Evaluation of extreme wave probability on the basis of long-term data analysis.

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Abstract. A method of calculation of wind wave height probability based on the significant wave height probability is described. An application of the method on the basis of long-term data analysis is presented. Examples of averaged annual and seasonal fields of extreme wave heights obtained by the above method are given.

1. Introduction

Highest risks of economic and environmental damage for the sea-based human activities, i.e., cargo shipments, fishery, oil production etc., are mostly connected with extreme weather conditions on sea surface among which strong storms are the foremost. It is especially difficult to predict emergency situations caused by extreme waves for those cases of the sea-based activities which require people's long stay at sea or prolonged use of the equipment in the ocean.

One of the methods to minimize such possible risks is use of climatic data based on long-term series of observations. At present there are archives consisting of the reanalysis data on surface waves based on wave forecast corrected by different methods, i.e., direct measurements using accelerometers and GPS-buoys; remote measurements by satellite-borne altimetry and various types of radars. The main characteristic of wave field included in the archive is significant wave height H_s defined as a mean value (trough to crest) of one third of the highest of all the waves (Ochi 2005). The value of H_s is calculated in the following way:

$$H_s = 4\left(\int_0^\infty \int_0^\infty S(k_x, k_y) dk_x dk_y\right)^{1/2}, \quad (1)$$

where k_x, k_y are wave numbers, while $S(k_x,k_y)$ is wave spectrum.

It is evident that significant wave height does not provide any information on real wave height for a given wave field. Extreme waves of the same height can appear with different probability for different values of H_s. For example, a wave 10 meters high can appear both in a wave field with H_s=10 m and in a wave field with H_s=5 м. Thus, H_s data are not enough to evaluate the probability of real wave heights.

Up to the present some vagueness of definition of extreme ('freak') wave has existed. The standard definition suggests that any wave exceeding two significant wave heights H_s can be considered as "freak" wave (Chalikov 2016). Firstly, such definition is not universal, since it does not define real dimensional wave height. This definition makes sense for theoretical investigations only, as the adiabatic equations of wave dynamics are self-similar, i.e., when transformed to the nondimensional form, they do not contain any nondimensional parameters, which makes the whole approach quite convenient due to its universal character. The non-adiabatic factors (for example, input energy) violate self-similarity; anyway, since the time scale of wave energy transformation is by many orders of magnitude larger than the wave periods, a nondimensional approach in theoretical investigations is still acceptable. However, practical conclusions of the theoretical investigation should be formulated in a dimensional form.

Secondly, the value $2H_s$ is large, which means that the definition refers to the full height of wave, i.e., trough-to-crest height. This definition is straightforward for linear monochromatic waves only when the trough-to crest height is simply equal to the doubled wave height above mean level. For a complicated multi-mode wave field the trough-to crest height should be defined as a vertical distance between crest and the closest wave trough. The question is what the term 'closest' means. The only way recommended in (Chalikov, 2009) (see also (Chalikov, 2016)) is based on the consideration of moving window with horizontal size L_w. Since extreme wave actually occurs at peak frequency, it is reasonable to assume $L_w=1.5L_p$ (where L_p is the wavelength in peak of spectrum). Coefficient 1.5 is introduced to take into account the variability of length of 'wave' (which in fact is a transient local superposition of several modes).

The nature of freak waves was investigated analytically (Onorato et al., 2009) and numerically (Chalikov, 2009). Recently it was found that the statistical properties of trough-to-crest wave height are quite different from those of the wave height above mean level. Paper (Chalikov and Babanin, 2016; Chalikov, 2017) shows that linear and nonlinear statistics of extreme waves (defined as trough-to-crest waves) are identical not only for broad spectrum but for one-dimensional wave field too. It means that generation of a trough-to-crest extreme wave is the result of simple superposition of linear modes, no matter how broad the spectrum is. This property is not found for the wave height above mean level. Thus, the statistical properties of trough-to-crest wave height can be investigated with no nonlinear modeling, but just by generation of large ensembles of superposition of linear modes with random phases and the spectrum prescribed. It is obvious that the problem of the trough-to-crest statistics becomes quite straightforward. Contrary to such approach, investigation of the statistics of wave height above mean level remains a subject of nonlinear wave theory.

