



1 **Forecasting experiments of a dynamical-statistical model**
2 **of the sea surface temperature anomaly field based on the**
3 **improved self-memorization principle**

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26 **Abstract:** With the objective of tackling the problem of inaccurate long-term El Niño
27 Southern Oscillation (ENSO) forecasts, this paper develops a new
28 dynamical-statistical forecast model of sea surface temperature anomaly (SSTA) field.
29 To avoid single initial prediction values, a self-memorization principle is introduced
30 to improve the dynamic reconstruction model, thus making the model more
31 appropriate for describing such chaotic systems as ENSO events. The improved
32 dynamical-statistical model of the SSTA field is used to predict SSTA in the
33 equatorial eastern Pacific and during El Niño and La Niña events. The long-term
34 step-by-step forecast results and cross-validated retroactive hindcast results of time
35 series T_1 and T_2 are found to be satisfactory, with a correlation coefficient of
36 approximately 0.80 and a mean absolute percentage error of less than 15%. The
37 corresponding forecast SSTA field is accurate in that not only is the forecast shape
38 similar to the actual field, but the contour lines are essentially the same. This model
39 can also be used to forecast the ENSO index. The correlation coefficient is 0.8062,
40 and the MAPE value of 19.55% is small. The difference between forecast results in
41 summer and those in winter is not high, indicating that the improved model can
42 overcome the spring predictability barrier to some extent. Compared with six mature
43 models published previously, the present model has an advantage in prediction
44 precision and length, and is a novel exploration of the ENSO forecast method.

45

46 **Keywords:** Dynamical-statistical forecast model; self-memorization principle; sea
47 surface temperature field; long-term forecast of ENSO



48 **1. Introduction**

49 The El Niño Southern Oscillation (ENSO), the well-known coupled atmosphere
50 –ocean phenomenon, was firstly proposed by Bjerknes (1969). The ENSO
51 phenomenon can influences regional and global climates, so the prediction of ENSO
52 has received considerable public interest (Rasmusson and Carpenter, 1982; Glantz et
53 al., 1991).

54 Over the past two to three decades, one might reasonably expect the ability to
55 predict warm and cold episodes of ENSO at short and intermediate lead times to have
56 gradually improved (Barnston et al., 2012). Many countries have been focusing on
57 ENSO forecasts since the 1990s, and the ENSO forecast has become one of the
58 important research topics in the International Climate Change and Predictability
59 Research plan. The U.S. International Research Institute for Climate and Society, the
60 U.S. Climate Prediction Centre, Japan Meteorological Agency, and European Centre
61 for Medium-Range Weather Forecasting have developed different coupled
62 atmosphere–ocean models to forecast ENSO (Saha et al., 2006; Molteni et al., 2007) .

63 The forecast models can generally be divided into two types (Palmer et al., 2004).
64 The first type is typified by a dynamic model, which mathematically expresses
65 physical laws that govern how the ocean and the atmosphere interact. The second type
66 is typified by a statistical model, which requires large a amount of historical data and
67 analyses the data to do forecasting (Chen et al., 1995; Moore et al., 2006).

68 Over the past three decades, ENSO predictions have made remarkable progress,
69 reaching a stage where reasonable statistical and numerical forecasts (Jin et al.,



70 2008) can be made 6–12 months in advance (Wang et al., 2009). . However, there are
71 three problems remaining to be resolved (Zhang et al., 2003a): (1) The current ENSO
72 predictions are mainly limited to the short term, such as annual and seasonal
73 predictions; (2) Although the representation of ENSO in coupled models has
74 advanced considerably during the last decade, several aspects of the simulated
75 climatology and ENSO are not well reproduced by the current generation of coupled
76 models. The systematic errors in SST are often very large in the equatorial Pacific,
77 and model representations of ENSO variability are often weak and/or incorrectly
78 located (Neelin et al. 1992; Mechoso et al. 1995; Delecluse et al. 1998; Davey et al.
79 2002). (3) Coupled models of ENSO predictions initialized from observed initial
80 states tend to adjust towards their own climatological mean and variability, leading to
81 forecast errors. The errors associated with such adjustments tend to be more
82 pronounced during boreal spring, which is often called the “spring predictability
83 barrier” (Webster et al., 1999). More efficient models are therefore desired (Belkin
84 and Niyogi, 2003; Weinberger and Saul, 2006). Therefore, the idea of combining
85 dynamical and statistical methods to improve weather and climate prediction has been
86 developed in many studies (Chou, 1974; Huang et al., 1993; Yu et al., 2014a; Yu et
87 al., 2014b). By introducing genetic algorithms (GAs), Zhang et al. (2006) inverted and
88 reconstructed a new dynamical-statistical forecast model of the tropical Pacific sea
89 surface temperature (SST) field using historic statistical data (Zhang et al., 2008).
90 However, there is one flaw in the forecast model: the time-delayed SST field. This is
91 because ENSO is a complicated system with many influencing factors. To overcome



92 information insufficiency in the forecast model, Hong et al. (2014) selected the
93 tropical Pacific SST, SSW and SLP fields as three modelling factors and utilized the
94 GA to optimize model parameters.

95 However, the above dynamical prediction equations which were proposed by
96 Hong et al.(2014), greatly depend on a single initial value, creating long-term
97 forecasts over 8 months that diverged significantly. These unsatisfactory results
98 indicate that this model needs to be improved. Cao (1993) first proposed the
99 self-memorization principle, which transforms the dynamical equations with the
100 self-memorization equations, wherein the observation data can determine the memory
101 coefficients. This method has been widely used in forecast problems in environmental,
102 hydrological and meteorological fields (Feng et al., 2001; Gu, 1998; Chen et al.,
103 2009). The method can avoid the question of initial conditions for the differential
104 equations, so it can be introduced here to improve the proposed dynamical forecast
105 model.

106 Therefore, an improved dynamical-statistical forecast model of the SST field
107 and its impact factors with a self-memorization function was developed. The
108 improved model can absorb the information from past observations.

109 This paper is organized as follows: Research data and forecast factors are
110 introduced in section 2. In Section 3 the reconstruction of the dynamical model of
111 SSTA field is described. To improve the reconstruction model, the self-memorization
112 principle is introduced in Section 4. Model forecast experiments are described in
113 Section 5, and conclusions are given in Section 6.



114 **2. Research data and forecast factors**

115 **2.1 Data**

116 The monthly average SST data from January 1951 to January 2010, 720 months
117 in total, were obtained from the UK Met Office Hadley Centre for the region
118 (30 °S-30 °N; 120 °E -90 °W). The sea areas provide important information on
119 ocean-atmosphere coupling in the East and West Pacific Ocean and the El Niño and
120 La Niña events. The reanalysis data and zonal winds were obtained from the National
121 Center for Environmental Forecast (NECP) of America and the National Center for
122 Atmospheric Research (NCAR) (Kalnay et al., 1996). The SOI data were obtained
123 from the Climate Prediction Center (CPC). The time series of all data were from Jan.
124 1951 to Jan. 2010.

