Evaluation of extreme wave probability on the basis of long-term data analysis.

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Abstract. A method of calculation of wind wave height probability based on the significant wave height probability is described (Chalikov and Bulgakov, 2017). The method can be also used for estimation of height of extreme waves of any given cumulative probability. The application of the method on the basis of long-term model data is presented. Examples of averaged annual and seasonal fields of extreme wave heights obtained by the above method are given. Areas where extreme waves can appear are shown.

Introduction

Highest risks of economic and environmental damage for sea-based human activities, i.e., cargo shipments, fishery, oil production etc., are mostly connected with extreme weather conditions on the sea surface among which strong storms are the foremost. It is especially difficult to predict emergency situations caused by extreme waves for those cases of sea-based activities which require people's long stay at sea or prolonged use of equipment in the ocean.

One of the methods to minimize possible risks is using of climate data based on long-term series of observations. At present there are archives consisting of the reanalysis data on surface waves based on wave forecasts corrected by different methods, i.e., direct measurements using accelerometers and GPS-buoys; remote measurements by satellite-borne altimetry and various types of radars. The main characteristic of wave field included in the archive is significant wave height H_s defined as a mean value (trough to crest) of one third of the highest of all the waves (Ochi 2005). The value of H_s is calculated in the following way:

$$H_s = 4\left(\int_0^\infty \int_0^\infty S(k_x, k_y) dk_x dk_y\right)^{1/2}, \quad (1)$$

where k_x, k_y are wave numbers, while $S(k_x,k_y)$ is the wave spectrum.

It is evident that knowledge on significant wave height is not sufficient to evaluate real wave height for a given wave field. Extreme waves of the same height can appear with different probability for different values of H_s. For example, a wave 10 meters high can appear both in a wave field with H_s=10 m and in a wave field with H_s=5 m. Or there can be waves with height 15 m. and 17 m in a wave field characterized by H_s 10 m. Thus, H_s data does not give enough information about the probability of real wave heights.

The nature of freak waves was investigated analytically (Onorato et al., 2009) and numerically (Chalikov, 2009). Recently it was found that the statistical properties of trough-to-crest wave height are quite different from those of the wave height above mean level. Papers (Chalikov and Babanin, 2016; Chalikov, 2017) show that linear and nonlinear statistics of extreme waves (defined as trough-to-crest waves) are identical not only for a broad spectrum but for one-dimensional wave field too. It means that generation of a trough-to-crest extreme wave is the result of simple superposition of linear modes, no matter how broad the spectrum is. This property is not found for the wave height above mean level. Thus, the statistical properties of trough-to-crest wave height can be investigated with linear modeling, just by generation of large ensembles of superposition of linear modes with random phases and the spectrum prescribed. Thus the problem of trough-to-crest statistics becomes quite straightforward. Contrary to such an approach, investigation of the statistics of wave height above mean level remains a subject of nonlinear wave theory. From the practical point of view, for floating objects the data on the full height (trough-to-crest) of wave are more important. However, the data on probability of wave height above mean level are important for fixed-construction offshore platform.

The theoretical probability distribution for wave crest height (or wave height above mean level) was suggested by Weibull (Weibull, 1951). Later it was studied on a basis of observational data in nature and wave channels (see review by Kharif et al. (2009)). Extended data for estimation of probability of wave height can be obtained with integration of nonlinear modes based on full equations for potential (irrotational) flow (Touboul and Kharif, 2010; Chalikov et al, 2009). Methods of probability calculations were considered in many papers (see, for example Bitner-Gregersen and Toffoli (2012); Dyachenko et al. (2016)).

The most popular method of trough-to-crest wave height detection is based on a zero-crossing technique. A direct method is based on use of moving windows, the method is applicable both for 1-D and 2-D cases.

Estimation of extreme waves nowadays mostly is made by analysis of data of significant wave height. Jiangxia Li (2018) analyzing long-term data considered that an extreme wave is a wave exceeding two significant wave heights. In Larsen et al. (2015) long-term wave dataset was analyzed by spectral method, it was shown that the spectrum of modeled significant wave height (through to crest) miss the energy for frequency more than 2.5x10^{-5} Hz (daily timescale and less). A spectral correction method was developed to fill in the missing variability of the modeled variable at high frequencies. In Lanli Guo and Jinyu Sheng (2015) the peaks-over-threshold method was used to estimate the extreme significant wave heights from 30-year wave simulations. In Samayam et al. (2017) estimation of extreme wave height (crest-mean level) was made by using methods of extreme value theory. The main advantage of the method of Chalikov and Bulgakov (2017) compared with methods mentioned above is that their method is based on results of direct modeling of wave fields.