The theoretical probability distribution for wave height was suggested by Weibull (1951). Later it was studied on a basis of observational data in nature and wave channels (see review by Kharif et al., 2009). Extended data for estimation of probability of wave height can be obtained with integration of nonlinear modes based on full potential equations (Touboul and Kharif, 2010; Chalikov et al, 2009). Methods of probability calculations were considered in many papers (see, for example Bitner-Gregersen and Toffoli, 2012; Mori and Janssen, 2005; Dyachenko at all, 2016). The most popular method of trough-to-crest wave height detection is based on zero-crossing technique. Direct method is based on use of moving windows, which is applicable both for 1-D and 2-D cases.

This paper is devoted to investigation of the statistics and geographical distribution of wave height above mean sea level.

2. Description of the method

In paper (Chalikov and Bulgakov, 2017) an algorithm for estimation of cumulative probability of waves exceeding a specific value of wave height above mean level (*P(h)* and *h* below) was developed using long-term data on H_s.

The algorithm was based on results of 3-D model of potential waves. The model used spectral definitions of fields, finite differences for vertical derivatives calculation, fourth-order Runge–Kutta scheme for time integration. Fourier resolution is 256X64 wave number, resolution in physical space is 1024*256 (more detail in (Chalikov et al., 2014)). The calculations were done for 350 units of nondimensional time, i.e., for 70,000 time steps. The initial conditions were generated on basic JONSWAP spectrum. Totally 50 experiments were made (more detail in (Chalikov and Bulgakov, 2017)). The results of the series of experiments were processed in the following way: Each wave field of surface height above mean level (η) reproduced by numerical model was normalized by the value of significant wave height corresponding to this field. ($\widetilde{H} = \eta/H_s$). Then, a nondimensional wave field was used for calculation of cumulative probability of nondimensional wave height $\tilde{P}(\widetilde{H})$. The distribution obtained was approximated by the following function:

$$\tilde{P}(\widetilde{H}) = \exp\,(-3.97\widetilde{H} - 4.02\widetilde{H}^2) \tag{2}$$

Note, that $\tilde{P}(\widetilde{H})$ is cumulative probability of the height of free surface above mean level. This probability for $\widetilde{H} = 1$ (the height of free surface equals significant wave height) is quite small (0.0003).

Also note, that η is variable of 3-D model of potential waves. It should be distinguished from h despites the fact that both (η and h) have the same physical sense.

The above expression can be used for the interval $0 \leq \widetilde{H} \leq 1.85$. The probability of wave higher than 1.85 (it's maximal value of $\widetilde{H}$ in data) can be considered as extremely low and therefore - neglected. It should be noted that approximation (2) was obtained with use of the precise 3-D model based on full nonlinear equations. The volume of data used for approximation (2) includes more than 4.5 billion values of η (number point in single field multiplied by number of record in experiment multiplied by number of experiments). Currently, this approximation is considered as universal for wind wave fields where cases of freak waves are most likely. Waves of other types of spectrum (swells) have a small steepness and don't influence on extreme wave generation.

The probability of wave exceeding specific height h, if significant wave height is in a small range dH_s around H_s, equals $\tilde{P}(\widetilde{H})$ for specific h/H_s multiplied by probability of H_s in this range ($\tilde{P}(\widetilde{H}) \cdot P(H_s)$), by the standard definition of conditional probability. Consequently, $P(h)$ can be determined as integral of $\tilde{P}(\widetilde{H}) \cdot P(H_s)$: over all possible value of H_s:

$$P(h) = \int_0^{H_{smax}} \tilde{P}\left(\frac{h}{H_s}\right) P(H_s) dH_s, \tag{3}$$

where $P(H_s)$ is probability distribution of H_s for a specific point, while H_{smax} is the maximum value of H_s in the dataset for a specific point. .
The data (Chawla et al., 2013) used were calculated with the latest version of WAVEWATCH III model (Tolman 2014) and GFS-2 wind analysis 2 (Sasha et al. 2014). The hindcasts cover the period from August 1999 to July 2015. The spatial resolution of the dataset fields is 0.5 × 0.5 degree. Calibration of the model and its validation are carried out using a great number of wave buoys.