125 **2.2 EOF deconstruction**

126 The sea surface temperature anomaly (SSTA) field can be calculated from the
127 SST field and can be deconstructed into time (coefficients)-space (structure) using the
128 EOF method. Detailed information on the EOF method can be seen in the related
129 references (Dommenget & Latif, 2002).

130 An empirical orthogonal function (EOF) analysis of smoothed anomalies was
131 performed, and the first two SSTA EOFs are shown in Figs. 1a and 1c. The principal
132 component (PC) time series corresponding to the first and second EOFs are shown in
133 Figs. 1b and 1d. The first EOF pattern, which accounted for 61.33% of the total SSTA
134 variance, represented the mature ENSO phase (El Niño or La Niña), and the
135 corresponding PC time series was highly correlated (with a correlation coefficient of



136 0.85) with the cold tongue index (SST anomaly averaged over 4 °S–4 °N, 180 °–90 °
137 W) over the whole period. The second EOF, accounting for 14.52% of the total
138 SSTA variance, indicated the ENSO signal beginning to decay. Compared with the
139 first mode, these were slightly attenuated in terms of the scope and intensity. The
140 above analysis is similar to the EOF analysis of the SSTA field in the previous studies
141 (Johnson et al., 2000; Timmermann et al., 2001). This indicates that the front two
142 variance contribution modes can describe the main characteristics of the SSTA field
143 and El Niño/La Niña. Therefore, we can choose the T_1, T_2 time series EOF
144 decomposition modes as the modelling objects.

145 **2.3 Selection of other prediction model factors**

146 The ENSO intensity impact factor is an important issue in the ENSO
147 prediction. Previous studies have found that teleconnection patterns, temperature,
148 precipitation, wind and SSH may affect the ENSO strength (Trenberth et al., 1998;
149 Webster, 1999; Ashok et al., 2001; Yoon and Yeh, 2010; Tomita and Yasunari, 1996).
150 For example, Trenberth et al. (1998) noted that the Pacific North American Oscillation
151 Index (PNA) and SOI in the Pacific Intertropical Convergence Zone (ITCZ) were all
152 closely related to ENSO. Liao et al. (2007) also noted that the decadal variation
153 during ENSO events had a close relationship with the SOI index. The vast majority of
154 studies (Tomita and Yasunari, 1996; Zhou and Wu, 2010) have concentrated on the
155 impacts of ENSO on the East Asian winter monsoon (EAWM). During the EAWM
156 season, ENSO generally reaches its mature phase and has the most prominent impact
157 on the climate. Wang et al. (1999a) and Wang et al. (1999b) suggested that the zonal



158 wind factors in the eastern and western equatorial Pacific played a critical role in the
159 transition phase of the ENSO cycle, which could excite eastward propagating Kelvin
160 waves and affect the SSTA in the equatorial Pacific.

161 Based on the above analysis, we selected four factors, which may be closely
162 related with the ENSO index (Niño 3.4) and were obtained as follows:

163 (1) The zonal wind in the eastern equatorial Pacific factor (u_1) was calculated
164 as the grid-point average of zonal wind in the area [$5^\circ\text{S} \sim 5^\circ\text{N}$, $150^\circ\text{W} \sim 90^\circ\text{W}$].

165 (2) The PNA teleconnection factor was obtained from the CPC.

166 (3) The SOI factor was obtained from the CPC.

167 (4) The EAWM index (EAWMI) factor was proposed by Yang et al. (2002),
168 which is defined by the meridional 850-hPa winds averaged over the region (20°
169 $\sim 40^\circ\text{N}$, $100^\circ \sim 140^\circ\text{E}$).

170 All the four data selected ranged from January 1951 to January 2010.

171 Actually, how many variables and which variables are used in our model
172 become a key issue to be resolved. We can introduce a stepwise regression principle
173 to choose more reasonable predictors (Yim et al., 2015), because the stepwise
174 procedure can help selecting statistically important predictors at each step. The
175 significance of each predictor selected was based on its significance in increasing the
176 regressed variance by the standard F test (Panofsky and Brier, 1968). A 95 %
177 statistical significance level was used as a criterion to select a new predictor at each
178 step. Once selected into the model, a predictor can only be removed if its significance
179 level falls below 95 % by the addition/removal of another variable. For example, for



180 the model of only one variable, because we forecast the ENSO index, we should
181 choose T_1 or T_2 as the variable. Considering that T_1 accounts for 61.33% of the total
182 SSTA variance, so we chose T_1 as the variable. For the model of two variables, there
183 are five factors (T_2, u_1 , PNA, SOI and EAWMI) which can be chosen for the second
184 variable. Taking advantage of the stepwise regression ideas and selecting statistically
185 important predictors by a standard F test, we can find the largest F test value among
186 the five factors. That is T_2 . Continuing this step, we can also select the reasonable
187 factors for the model of three variables. Based on this thought, when the number of
188 variables is determined, we can choose the most statistically important variables to
189 reconstruct the prediction model. The forecast results of these models can be seen in
190 table 1.

191 From table 1, the forecast results of all six models are satisfactory, where the
192 temporal correlations of the models are all greater than 0.60 and the root mean square
193 errors are all less than 0.81. Among all six models, the forecast results of four
194 variables are the best for the following reasons:

195 (1) In general, the amount of parameters is less than 10% of the sample size,
196 which can avoid over-fitting (Tetko et al., 1995). The number of parameters
197 $a_1, a_2, \dots, a_{14}, b_1, b_2, \dots, b_{14}, c_1, c_2, \dots, c_{14}, d_1, d_2, \dots, d_{14}$ of the model of four variables $T_1, T_2, SOI, EAWMI$ is 56,
198 but we deleted the parameters which contributed little to the prediction. That means
199 that there are 56 parameters in equation (1) in section 3, but there are only 34
200 parameters in equation (3) in section 3 which is our final prediction equation. In
201 section 5.1, because p is identified as 6, the number of parameters of the



202 self-memorization function β_i is 28. Therefore, the total number of parameters in the
203 model of four variables is 62, which is less than 10% of the sample size (720 months).
204 The number of parameters $a_1, a_2, \dots, a_{20}, b_1, b_2, \dots, b_{20}, c_1, c_2, \dots, c_{20}, d_1, d_2, \dots, d_{20}, e_1, e_2, \dots, e_{20}$ of the model
205 of five variables $T_1, T_2, SOI, EAWMI, u_1$ is 100. Although the parameters which contributed a
206 little were deleted, the number was still 72, and the number of self-memorization
207 parameters was 30 (p determined as 5). Thus, the total number of parameters in the
208 model of five variables was 102, which was more than 10% of the sample size (720
209 months). This will cause an overfitting problem. Hence, when we selected the model
210 of five or six variables which entailed large amounts of computation that made
211 precision difficult, and too many parameters caused an overfitting phenomenon. That
212 is why the forecast results of five or six variables were worse than those of four
213 variables.