This paper is devoted to investigation of the statistics and geographical distribution of wave height above mean sea level.

1. Description of the method

In Chalikov and Bulgakov (2017) an algorithm for estimation of cumulative probability of waves $P(h)$ exceeding a specific value of wave height above mean level (h) was developed using long-term data on H_s. The description of the method is given below.

The probability of a wave exceeding a specific height h, if significant wave height is in a small range dH_s around H_s, equals $\tilde{P}(\tilde{H})$ for specific $\tilde{H} = h/H_s$ multiplied by probability of H_s in this range ($\tilde{P}(\tilde{H}) \cdot P(H_s) dH_s$), by the standard definition of conditional probability. Consequently, $P(h)$ can be determined as the integral of $\tilde{P}(\tilde{H}) \cdot P(H_s)$: over all possible values of H_s:

$$P(h) = \int_0^{H_{smax}} \tilde{P}(\tilde{H})\, P(H_s) dH_s, \qquad (2)$$

where $P(H_s)$ is probability distribution of H_s for a specific point, while H_{smax} is the maximum value of H_s in the dataset for a specific point.

The model H_s data used for $P(H_s)$ were calculated with the latest version of the WAVEWATCH III model (Tolman, 2014) and GFS-2 wind analysis 2 (Sasha et al.,2014). The hindcasts cover the period from August 1999 to July 2015. The spatial resolution of the dataset fields is 0.5 × 0.5 degree. Calibration of the model and its validation are carried out using a great number of wave buoys. The data and results of its validation are described in Chawla et al. (2013).

The approximation of $\tilde{P}(\widetilde{H})$ was based on results of a 3-D model of potential (irrotational) flow. The model used spectral definitions of fields, finite differences for vertical derivatives calculation and a fourth-order Runge–Kutta scheme for time integration. Fourier resolution is 256*64 wave number, resolution in physical space is 1024*256 (more detail in Chalikov et al. (2014)). The calculations were done for 350 units of nondimensional time, i.e., for 70,000 time steps. The initial conditions were generated on basic a JONSWAP spectrum. Model runs were calculated under a condition that input energy from wind to waves equals wave energy dissipation. This condition corresponds to fully developed wind waves. Totally 75 experiments were made (more detail in Chalikov et al. (2014); Chalikov and Bulgakov (2017)).

The results of the series of experiments were processed in the following way: each wave field of surface height above mean level (η) reproduced by the numerical model was normalized by the value of significant wave height corresponding to this field ($\widetilde{H} = \eta/H_s$). (Note that η is a variable of 3-D model of potential (irrotantional) waves. It should be distinguished from h despites the fact that both η and h have the same physical sense.) Then, a nondimensional wave field was used for calculation of cumulative probability of nondimensional wave height $\tilde{P}(\widetilde{H})$. The distribution obtained was approximated by the following function:

$$\tilde{P}(\widetilde{H}) = \exp(-3.97\widetilde{H} - 4.02\widetilde{H}^2) \qquad (3)$$

Note that $\tilde{P}(\widetilde{H})$ is cumulative probability of the height of the free surface above mean level. This probability for $\widetilde{H} = 1$ (the height of free surface equals significant wave height) is quite small (0.0003).

The above expression can be used for the interval $0 \leq \widetilde{H} \leq 1.85$. The probability of a wave higher than 1.85 (the maximal value of $\widetilde{H}$ in data) can be considered as extremely low and therefore is neglected. It should be noted that approximation (3) was obtained with use of the precise 3-D model based on full nonlinear equations. The volume of data used for approximation (3) includes more than 4.5 billion values of η (number of points in single field multiplied by number of records in experiment multiplied by number of experiments). Currently, this approximation is considered as universal for wind wave fields where cases of freak waves are most likely. Waves of other types of spectrum (swells) have a small steepness and do not influence extreme wave generation except in cases when long-wave currents can steepen shorter waves.