The examples of the calculations using the method (2-3) are given in (Chalikov and Bulgakov, 2017) where space distribution of the extreme wave probability was investigated. The method 2-3 can be also used for estimation of height of extreme waves of any given cumulative probability.

3. Results

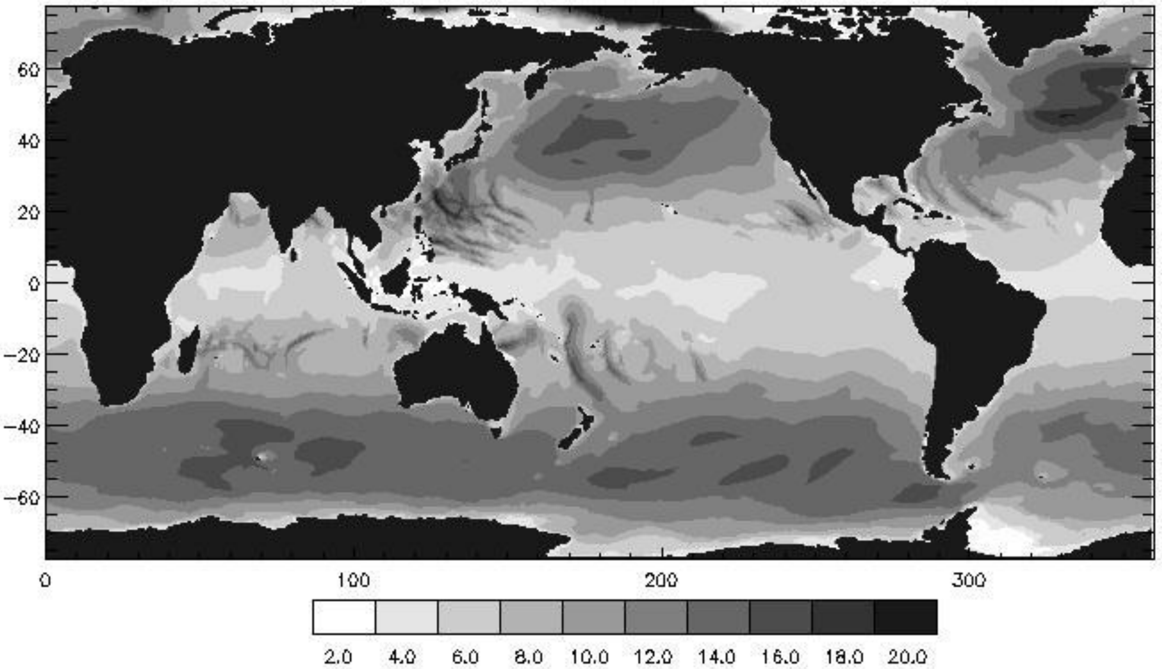

Fig.1 — Wave heights (m) with cumulative probability 10^{-7}, annual average

Figure 1 shows an average annual field of wave heights with cumulative probability 10^{-7}. It can be seen that waves with the height up to 20 m can appear with such probability, some of the extreme waves (16 m and more) being found in the areas of active navigation (eastern part of the Atlantic Ocean, East China Sea, Philippine Sea, Yellow Sea, south-western part of the Pacific Ocean).