214 (2) The models of one, two and three variables can avoid the overfitting problem,
215 but too few variables will result in too few reconstruction parameters, causing
216 important information missing from the model. Especially, when the model of one or
217 two variables was considered, we only studied the self-memorization of the ENSO
218 system but did not consider the mutual-memorization between factors. Thus,
219 equations of our model only contained a self-memory term, not an exogenous effect
220 term. That is why the forecast results of one, two and three variables were worse than
221 those of four variables.

222 Based on the above analysis, we finally chose T_1, T_2, SOI and EAWMI as
223 predictors for the model.



224 **3. Reconstruction of dynamical model based on GA**

225 Takens' delay embedding theorem (Takens, 1981) provides the conditions under
226 which a smooth attractor can be constructed from observations made with a generic
227 function. Later results replaced the smooth attractor with a set of arbitrary
228 box-counting dimensions and the class of generic functions with other classes of
229 functions. Takens had shown that if we measured any single variable with sufficient
230 accuracy for a long period of time, it would be possible to construct the underlying
231 dynamical structure of the entire system from the behavior of that single variable
232 using delay coordinates and the embedding procedure. It was therefore possible to
233 construct a dynamical model of system evolution from the observed time series.
234 Introducing this idea here, four time series of the T_1 , T_2 , SOI and EAWMI factors
235 were chosen to construct the dynamical model.

236 The basic idea of statistical-dynamical model construction is discussed in
237 Appendix A and was introduced in our previous work (Zhang et al., 2006; Hong et al.,
238 2014).

239 A simplified second-order nonlinear dynamical model can be used to depict the
240 basic characteristics of atmosphere and ocean interactions (Fraedrich, 1987). Suppose
241 that the following nonlinear second-order ordinary differential equations are taken as
242 the dynamical model of reconstruction. In the equations, x_1, x_2, x_3, x_4 were used to
243 represent the time coefficient series of T_1 , T_2 , SOI and EAWMI.



$$\begin{aligned}
 \frac{dx_1}{dt} &= a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_1^2 + a_6x_2^2 + a_7x_3^2 + a_8x_4^2 + a_9x_1x_2 + a_{10}x_1x_3 + a_{11}x_1x_4 + a_{12}x_2x_3 + a_{13}x_2x_4 + a_{14}x_3x_4 \\
 \frac{dx_2}{dt} &= b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_1^2 + b_6x_2^2 + b_7x_3^2 + b_8x_4^2 + b_9x_1x_2 + b_{10}x_1x_3 + b_{11}x_1x_4 + b_{12}x_2x_3 + b_{13}x_2x_4 + b_{14}x_3x_4 \\
 \frac{dx_3}{dt} &= c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_1^2 + c_6x_2^2 + c_7x_3^2 + c_8x_4^2 + c_9x_1x_2 + c_{10}x_1x_3 + c_{11}x_1x_4 + c_{12}x_2x_3 + c_{13}x_2x_4 + c_{14}x_3x_4 \\
 \frac{dx_4}{dt} &= d_1x_1 + d_2x_2 + d_3x_3 + d_4x_4 + d_5x_1^2 + d_6x_2^2 + d_7x_3^2 + d_8x_4^2 + d_9x_1x_2 + d_{10}x_1x_3 + d_{11}x_1x_4 + d_{12}x_2x_3 + d_{13}x_2x_4 + d_{14}x_3x_4
 \end{aligned}
 \tag{1}$$

Based on the parameter optimization search method of GA in Appendix A, the time coefficient series of T_1 , T_2 , SOI and EAWMI from January 1951 to April 2008 are chosen as the expected data to optimize and retrieve model parameters. To avoid the overfitting problem, we used $x_{nor} = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$ to normalize the raw value of each of the four predictors, then we used the normalized value to model and forecast. Finally, we made forecast results revert back to the raw data magnitude by $x = x_{nor}(x_{\max} - x_{\min}) + x_{\min}$.

After eliminating weak items with small dimension coefficients, the nonlinear dynamical model of the first time series T_1 , the second time series T_2 , SOI and EAWMI can be reconstructed as follows:

$$\begin{aligned}
 \frac{dx_1}{dt} &= F_1 = -0.3328x_1 + 1.2574x_2 - 0.3511x_3 - 0.0289x_1^2 + 3.1280x_3^2 + 0.0125x_1x_2 + 2.7805x_1x_3 - 1.5408x_2x_4 \\
 \frac{dx_2}{dt} &= F_2 = 1.0307x_1 - 3.1428x_2 + 0.3095x_4 + 4.2301x_1^2 - 1.2066x_2^2 + 2.5024x_4^2 - 0.2891x_1x_3 + 0.7815x_1x_4 - 0.4266x_3x_4 \\
 \frac{dx_3}{dt} &= F_3 = -2.3155x_1 + 3.2166x_3 + 1.5284x_4 - 1.4527x_2^2 - 0.0034x_3^2 - 4.1206x_4^2 - 0.0025x_1x_4 + 0.0277x_2x_3 + 1.2860x_2x_4 \\
 \frac{dx_4}{dt} &= F_4 = 0.4478x_2 - 0.0268x_4 + 0.8995x_1^2 - 2.3890x_3^2 + 0.2037x_4^2 + 1.3035x_1x_2 + 2.0458x_1x_4 - 2.0015x_2x_4
 \end{aligned}
 \tag{2}$$



259 The appropriate model coefficient estimates determine the robustness of the
260 model and the accuracy of forecast results. We should now judge whether the model
261 coefficients are appropriate or not.

262 First, the largest Lyapunov exponent (LLE) is one of the indexes that can
263 represent the characteristics of chaotic systems. The final Lyapunov exponents of Eq.
264 (2) were [0.0433, 0.0012, -0.1285], containing both a negative Lyapunov exponent
265 and two positive Lyapunov exponents, which demonstrate that our dynamic system is
266 indeed a chaotic system.