The spatial distribution of extreme wave probability was investigated, based on (3) from Chalikov and Bulgakov (2017) together with the spatial distribution of significant wave height from Chawla et al. (2013). In this paper results of an application of this method are considered. We show global fields of wave height with cumulative probability 10^{-7} so calculated.

2. Results

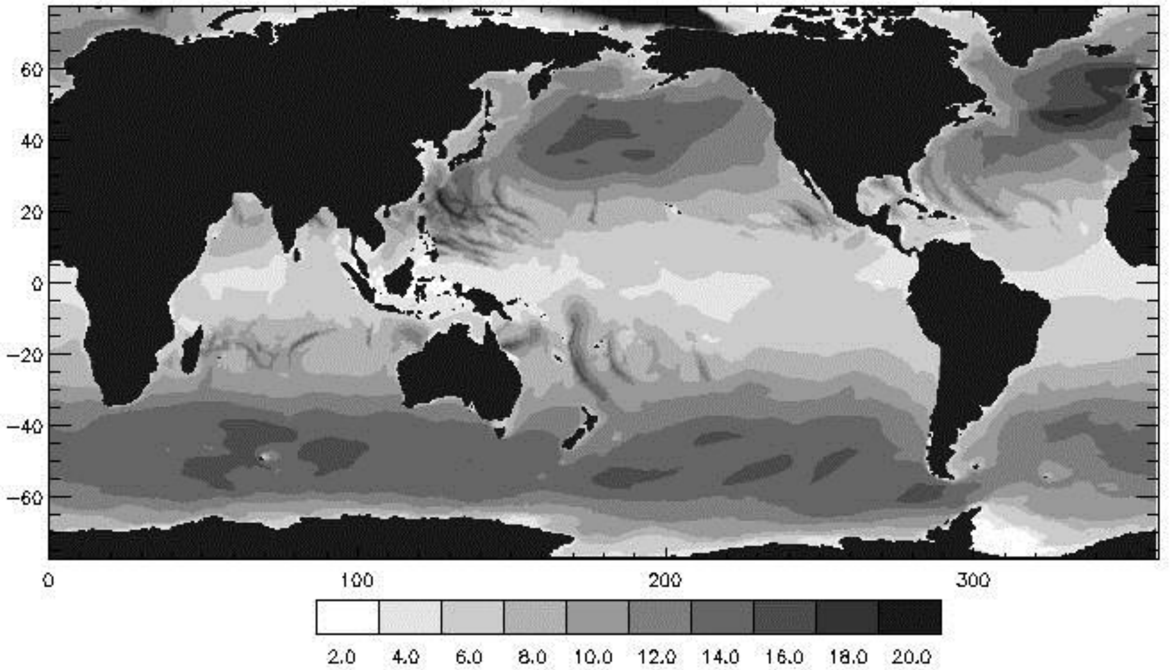

Fig.1 — Wave heights (m) above mean level with cumulative probability 10^{-7}, annual average

Figure 1 shows an average annual field of wave heights with cumulative probability 10^{-7}. It can be seen that waves with height up to 20 m above mean level can appear with such probability, some of the extreme waves (16 m and more) being found in areas of active navigation (eastern part of the Atlantic Ocean, East China Sea, Philippine Sea, Yellow Sea, south-western part of the Pacific Ocean).