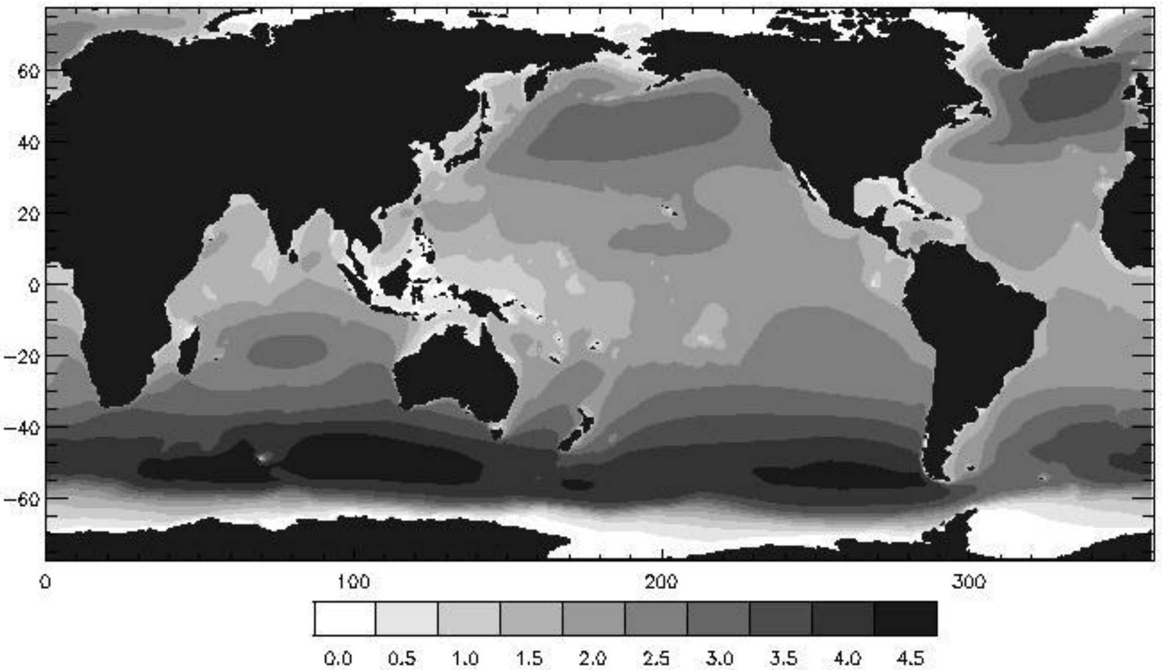

Figure 2 – Average annual significant wave height (m).

Distribution of mean annual significant wave height calculated by data (Chawla et al., 2013) is shown in Fig. 2. As seen, the maximum value of significant wave height does not exceed 5 m, while the height of real extreme wave can reach 20 m. The data in Fig 1 have a more complicated structure, due, for example, to the periods with strong wind along trajectories of tropical storms. Consequently, the calculations of distribution of real wave height should be done for shorter periods, i.e., for seasonal or monthly averaged data on significant wave heights.