267 Second, we calculated the equilibrium roots of Eq. (2). Only the third
268 equilibrium was adjudged to be stable, based upon higher-order terms within the
269 Taylor series, the indices of which were mostly in accordance with the actual weather
270 system. The indices in the unstable equilibria could not accurately describe the actual
271 weather. Based on these two aspects, we can see that the model coefficient estimates
272 were reasonable and reflected the dynamical characteristics of the model.

273 The model required testing. Because the training period was from January 1951
274 to April 2008, we chose T_1 , T_2 , SOI and EAWMI of May 2008, which were not used
275 as initial forecast data in the modeling. Next, the Runge–Kutta method was used to do
276 the numerical integration of the above equations, and every step of the integration was
277 regarded as 1 month's worth of forecasting results. As a result, forecast results of four
278 time series over a period of 20 months were obtained. Here, the focus was on the
279 forecast results of T_1 and T_2 , as shown in Fig.2.

280 From Fig. 2, forecast performance of T_1 and T_2 within 5 months was better.



281 Using T_1 as an example, at this time, the temporal correlation between model
282 predictions and corresponding observations was 0.8966 and the mean absolute
283 percentage error (MAPE) (Hu et al., 2001), $MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{D_e(i) - D_0(i)}{D_0(i)} \right| \times 100$, was
284 8.32%. However, after 5 months, MAPE increased rapidly, and was 31.29% at 10
285 months. The model forecast then significantly diverged from observations, and the
286 forecast became inaccurate. After 10 months, the forecast results became increasingly
287 worse, which indicated that the forecast of the model after 5 months was unacceptable.
288 The forecast results of T_2 were similar to those of T_1 .

289 The model's skill should be further assessed by cross-validated retroactive
290 hindcasts of the time series. As in the above example, omitting a portion of the time
291 series (12 months, January 1951 to January 1952) from observations, we trained the
292 model based on the data from February 1952 to December 2010, and then predicted
293 the omitted segments (12 months, January 1951 to January 1952). We then repeated
294 this procedure by moving the omitted segment along the entirety of the available time
295 series. Each experiment have used the different training sample and have established
296 the different model equation (but the method is the same). Finally, we obtained
297 cross-validated retroactive hindcast results of T_1 and T_2 , as shown in Fig. 3. Figure
298 3 is combined results of the 60 forecast experiments.

299 As Fig. 2, the forecast performance of T_1 and T_2 in Fig. 3 was not satisfactory.
300 The model forecast significantly diverged from observations, and the forecast became
301 inaccurate. The temporal correlations of T_1 and T_2 between model predictions and
302 corresponding observations were 0.3411 and 0.4176, respectively. Additionally, the



303 mean absolute percentage errors (MAPE) of T_1 and T_2 were 65.42% and 57.56%,
304 respectively. This indicates that the forecast of the model in the long -term was
305 inaccurate and unacceptable.

306 The forecast result may be inaccurate when the integral forecasting time is long.
307 There will be a significant divergence which will cause an ineffective forecast. To
308 improve the forecast accuracy, the forecast not only depends on the integral equation
309 but also on a single initial value. Choosing the different initial value will cause
310 different forecast accuracy. For example, in a total of 60 cross-validated retroactive
311 hindcasts examples, the minimum MAPE was 37.65%, while the maximum MAPE
312 was 89.88%. A forecast, depending on a single initial value, will cause instability of
313 the forecast results. These two problems are addressed by introducing the
314 self-memorization principle in the next section.

315

316 **4. Introduction of self-memorization dynamics to improve the** 317 **reconstructed model**

318 In the above discussion, it was shown that the accuracy of the forecast results of
319 equation (2) were unsatisfactory. To improve long-term forecasting results, the
320 principle of self-memorization can be introduced into the mature model (Gu 1998;
321 Chen et al., 2009). The principle of self-memorization dynamics (Cao, 1993; Feng et
322 al., 2001) can be seen in Appendix B.

323 Based on Eq. (B10) in Appendix B, the improved model can be expressed as



324 follows:
$$\begin{cases} x_{1t} = \sum_{i=-p-1}^{-1} \alpha_{1i} y_{1i} + \sum_{i=-p}^0 \theta_{1i} F_1(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \\ x_{2t} = \sum_{i=-p-1}^{-1} \alpha_{2i} y_{2i} + \sum_{i=-p}^0 \theta_{2i} F_2(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \\ x_{3t} = \sum_{i=-p-1}^{-1} \alpha_{3i} y_{3i} + \sum_{i=-p}^0 \theta_{3i} F_3(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \\ x_{4t} = \sum_{i=-p-1}^{-1} \alpha_{4i} y_{4i} + \sum_{i=-p}^0 \theta_{4i} F_4(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \end{cases} \quad (3)$$

325 where y_i is replaced by the mean of two values at adjoining times; i.e.,
 326 $y_i \equiv \frac{1}{2}(x_{i+1} + x_i)$; F is the dynamic core of the self-memorization equation, which
 327 can be obtained from Eq. (2); and α and θ are the memory coefficients, the formula
 328 for which can be found in Appendix B.

329 If the values of α and θ can be obtained, Eq. (3) can be used to obtain the
 330 results of final prediction. The memory coefficients α and θ in Eq. (3) were
 331 calibrated using the least-squares method with the same data (January 1951 to April
 332 2008) as those used in Section 3. Eq. (3) can be deconstructed as follows (M is the
 333 length of the time series):

334
$$X = \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ \vdots \\ x_{1M} \end{bmatrix}, \alpha = \begin{bmatrix} \alpha_{-p-1} \\ \alpha_{-p} \\ \vdots \\ \vdots \\ \alpha_{-1} \end{bmatrix}, Y = \begin{bmatrix} y_{-p-1,1} & y_{-p,1} & \cdots & y_{-1,1} \\ y_{-p-1,2} & y_{-p,2} & \cdots & y_{-1,2} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ y_{-p-1,M} & y_{-p,M} & \cdots & y_{-1,M} \end{bmatrix}, \Theta = \begin{bmatrix} \theta_{-p} \\ \theta_{-p+1} \\ \vdots \\ \vdots \\ \theta_0 \end{bmatrix},$$

335
$$F = \begin{bmatrix} F_{-p,1} & F_{-p+1,1} & \cdots & F_{0,1} \\ F_{-p,2} & F_{-p+1,2} & \cdots & F_{0,2} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ F_{-p,M} & F_{-p+1,M} & \cdots & F_{0,M} \end{bmatrix}$$



336 The matrix equation is:

$$337 \quad X = Y\alpha + F\theta \quad (4)$$

$$338 \quad \text{where } Z = [Y:F], \quad W = \begin{bmatrix} \alpha \\ \vdots \\ \theta \end{bmatrix}.$$

339 Eq. (4) can be written as:

$$340 \quad X = ZW \quad (5)$$

341 The memory coefficients vector W can be calibrated using the least squares
342 method:

$$343 \quad W = (Z^T Z)^{-1} Z^T X \quad (6)$$

344 The memory coefficients a, θ can be obtained from Eq. (6). We then made a
345 prediction using the self- memorization equation (3), which used the p values before
346 t_0 .