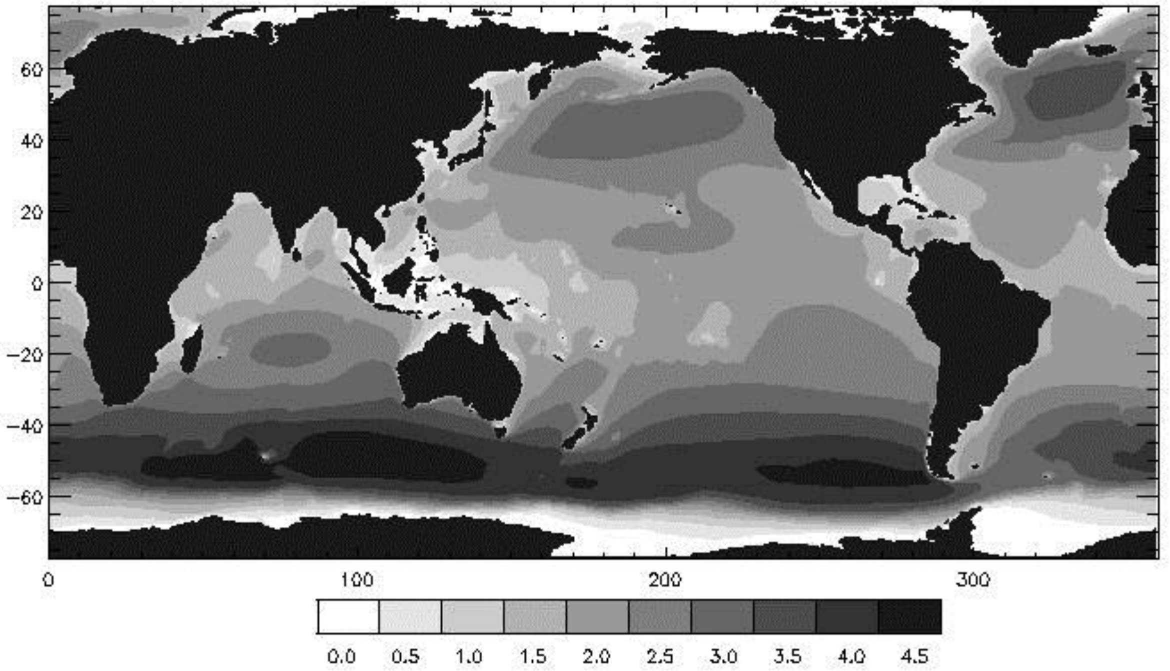

Figure 2 – Average annual significant wave height (m).

The distribution of annual average significant wave height provided by the model (Chawla et al 2013) is shown in Fig. 2. As seen, the maximum value in the field of annual average significant wave height does not exceed 5 m (southern area of Indian and Pacific oceans), while the height of real extreme wave can reach 16 m there. The data in Fig 1 have a more complicated structure, due, for example, to the periods with strong wind along trajectories of tropical storms. Consequently, the calculations of distribution of real wave height should be done for shorter periods, i.e., for seasonal or monthly averaged data on significant wave heights.

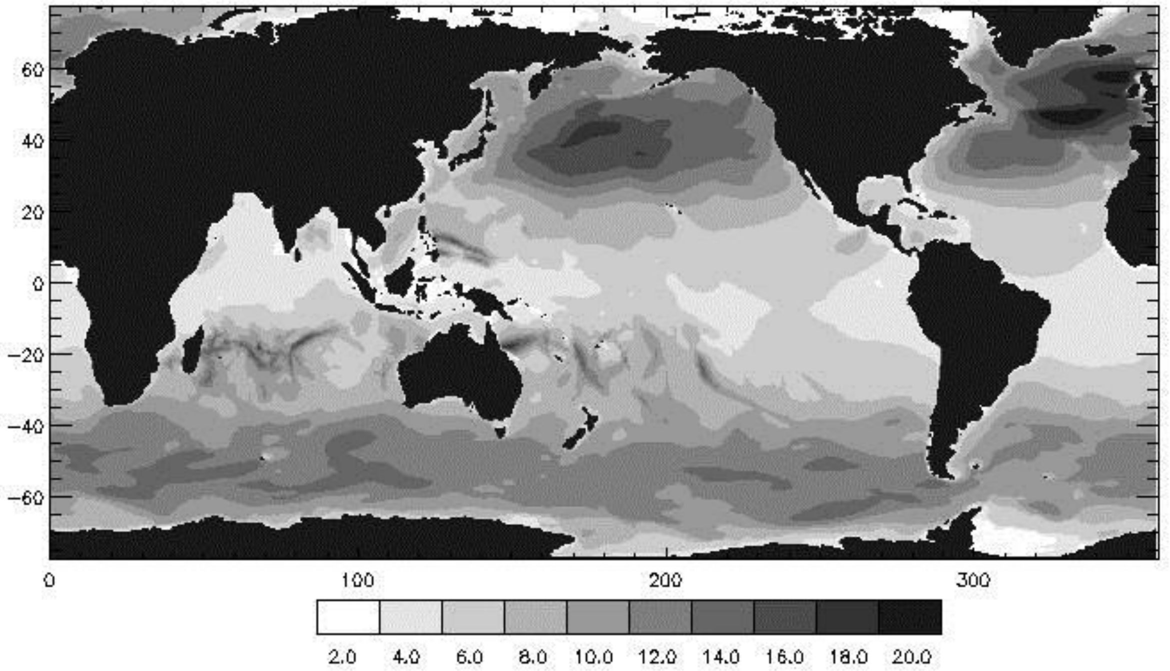

Figure 3. – Wave height above mean level (m) with cumulative probability 10^{-7} for December-February.