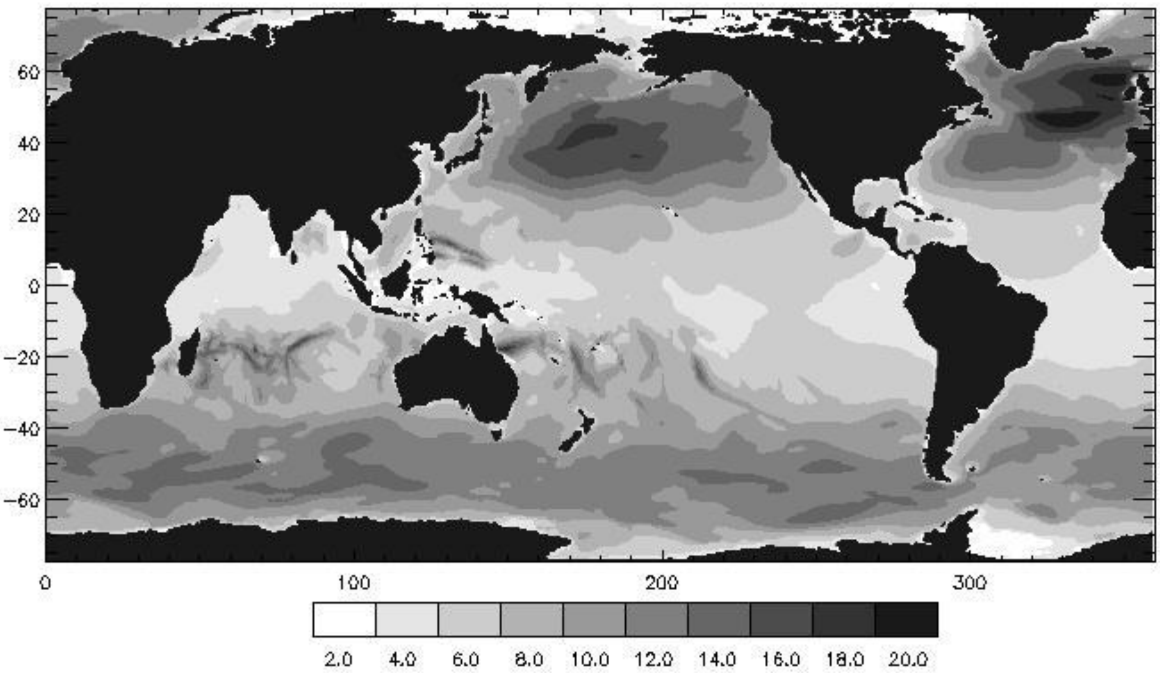

Figure 3. – Wave height (m) with cumulative probability 10^{-7} for December-February.

In Fig. 3 the field of wave height with probability 10^{-7} calculated by the data (Chawla et al., 2013)], averaged for December-February, is shown. When comparing Figs 3 and 1 it is seen that in mid-latitudes of the Northern Hemisphere wave heights become higher. In some areas appearance of extreme wave heights exceeding 16 m is possible. At the same time there are actually no extreme waves in the eastern part of the Arctic Ocean, which is connected with seasonal ice formation in the area. In equatorial and tropical areas of the World Ocean wave heights are less in winter (Northern Hemisphere), as compared with the average annual wave heights. It should be noted that in the western part of the Atlantic Ocean tracking trajectories of hurricanes disappeared while the number of such trajectories increased in the Indian Ocean.