347 The coefficients in F and W were used with the same training data from January
348 1951 to April 2008. In the forecast examples, we trained both the coefficients in F and
349 W at the same time, but in the paper we describe them separately to facilitate the
350 reader for better understanding.

351 **5. Model prediction experiments**

352 **5.1 Forecast of time series T_1 and T_2**

353 The training sample for the model was from January 1951 to April 2008. Here, from
354 Eq. (3), the forecast results using T_1, T_2 , SOI and EAWMI factors can be calculated, called
355 as step-by-step forecast.

356 When the retrospective order p is confirmed, step-by-step forecasts can be



357 carried out. For example, when the T_1, T_2 , SOI and EAWMI values of May 2008 were
358 forecast, y_i was obtained from the previous $p + 1$ time of T_1, T_2 , the SOI and the
359 EAWMI data, and $F_i(x_{1i}, x_{2i}, x_{3i}, x_{4i})$ was obtained from the previous p times of
360 T_1, T_2 , the SOI and the EAWMI data. All four equations were integrated simultaneously.
361 Taking these in Eq. (3), we can get the T_1, T_2 , SOI and EAWMI values of May 2008,
362 which these can be taken as the initial values for the next prediction step. Then, the
363 T_1, T_2 , SOI and EAWMI values from June 2008 and so on, can be generated.

364 5.1.1 Determination of p

365 Based on the self-memorization principle, the self-memorization of the system
366 determines the retrospective order p (Cao, 1993). If the system forgets slowly,
367 parameters a and θ will be small and the p value should be high. The SSTA field
368 forecasts were on a monthly scale, the change of which was slow in contrast to
369 large-scale atmospheric motion. So parameters a and θ were small, and generally,
370 the p value was in the range 5 to 15.

371 The retrospective order p was obtained by a trial calculation method. We selected
372 the p values in the range 4 to 16 to construct the model. The correlation coefficients
373 (CC) and MAPE of long-term fitting test (from February 1951 to December 2010) are
374 shown in Table 2, which can be used as the standard to determine the retrospective
375 order p .

376 Table 2 indicates that when $p = 6$, the MAPE values of long-term fitting test
377 were the smallest and the correlation coefficients were the largest. Also, when p from
378 5 to 9, CCs were all more than 0.58 and the forecast results were all good, which is



379 consistent with our interpretation of the physical mechanisms in section 6.2 below.
 380 SOI and EMWMI were 5-12 months lead relationships with SST (Xu et al., 1993;
 381 Chen et al, 2010; Wang et al., 2003). Using a cumulative period of SOI , EMWMI 5-8
 382 months ahead as initial values can help improve the final forecast results. Our results
 383 in table 2 are consistent with the actual physical ENSO process. Therefore, we
 384 selected the retrospective order as $p=6$.

385 Then, the prediction experiments can be carried out, based on improved
 386 self-memorization Eq. (3).

387 The improved self-memorization equation of T_1, T_2 , SOI and EAWMI can then be
 388 established. After the differential equation was discretely dealt with, the memory
 389 coefficients were solved by the least-squares method given in section 4 (Training
 390 period is January 1951 to April 2008). Finally, the improved prediction equation of
 391 T_1, T_2 , SOI and EAWMI, based on the self-memorization principle, can be expressed
 392 as:

$$\left\{ \begin{array}{l}
 x_{1t} = \sum_{i=-7}^{-1} \alpha_{1i} y_{1i} + \sum_{i=-6}^0 \theta_{1i} F_1(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \\
 x_{2t} = \sum_{i=-7}^{-1} \alpha_{2i} y_{2i} + \sum_{i=-6}^0 \theta_{2i} F_2(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \\
 x_{3t} = \sum_{i=-7}^{-1} \alpha_{3i} y_{3i} + \sum_{i=-6}^0 \theta_{3i} F_3(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \\
 x_{4t} = \sum_{i=-7}^{-1} \alpha_{4i} y_{4i} + \sum_{i=-6}^0 \theta_{4i} F_4(x_{1i}, x_{2i}, x_{3i}, x_{4i})
 \end{array} \right. \quad (7)$$

394 where



$$395 \quad \alpha = [\alpha_{ij}] = \begin{bmatrix} 0.0315 & -2.113 & 0.0284 & 2.1468 & 0.0688 & -0.7014 & 1.3248 \\ 0.4088 & -1.887 & -1.0233 & 1.5485 & 0.9028 & 1.0255 & -0.6443 \\ -0.9088 & -0.2557 & 0.9671 & -0.0054 & 1.0568 & 2.9764 & -0.5234 \\ 0.2088 & -1.0567 & 0.4891 & -0.5066 & -0.4890 & 1.4555 & 1.0966 \end{bmatrix}$$

$$(i = 0, 1, \dots, 4; j = -7, -6, \dots, -1)$$

$$396 \quad \theta = [\theta_{ij}] = \begin{bmatrix} 0.0485 & 0.0425 & -1.7688 & 0.8543 & 2.8901 & -0.1788 & -0.9066 \\ 0.07642 & 0.0941 & -1.2466 & -0.2288 & 0.1097 & 2.3221 & -1.4228 \\ -0.5288 & 1.2368 & -0.5568 & -0.0155 & 0.2886 & -0.1560 & 1.2775 \\ 1.5335 & -0.2887 & -0.5336 & -0.6072 & -0.5611 & 1.0225 & -1.0625 \end{bmatrix}$$

$$(i = 0, 1, \dots, 4; j = -6, -5, \dots, 0)$$

397 The step-by-step forecast was performed. The retrospective order $p = 6$ means
 398 that earlier seven observation data ($p + 1 = 7$) should be used during the forecasting
 399 process. The forecast results per month were saved for the next period predictions.

400 5.1.2 Long-term step-by-step forecasts of T_1 and T_2

401 To test the actual forecast performance of the above improved model, long-term
 402 step-by-step forecasts of T_1 and T_2 from May 2008 to December 2010 for 20 months
 403 were carried out, as shown in Fig. 4. The forecast results of T_1 and T_2 were good.
 404 Within 8 months, the correlation coefficients of T_1 and T_2 were 0.9163 and 0.9187.
 405 MAPEs of T_1 and T_2 were small, only 5.86% and 6.78%. The forecast time series
 406 from 8 months to 14 months gradually diverged, but the trend was acceptable. The
 407 correlation coefficients of T_1 and T_2 reached 0.8375 and 0.8251, and MAPEs of T_1
 408 and T_2 were 8.32% and 9.11%. After 14 months, forecast began to diverge and the
 409 error started to increase, but the correlation coefficients of T_1 and T_2 remained
 410 about 0.6899 and 0.6782, and MAPEs reached 18.31% and 19.44%, which can be
 411 acceptable.