Fig. 3 shows the field of wave height above mean level with probability 10^{-7} averaged for December-February. When comparing Figs 3 and 1 it is seen that in mid-latitudes of the Northern Hemisphere wave heights become higher. In some areas appearance of extreme wave heights exceeding 16 m is possible. At the same time there are actually no extreme waves in the eastern part of the Arctic Ocean, which is connected with seasonal ice formation in the area. In equatorial and tropical areas of the World Ocean wave heights are less in winter (Northern Hemisphere), as compared with the average annual wave heights. It should be noted that in the western part of the Atlantic Ocean trajectories of hurricanes disappeared while the number of such trajectories increased in the Indian Ocean.

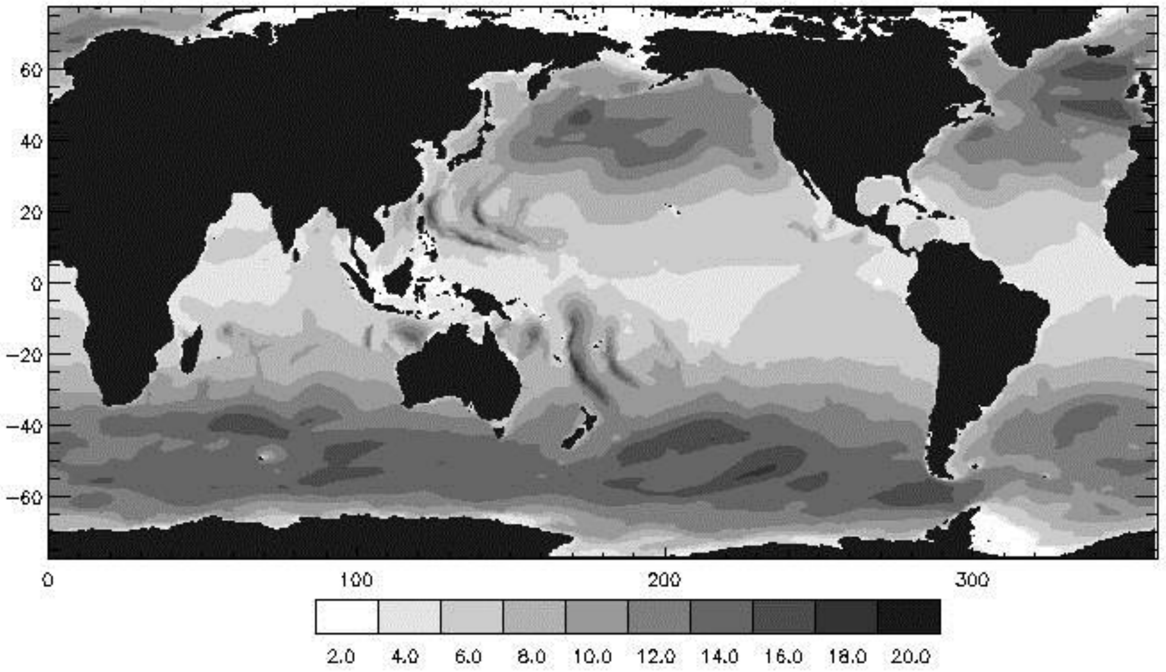

Figure 4. – Wave height above mean level (m) with cumulative probability 10^{-7} averaged for March-May.

An increase of wave heights over March-May can be seen (Fig.4) in the Southern Hemisphere. Actually all the area of mid-latitudes from the latitude of 40 degrees S. to the latitude of 60 degrees S. is characterized by probability $>10^{-7}$ of wave heights above mean level exceeding 14 m. In mid-latitudes of the Northern Hemisphere in spring, wave-height values are more than the average annual values, though less than the winter values, while in some areas (Atlantic Ocean near Iceland, Pacific Ocean near Bering Sea) appearance of waves exceeding 14 m in height above mean level is quite possible.