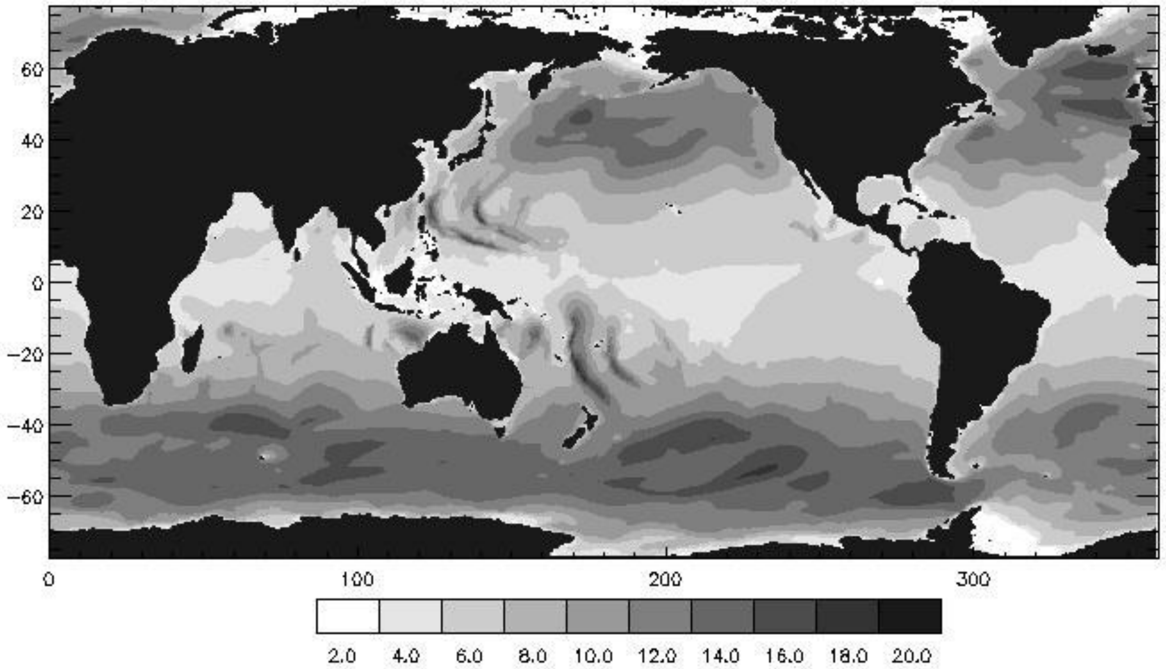

Figure 4. – Wave height (m) with cumulative probability 10^{-7} averaged for March-May.

An increase of wave heights over March-May can be registered (Fig.4) in the Southern Hemisphere. Actually all the area of mid-latitudes from the latitude of 40 degrees S. to the latitude of 60 degrees S. is characterized by probability $>10^{-7}$ of wave heights over 14 m. In mid-latitudes of the Northern Hemisphere in spring wave height values are more than the average annual values, though less than the winter values, while in some areas (Atlantic Ocean near Iceland, Pacific Ocean near Bering Sea) appearance of waves exceeding 14 m in height is quite possible.

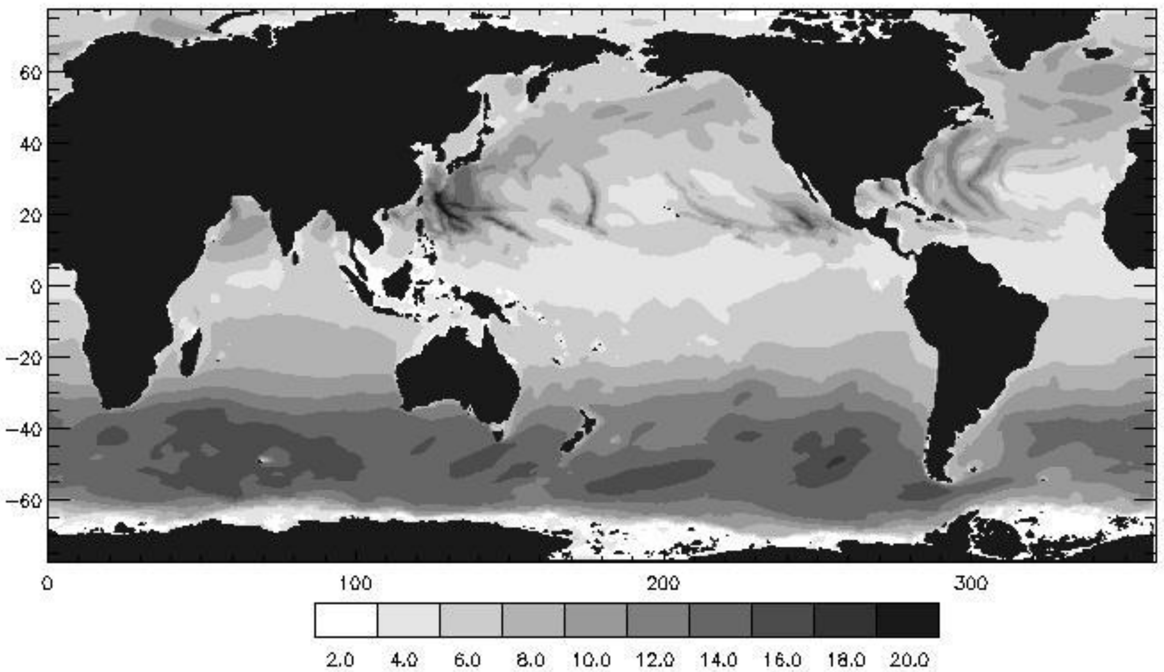

Fig.5 –Wave heights (m) with the cumulative probability of 10^{-7}, for June-August

Summer months (Fig.5) are characterized by general decrease of extreme wave probability. It is especially noticeable in the northern areas of the Atlantic and Pacific Oceans.