412 5.2 Cross-validated retroactive hindcasts of time series T_1 and T_2



413 As in section 3, the model's skill should be further assessed by cross-validated
414 retroactive hindcasts of the time series. Because our step-by-step forecasts need the
415 earlier seven observation data ($p + 1 = 7$), we can obtain cross-validated retroactive
416 hindcast results of T_1 and T_2 from August 1951 to December 2010, as shown in Fig.
417 5.

418 From Fig. 5, the forecast performance of T_1 and T_2 was good. The
419 correlation coefficients of T_1 and T_2 were 0.7124 and 0.7036, respectively. The
420 MAPEs of T_1 and T_2 were small, only 19.57% and 19.79%, respectively. The peaks
421 and valleys of T_1 and T_2 were also forecasted accurately. The forecast results
422 indicated that the cross-validated retroactive hindcast results of T_1 and T_2 were close
423 to the observed values. Compared to Fig. 3, the improved model had better forecast
424 abilities than the original model.

425 Many researchers (Zhang et al., 2003b; Smith, 2004) have used Oceanic Niño
426 Index (ONI) which is used by the U.S. NOAA Climate Prediction Center to determine
427 the El Niño and La Niña years. It defined that the ONIs of five consecutive months in
428 winter were all more than 0.5 (less than -0.5) is the El Niño (La Niña) year. Based on
429 the above criterion, we can divide the total 60 years (1951-2010) into three categories.
430 It includes the 18 examples of El Niño year (such as 1958, 1964, 1966, etc.), 22
431 examples of La Niña year (such as 1951, 1955, 1956, etc.) and the remaining 20
432 experiments of the neutral year. Since the details in Fig.5 is not clear, we list the
433 forecast results of 60 experiments (including 18 El Niño examples, 22 La Niña
434 examples and 20 Neutral examples) in table 3.



435 From table 3, the average of CC of both T_1 and T_2 of 60 experiments within
436 6 months was more than 0.84 and MAPE was less than 8%. The average of CC within
437 12 months was more than 0.74 and MAPE was less than 12%. According to the
438 literature (Barranel et al., 1999), when MAPE was less than 15%, which means the
439 error was not great and the forecast results were good. Obviously, the forecast results
440 of El Niño / La Niña experiments were a little worse than those of neutral examples,
441 which means the forecast ability of our model for the abnormal situation was a little
442 worse than those for the normal situation. But even for El Niño / La Niña experiments,
443 the average of CC was still more than 0.7 and MAPE was less than 15%, which
444 means the error was not too large and was still within an acceptable range.

445 5.3 Forecast of the SSTA field

446 When we obtained the forecast results of the time coefficient series T_1 and T_2 ,
447 we submitted them into the following equation to reconstruct the forecast SSTA field:

$$448 \hat{x}_t = \sum_{n=1}^2 E_n \bullet T_{nt}, t = 1, 2, \dots, 12 \quad (8)$$

449 where E_n , T_{nt} are the EOF space fields and forecast time coefficients,
450 respectively, and \hat{x}_{ij} is the forecast SSTA field reconstructed by EOF.

451 After reconstruction of the space mode (treated as constant) and time coefficient
452 series (model prediction), the forecast of the SSTA fields was obtained, based on the
453 forecast results of T_1 and T_2 in Section 5.2. For economy of space, we cannot draw
454 all of the forecasted SSTA fields, so we selected a strong El Niño event (December
455 1997), a strong La Niña event (December 1999) and a neutral event (November 2002)
456 as examples.

457 Fig. 6 shows the forecast SSTA field during a strong El Niño event. From the



458 actual SSTA field in December 1997 (Fig. 6a), an obvious warm tongue structure
459 occurred in the area of [10 °S~5 °N, 90 °W~150 °W] in the Eastern Equatorial Pacific,
460 and a warm anomalous distribution arose in the west Pacific, which indicated a weak
461 El Niño event. The forecasted SSTA field of December 1997 is shown in Fig. 6b.
462 Although the range of warm tongue was a litter bigger than the actual situation, the
463 forecast shape was similar to the actual field and also the contour lines were similar.
464 The average MAPE between the forecast field and the actual field is 8.56%, which
465 was controlled within 10%. The forecast results of the improved model event were
466 quite good for the El Niño event.

467 Fig.7 shows the forecasted SSTA field of a strong La Niña event. From the actual
468 SSTA field in December 1999 (Fig. 7a), an obvious cold pool occurred in the area of
469 [10 °S~10 °N, 120 °W~180 °W] in the Equatorial Pacific, which covered the Niño3.4
470 area. This SSTA field presented a strong strength La Niña event. The forecast SSTA
471 field from December 1999 is shown as Fig. 7b. Although the strength of the cold pool
472 was weaker than the actual situation, the forecast shape was similar to that of the
473 actual field. The average MAPE between the forecast field and the actual field was
474 9.69%. The errors were larger than that of the El Niño event, but they can be
475 controlled within 10%, which is acceptable.

476 Fig. 8 shows the forecasted SSTA field of a neutral event. From the actual SSTA
477 field in November 2002 (Fig. 8a), a warm pool occurred in the area of [10 °S~10 °N,
478 120 °W~180 °W] in the Equatorial Pacific, which covered the Niño3.4 area. However,
479 the warm pool was small and weak, which represented a neutral event. The forecasted



480 SSTA field from November 2002 is shown in Fig. 8b. Comparing Figures 6, 7 and 8,
481 we can see that the forecasted SSTA field of a neutral event was a little worse than
482 that of the El Niño and La Niña events. The forecasted shape of the SSTA field
483 basically described the actual situation, but the warm pool in the Niño3.4 area was
484 stronger and bigger than that of the actual situation, which indicated a borderline El
485 Niño event. The average MAPE between the forecasted field and the actual field was
486 14.50%, which was big but can be accepted.

487 We obtained the average values of MAPE of 18 El Niño events, 22 La Niña
488 events and 20 neutral events, which were 9.52%, 9.88% and 14.67%, respectively,
489 representing a good SSTA field forecasting ability of our model.

