplicable to a very small and large fetch.

910

911 **5. Discussion**

912

913 A model based on the full three-dimensional equations of potential motion with free
 914 surface was used for simulation of development of wave fields. The model is written in the
 915 surface-following nonstationary non-orthogonal coordinate system. The details of numerical
 916 scheme and the results of validation of the model were described in (Chalikov et al., 2014). The
 917 main difference between the given model and HOS model (Ducroset et al., 2017) is that our
 918 model is based on direct solution of 3-D equations for velocity potential. This approach is similar
 919 to that developed at Technical University of Denmark (TUD, see Engsig-Karup et al., 2009).
 920 Actually, the models developed at TUD are directed to solution of a variety of problems
 921 including such problems as modeling of wave interaction with submerged objects and simulation
 922 of wave regime in the basins with real shape and topography.

923 In the current paper a three-dimensional model was used for simulation of development
 924 of wave field under the action of wind and dissipation. The input energy is described by single
 925 term, i.e., surface pressure p in Eq. (4). It is traditionally assumed that the complex pressure
 926 amplitude in Fourier space is linearly connected with the complex elevation amplitude with a
 927 complex coefficient called β – function. Such simple formulations can be imperfect. Firstly, it is
 928 assumed that wave field is represented by superposition of linear modes with slowly changing
 929 amplitudes and phase velocity obeying the linear dispersive relation. This assumption is valid
 930 only for a low-frequency part of spectrum. In reality, the amplitudes of medium and high-
 931 frequency modes undergo fluctuations created by reversible interactions. A solid dispersion
 932 relation does not connect their phase velocities with wave number. Besides, it is also quite
 933 possible that the suggestion of linearity of the connection between pressure and elevation
 934 amplitudes is not precise, i.e., β – function can depend on amplitudes of modes.

935 We are not familiar with any observational data that can be used for formulation of
936 statistically provided scheme for calculation of the input energy to waves. The only method that
937 can give more or less reliable results is mathematical modeling of the statistical structure of
938 turbulent boundary layer above a curvilinear moving surface, of which characteristics satisfy
939 kinematic conditions. The method described above is based on several millions values of
940 pressure referred strictly to surface. As a whole, the problem of boundary layer seems even more
941 complicated than the wave problem itself. Some early attempts to solve this problem were made
942 on the basis of the finite difference two-dimensional model of boundary layer written in the
943 simple surface following coordinate (see review Chalikov, 1986). Waves were assigned as a
944 superposition of linear modes with random phases corresponding to the empirical wave
945 spectrum. This approach was not quite accurate since it did not take into consideration the
946 nonlinear properties of surface (for example, the sharpness of real waves and the absence of
947 dispersive relation for waves of medium and high frequencies. The next step was formulation of
948 coupled models for boundary layer and potential waves, both written in the conformal
949 coordinates (Chalikov and Rainchik, 2014). The calculations showed that pressure field consists
950 mostly of random fluctuations not directly connected with waves. A small part of these
951 fluctuations is in phase with surface disturbances. The calculated values of β in Eq. (13) have
952 large dispersion. However, since the volume of data was very large, the shape of β -function was
953 found with high-level accuracy. Probably, approximation of β used in the current work can be
954 considered as most adequate. We are planning additional investigations based on coupled wind-
955 wave models. The next step in investigations of Wave boundary Layer (WBL) should use a
956 three-dimensional LES approach. Note that even availability of large volume of data on the
957 structure of WBL does not make the problem of parameterization of wind input in spectral wave
958 models easily solvable, since the pressure is characterized by a broad continuous spectrum
959 created by nonlinearity.

960 The wave breaking is obviously even more complicated than the input energy.
961 Nevertheless, this problem can be simplified, if common ideas used in the numerical fluid
962 mechanics are accepted. For example, in LES modeling the more or less artificial viscosity is
963 introduced to prevent too large local velocity gradients. It is a fact that the numerical instability
964 terminating computations precedes wave breaking. Hence, the scheme should prevent breaking
965 approach to preserve stability of the numerical scheme. Hence, a wave model should contain the
966 algorithms preventing appearance of too large slopes. The criterion of breaking is introduced not
967 for recognizing of the breaking itself, but for the choice of places where it might happen (or,
968 unfortunately, might not happen). Finally, the algorithm should produce local smoothing of
969 elevation (and surface potential). The algorithm should be highly selective so that ‘breaking’
970 would occur within narrow intervals and not affect the entire area. The exact criteria of breaking
971 events (most evident of them is the breaking itself) cannot be used for parameterization of
972 breaking since in coordinate system (1) the numerical instability occurs long before breaking. In
973 our opinion, the most sensitive parameter indicating potential instability is the curvilinearity
974 (second derivative) of elevation.