Also, wave heights slightly decreased in the Southern Hemisphere. It should be noted that storm tracks appear off the eastern coast of North America and disappear in the southern part of the Pacific Ocean. Besides, quite distinct trajectories of storms appeared in the eastern part of the Pacific Ocean. Small wave heights can be observed in the Arctic Ocean, in the area free from ice.

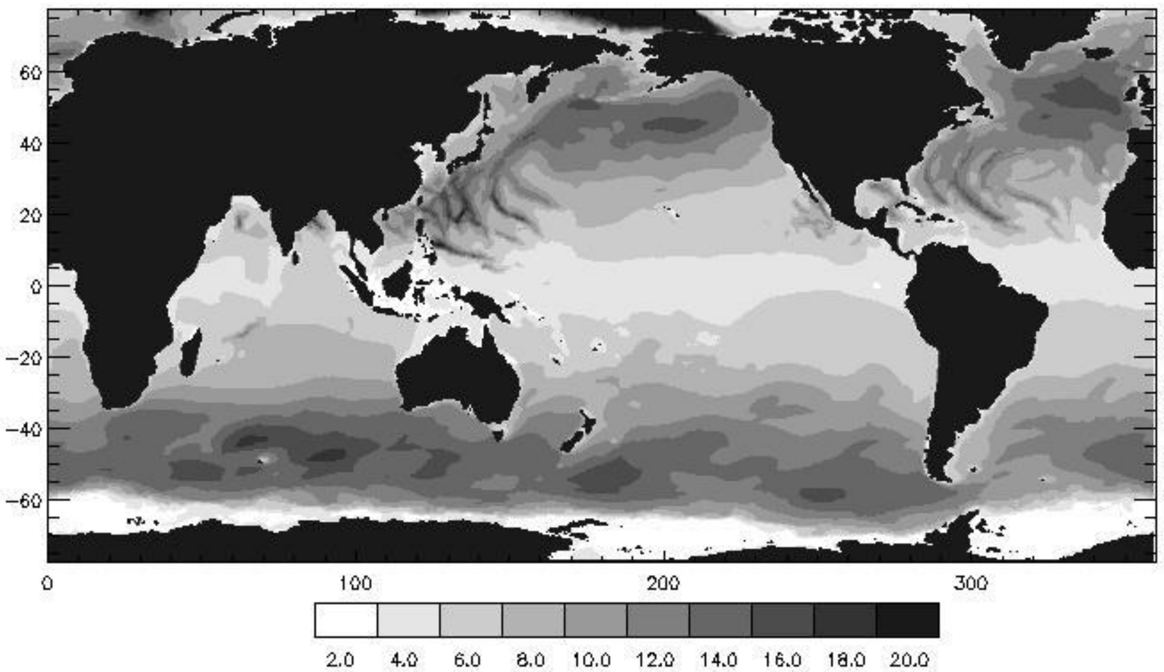

Figure 6. – Wave heights (m) with cumulative probability 10^{-7}, averaged for September - November

During autumn months (Fig. 6) an increase of wave heights is observed in the Arctic Ocean, the extreme wave height values sometimes reaching 20 m. Among other peculiarities is an increase of wave-free area in polar latitudes of the Southern Hemisphere, which is obviously connected with seasonal ice formation.

It is quite evident that the average monthly fields of cumulative wave-height probability will allow us to obtain more exact information on the areas of extreme wave probability.

The above approach can be used with different values of cumulative probability. It is not expedient to use the values less than 10^{-9} as this value is outside the range of validity for equation (2) ($\tilde{P}(1.85)$ is approximately 10^{-9}). Mapping of wave heights with the cumulative probability exceeding 10^{-7} may have a certain practical importance (ensuring of safety cargo shipments for instance). Hence, on the whole, the method considered is suitable for estimation of extreme values of wave heights with minor probability.

4. Conclusions

The paper describes a method of calculation of extreme wave probability, based on long-term wave hindcast data on significant wave height. Such method can be used for estimation of probability of extreme waves, which is important for designing of engineering constructions, as well as for other purposes. The maps of global distribution of wave heights of certain probability for main seasons illustrate the method.

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