490 **5.4 Forecast of ENSO index**

491 The ENSO index can be represented as the sea surface temperature anomaly
492 (SSTA) in the Niño-3.4 region (5°N - 5°S , 120° - 170°W) and the ENSO index
493 forecast was the 3-month forecast (Barnston et al. 2012). So we also can pick up the
494 ENSO index from the above forecasted SSTA field. The forecast results of the ENSO
495 index within 20 months can also be obtained. The definition of lead time can be seen
496 in the reference (Barnston et al. 2012). Therefore, similar to the forecast experiment in
497 section 5.1, a succession of running 3-month mean SST anomalies with respect to the
498 climatological means for the respective prediction periods, averaged over the Niño 3.4
499 region, can be obtained, as demonstrated in Fig. 9.

500 The forecast results within lead times of 18 months are shown in Fig. 9, which
501 demonstrate that the forecast results of the ENSO index are good. Within lead time of



502 12 months, the correlation coefficient was 0.8985 and the MAPE value was small,
503 only 8.91%. In addition, the borderline La Niña event in 2008–2009 was predicted
504 well. After lead times of 12 months, forecasts began to diverge and the errors started
505 to increase. Although the correlation coefficient remained approximately 0.61, MAPE
506 reached 18.58%. Therefore, a moderate strength El Niño event that occurred in
507 2009/10 was not predicted.

508 We should give more examples to test the ENSO prediction ability of our model.
509 As in section 5.3, we can divide 60 examples as three types, which are examples of
510 El Niño year, La Niña year and neutral year. Finally, we can obtain the forecast results
511 of different types of examples in different lead times, as shown in table 4.

512 From table 4, the average CC of 60 experiments was 0.712 and the average
513 MAPE was 7.62% within 12 months for all seasons of lead time, which indicates that
514 the overall ENSO forecast ability of our model was good. The forecast results of the
515 El Niño examples were significantly worse than those of La Niña examples, while the
516 forecast results of La Niña examples were significantly worse than those of neutral
517 examples, which show the model forecast ability of the abnormal state was worse than
518 the normal state of the ENSO index. Even for the forecast results of El Niño examples,
519 the average CC was still above 0.6 and the average MAPE can be controlled below
520 10%, which means the forecast results were still in the acceptable range. Our model
521 not only accurately predicted the stronger El Niño and La Niña phases but also the
522 neutral states. But the forecast results in summer were a little worse than those in
523 winter, as shown in Fig.10.



524 The ENSO forecast often had a spring predictability barrier (Webster, 1999),
525 which was most prominent during decades of relatively poor predictability
526 (Balmaseda et al., 1995). To test our model, the skill should be computed over the
527 entire time series and separately for seasonal subsets of the time series. The average
528 cumulative correlation coefficient and MAPE of winter were compared with those of
529 summer, as shown in Fig.10. The average cumulative correlation and average
530 cumulative MAPE values between the forecast values and the actual values changed
531 with time, from which good trends of forecast results can be seen. As long as the
532 forecast time increased, the cumulative MAPE increased and the correlation decayed
533 gradually. The forecast results appeared to diverge. Although the forecast results of
534 the present model in the summer were worse than in the winter, the margin was not
535 high, which means the model can overcome the “spring predictability barrier,” to
536 some extent.

537 **5.5 Compared with six mature models**

538 Barnston et al. (2012) compared many ENSO forecast models. Based on his
539 research, we selected four high quality dynamical models, including ECMWF, JMA,
540 the National Aeronautics and Space Administration Global Modelling and
541 Assimilation Office (NASAGMAO) and the National Centre for Environmental
542 Prediction Climate Forecast System (NCEP CFS; Version1). Two high quality
543 statistical models also be selected, including the University of California, Los Angeles
544 Theoretical Climate Dynamics (UCLA-TCD) multilevel regression model and the
545 NOAA/NCEP/CPC constructed Analogue (CA) model. The detail of the above



546 models can be seen in these references (Reynoldset al., 2002; Luo et al., 2005;
547 Barnston et al., 2012).

548 We then compared the forecast ability of the above six models with that of our
549 mode. All of the experiments of our model and six other models were conducted
550 under the same conditions using the same historical data for modelling and the same
551 initial values to forecast. In the CPC website, there are detailed explanations of six
552 models' training samples and the initial values. So we do not need to install all these
553 models on their own machines and run them for forecasting. We just made training
554 samples and initial values of our model were the same with those of selected six
555 models. At an 8-month lead time, the correlation ability of our model for all seasons
556 combined was 0.613 (Fig. 11). In brief, the forecast ability of the ECMWF model was
557 slightly better than that of our model but the ability of the other 5 models was worse
558 than that of our model. While, in regard to the forecast length, the temporal
559 correlation within 12 months of our model is greater than 0.6, which was superior to
560 the ECMWF model. In addition, the forecast results of the UCLA-TCD model and the
561 CPC CA model reduced quickly after 5-month lead times, so the forecast ability of
562 our model was more stable than them.

563 The root mean square error (RMSE) was also examined to assess the
564 performance of discrimination and calibration. Barnston et al. (2012) believed that all
565 seasonal RMSE values contributed equally to a seasonally combined RMSE. So we
566 drew figure 12 to show seasonally combined RMSE.

567 From Fig. 11 and Fig. 12, we can see the highest correlation tend to have



568 lower RMSE. So the RMSE of our model was slightly higher than that of ECMWF
569 model, but it was much lower than those of the other 5 models.

570 **6. Conclusions and discussion**

571 **6.1 Conclusions**

572 A new forecasting model of the SSTA field was proposed based on a dynamic
573 system reconstruction idea and the principle of self-memorization. The approach of
574 the present paper consisted of the following steps:

575 (1) The SST field can be time (coefficients)-space (structure) deconstructed
576 using the EOF method. Take T_1 , T_2 , SOI and EAWMI and consider them as
577 trajectories of a set of four coupled quadratic differential equations based on the
578 dynamic system reconstruction idea. The parameters of this dynamic model were
579 estimated using a GA.

580 (2) The forecast results of the dynamic model can be improved by the
581 self-memorization principle. The memory coefficients in the improved
582 self-memorization model were obtained using the GA method.

583 (3) The long-term step-by-step forecast results and cross-validated
584 retroactive hindcast results of time series T_1 and T_2 are all found to be good, with a
585 correlation coefficient of approximately 0.80 and a mean absolute percentage error of
586 less than 15%.

587 (4) The improved model was used to forecast the SSTA field. The
588 forecasted SSTA fields of three types of events are accurate. Not only is the forecast
589 shape similar to the actual field but also the contour lines are similar.