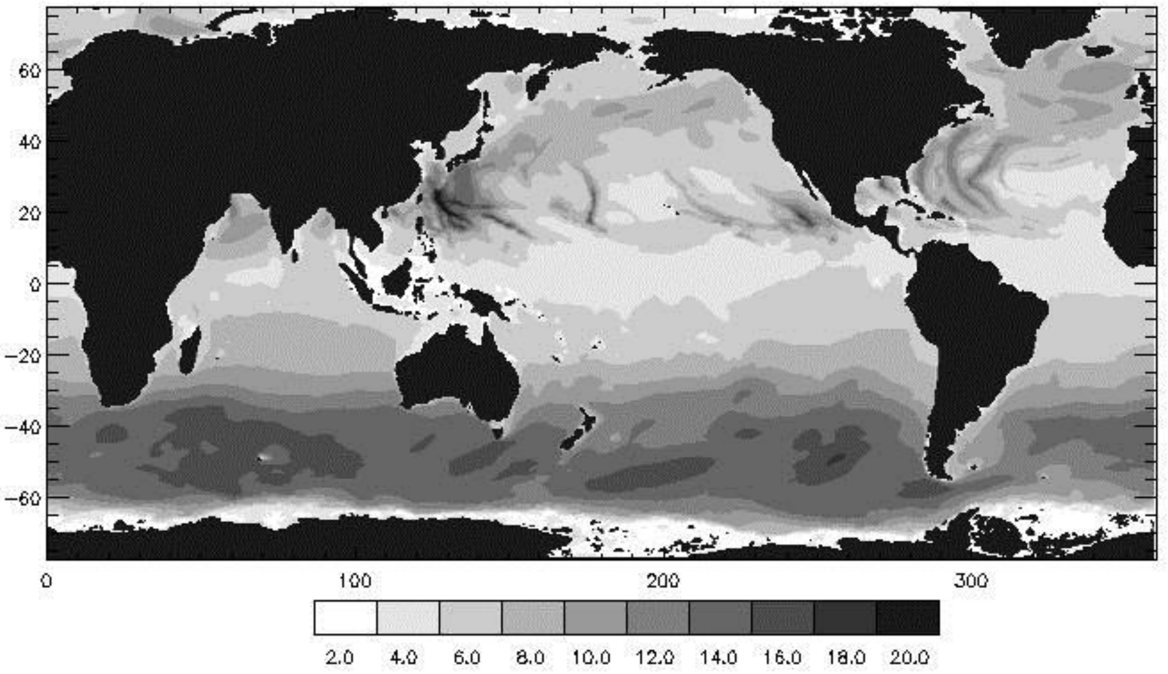

Fig.5 –Wave heights above mean level (m) with cumulative probability of 10^{-7}, for June-August

Summer months (Fig.5) are characterized by a general decrease of extreme wave probability. It is especially noticeable in the northern areas of the Atlantic and Pacific Oceans. Also, wave heights slightly decreased in the Southern Hemisphere. It should be noted that storm tracks appear off the eastern coast of North America and disappear in the southern part of the Pacific Ocean. Besides, quite distinct trajectories of storms appear in the eastern part of the Pacific Ocean. Small wave heights can be observed in the Arctic Ocean, in the area free from ice.