975 In the current work, the breaking is parameterized by diffusion algorithm with the
976 nonlinear coefficient diffusion providing high selectivity of smoothing. We admit that such
977 approach can be realized in many different forms. The same situation is observed in a problem of
978 turbulence modeling for parameterization of subgrid scales. Note, that breaking dissipation in
979 phase resolving models is included in more realistic manner, than in spectral models. For

980 example, the breaking is simulated in physical space, what allows to reduce the height and
981 energy of nonlinear waves, composed of several modes, In spectral models dissipation is
982 distribute more or less arbitrarily over entire spectrum. Spectral models sometimes include the
983 additional dissipation of short waves due to their modulation by long waves (Young and
984 Babanin, 2006; Babanin et al., 2010). In phase resolving models this process have been included
985 explicitly.

986 We can finally conclude that the physics included in the wave model is still based on a
987 shaky ground. Nevertheless, the result of the calculations looks quite realistic, which convinces
988 us that the approach deserves further development.

989 The numerical models of waves similar to that considered in the paper have a lot of
990 important applications. Firstly, they are a perfect tool for development of physical
991 parameterizations schemes in spectral wave models. Secondly, the direct model can be used in
992 future for numerical simulation of wave processes in the basins of small and medium size. These
993 investigations can be based on HOS model (Ducrozet et al., 2016) or the model used in the
994 current paper. However, the most universal approach seems to be developed at the Technical
995 University of Denmark (see Engsig-Karup, 2009). Any model used for a long-term simulation of
996 wave field evolution should include the algorithms describing transformation of energy, similar
997 to those considered in the current paper.

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1178 **Figure captions**

1179

1180 **Figure 1.** Real (dashed curve) and imaginary (solid curve) parts of β -function.

1181

1182 **Figure 2.** Probability of curvilinearity $\eta_{\xi\xi}$. Thick curve calculated with full 3-D model; thin
1183 curve is a probability calculated over ensemble of linear modes with the same spectrum.

1184

1185 **Figure 3.** Evolution of integral characteristics of solution, rate of evolution of integral energy
1186 multiplied by 10^7) due to: 1 – tail dissipation D_t (Eqs. 19-23); 2 – breaking dissipation D_b (Eqs.
1187 24-27); 3 – input of energy from wind I (Eqs. 14-18); 4 – balance of energy $I + D_t + D_b$. Curve
1188 5 shows the evolution of wave energy $10^5 E$. Vertical bars of grey color show the instantaneous
1189 values; thick curve shows the smoothed behavior.

1190

1191 **Figure 4.** The wave spectra $S_h(r)$ integrated over angle ψ in the polar coordinates and
1192 averaged over consequent intervals of length about 100 units of nondimensional time t . The
1193 spectra are growing and shifting from right to left.

1194

1195 **Figure 5.** Sequence of 3-D images of $\lg_{10}(S(k,l))$ where each panel corresponds to single
1196 curve in Fig. 3. The left side refers to wave number $l(-M_y \leq l \leq M_y)$ and front side – to
1197 $k(0 \leq k \leq M)$. The numbers indicate end of time interval expressed in hundreds of
1198 nondimensional time units.

1199

1200 **Figure 6.** Sequence of 2-D images of $\lg_{10}(S(k,l))$ averaged over consequent seven periods
1201 length $\Delta t = 200$. Numbers indicate the period of averaging (first panel marked 0, refers to initial
1202 conditions). Horizontal and vertical axes correspond to wave numbers k and l correspondingly

1203

1204 **Figure 7.** The spectrum of energy input $I(r)$ integrated over angle ψ in the polar coordinates
1205 and averaged over consequent intervals of length about 100 units of nondimensional time t .

1200 **Figure 8.** Tail dissipation spectra $D_t(r)$ integrated over angle ψ in the polar coordinates and
1201 averaged over consequent intervals of length about 100 units of nondimensional time t .

1202 **Figure 9.** Breaking dissipation spectra $D_b(r)$ integrated over angle ψ in the polar coordinates
1203 and averaged over consequent intervals of length about 100 units of nondimensional time t .

1204 **Figure 10.** Example of energy input due to breaking $D_b(x)$.

1205 **Figure 11.** Evolution of number of wave breaking events N_b expressed in percentage of the
1206 number of grid points $N_x \times N_y$.

1207 **Figure 12.** Sequence of wave spectra $S_h(r)$ (thick curves) and nonlinear input term $N(r)$ (thin
1208 curves) averaged over consequent eight periods of length $\Delta t = 100$ starting from 6th period.

1209 **Figure 13.** Time evolution of ratio E_l / E_k

1210 **Figure 14.** Time evolution of: weighted frequency ω_w (1) (Eq. 34); spectral peak frequency
1211 $\omega_p = k_p^{1/2}$ (2); full energy E (3) (Eq. 28). Thin curves are empirical a distance passed by the
1212 spectral peak.