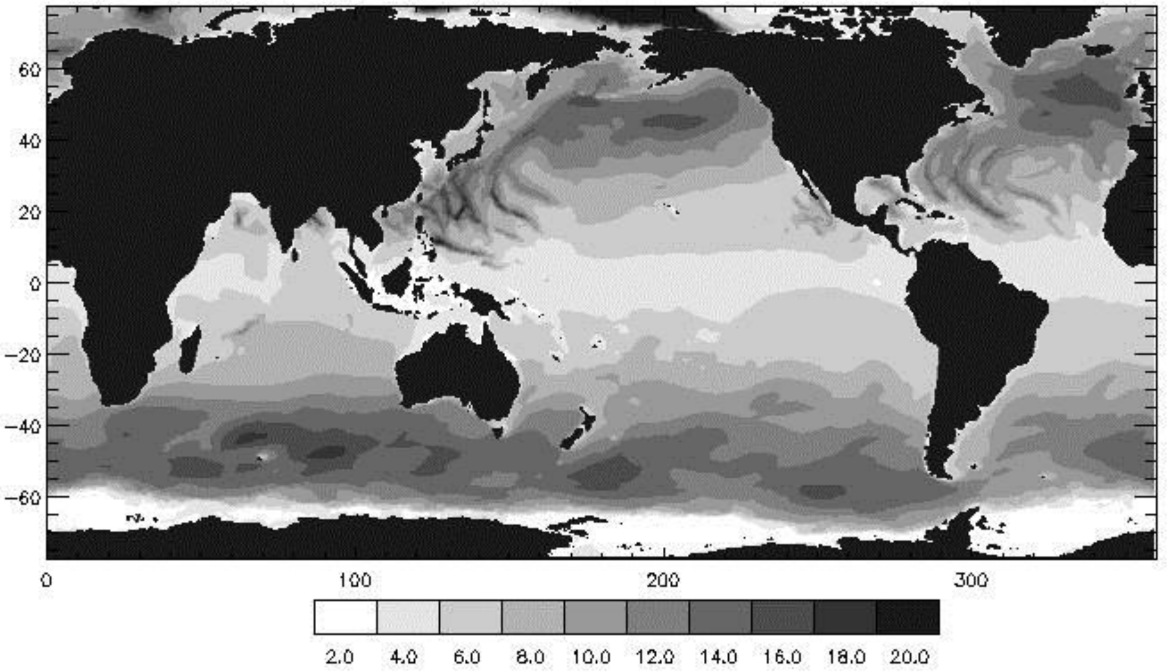

Figure 6. – Wave heights above mean level (m) with cumulative probability 10^{-7}, averaged for September -November

During autumn months (Fig. 6) an increase of wave heights is observed in the Arctic Ocean, values of the extreme wave height above mean level sometimes reaching 20 m. Among other features is an increase of the wave-free area in polar latitudes of the Southern Hemisphere, which is obviously connected with seasonal ice formation.

It is quite evident that the average monthly fields of cumulative wave-height probability will allow us to obtain more exact information on the areas of extreme wave probability.

3. Discussion and conclusions

The paper describes a method of calculation of extreme wave probability, based on (I) wave-model runs for its relation to significant wave-height (Chalikov and Bulgakov 2017) and (II) H_s spatial distribution from 16-year hindcast data (Chawla et al. 2013).

The method uses the results of massive numerical simulation with 3-D irrotational wave models (Chalikov et al., 2014). Initial conditions for each run were assigned by the JONSWAP spectrum, but for each run random phases were different. Such details of the initial spectrum are not too important. The ensemble modeling is used to eliminate the effects of reversible nonlinear interactions causing down shifting that can influence the statistics. To be sure that the simulated process can be treated as quasi-stationary; the time of integration was chose to be relatively short, viz. 350 units of nondimensional time. The rich statistics were obtained by multiple repetitions of runs with the same initial spectrum. The total number of records used for construction of approximation (3) was 4,587,520,000.

The wave spectrum during integration undergoes fluctuations: amplitudes grow with increase of wave number due to reversible nonlinear interactions. However, the averaged spectrum remains similar in different runs and more or less close to the spectrum assigned in initial conditions, confirming quasi-stationarity and some universality of the approximation (3) to wave height probability. This approximation fills the gap between more or less known statistics on significant wave height and unknown statistics of real waves.

The method can be used for estimation of probability of extreme waves, which is important for designing of engineering constructions. The approach here can be used to evaluate the height of waves of any given cumulative probability. It is not expedient to use the values less than 10^{-9} which are outside the range of validity for equation (3) ($\tilde{P}(1.85)$ is approximately 10^{-9}). Hence, on the whole, the method considered is suitable for estimation of extreme values of wave heights having small probability.

The maps of global distribution of wave heights of probability 10^{-7} for the main seasons illustrate the approach of the method. Estimation of the return period of a wave with specific cumulative probability is quite a sophisticated problem. It will be subject of the next work.

We do not state that results of this paper solve completely the problem of treating data on significant wave height in terms of real wave height (above mean level). The most difficult unresolved problem is the problem of estimating of confidence intervals which needs further extensive simulations and analysis.

The authors are thankful to Professor D. Chalikov for his useful consultations.

The investigation was fulfilled with financial support of Russian Scientific Fund (Project No 16-17-00124).

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