590 (5) The improved model was also used to forecast the ENSO index. The
591 average correlation coefficient of 60 examples within 12 months is 0.712, and the
592 MAPE value is small, only 7.62%, which proves that the improved model has better
593 forecasting results of the ENSO index. Although the forecast results of the model in
594 the summer were worse than in the winter, the margin was not high, which means that
595 the model can overcome the spring predictability barrier to some extent. Finally,
596 compared with the six mature models, the new dynamical-statistical forecasting
597 model has a scientific significance and practical value for the SST in the eastern
598 equatorial Pacific and El Niño/La Niña event predictions.

599 **6.2 Discussion**

600 Because the formula of our model includes a linear combination of 4 variables
601 (t_1, t_2 , SOI, EAWM), statistical forecasting requires independence between predictors.
602 We can calculate the correlation coefficients between variables, as shown in table 5.
603 In fact, as Table 5 shows, the correlation coefficients between the factors were all less
604 than 0.45, indicating the independence between factors. So this does not generate too
605 much redundancy and can avoid an overfitting problem, which can destroy the
606 stability of the model.

607 The introduction of self-memorization essentially introduces a lot of new
608 coefficients, which may cause an overfitting problem. Because we have selected a
609 model of four variables, there is a total of 62 parameters. In order to avoid the
610 overfitting problem, the sample sizes are more than 10% of the amount of parameters.
611 So our sample size is greater than 620 data to avoid the overfitting problem. If we



612 choose the model of three variables, the parameters in which will be less, the sample
613 size in this situation can be less. But the forecast results may be a little worse, based
614 on the analysis in section 2.3. So the length of training samples is related to the
615 number of parameters of our model.

616 Also, we have tried to detrend our data before the model constructed. But we
617 found the results didn't change too much. That is mean our model is not very
618 sensitive to climate change, so the detrended data has little effect for our model to
619 improve the forecast effect.

620 Compared with the original model, why the improved model has good forecast
621 results and can overcome the spring predictability barrier to some extent are as follow:
622 Recently, many studies have pointed out that spring is the most unstable season of the
623 air - sea interaction and the error is likely to develop or grow in the spring, resulting in
624 the spring predictability barrier (Zhang et al, 2012; Philander et al., 1992). When the
625 original model uses the indexes in summer as the initial values to predict, the SOI
626 factor representing the air-sea interaction is most unstable in the spring and the
627 EMWMI factor does not have much influence on ENSO in summer, so the forecast
628 results using the indexes in summer as the initial values are certainly much worse than
629 those using the indexes in the winter as the initial values. That is why our original
630 model does not overcome the spring predictability barrier.

631 However, the introduction of the self-memorization dynamics principle can help
632 our model overcome the spring predictability barrier to some extent. Although the
633 lead time is still summer (such as JJA), the information of the initial value actually



634 contains the previous $p + 1$ month (in this case $p = 6$, which contains the information
635 of the previous seven months, including the information of T_1, T_2 , SOI, EMWMI
636 factor in winter (January, February), spring (March, April, May) and summer (June
637 and July)). From the dynamical analysis, in this situation, the information and
638 interaction relationship of four factors have been a long period (from winter to
639 summer) accumulated, containing much air-sea interaction processes and winter
640 monsoon continued abnormal information, so the forecast results of our improved
641 model will be much better than the original model which simply uses only one initial
642 value. That is why the improved model overcomes the spring predictability barrier to
643 some extent.

644 The forecast results of our model are good, but it still has some problems:

645 (1) Although the reason why the improved model has good forecast results has
646 discussed in the section 6.2, the deep physical mechanisms that the proposed model
647 has dealt with is not very clear, so its dynamical characteristics should be further
648 analysed.

649 (2) The experiments in the present study have proven that the forecasting results
650 of the improved model are good for large-scale systems, such as ENSO events, and
651 the forecasting period has been extended. However, for small-scale systems, such as
652 Hurricanes, whether the forecast results could be improved using the present
653 improved model needs to be further verified.

654 (3) Our paper focuses primarily on these defined indices with T_1, T_2 to
655 reconstruct a prediction model. Maybe, we can select variables (predictor) based on



656 EOF analysis and our model may be a more physically oriented model. Maybe we can
657 learn from Yim et al (2013; 2015) to draw correlation maps between these fields and
658 the SSTA field and select the predictors from physical considerations. All these above
659 questions require that a lot of experiments to be carried out.

660 These items will be our future work.

661

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669

670 **APPENDIX A: THE PRINCIPLE OF DYNAMICAL MODEL** 671 **RECONSTRUCTION**

672 Suppose that the physical law of a nonlinear system going by over time can be
673 expressed as the following difference form:

$$674 \frac{q_i^{(j+1)M} - q_i^{(j-1)M}}{2\Delta t} = f_i(q_1^{jM}, q_2^{jM}, \dots, q_i^{jM}, \dots, q_N^{jM}) \quad j = 2, 3, \dots, M-1 \quad (\text{A1})$$

675 where f_i is the generalized nonlinear function of $q_1, q_2, \dots, q_i, \dots, q_N$, N is the number
676 of variables, and M is the length of observed data. $f_i(q_1^{jM}, q_2^{jM}, \dots, q_i^{jM}, \dots, q_N^{jM})$ can be assumed
677 to contain two parts: G_{jk} representing the expanding items which contain variable



678 q_i, P_{ik} just representing the corresponding parameters which are real numbers
 679 ($i = 1, 2, \dots, N, j = 1, 2, \dots, M, k = 1, 2, \dots, K$).

680 It can be supposed as follows:

681
$$f_i(q_1, q_2, \dots, q_n) = \sum_{k=1}^K G_{jk} P_{ik} \quad (A2)$$

682 $D = GP$ is the matrix form of Eq.(A2), in which

683
$$D = \begin{Bmatrix} d_1 \\ d_2 \\ \dots \\ d_M \end{Bmatrix} = \begin{Bmatrix} \frac{q_i^{3\Delta t} - q_i^M}{2\Delta t} \\ \frac{q_i^{4\Delta t} - q_i^{2\Delta t}}{2\Delta t} \\ \dots \\ \frac{q_i^{M\Delta t} - q_i^{(M-2)\Delta t}}{2\Delta t} \end{Bmatrix}, \quad G = \begin{Bmatrix} G_{11}, G_{12}, \dots, G_{1K} \\ G_{21}, G_{22}, \dots, G_{2,K} \\ \dots \\ G_{M1}, G_{M2}, \dots, G_{M,K} \end{Bmatrix}, \quad P = \begin{Bmatrix} P_{i1} \\ P_{i2} \\ \dots \\ P_{iK} \end{Bmatrix} \quad (A3)$$

684 Parameters of the above equation can be determined through inverting the
 685 observed data. Vector P which satisfies the above equation can be solved, based on a
 686 given vector D. Assuming q is unknown, it is a nonlinear system. However, assuming
 687 P is unknown, it is a linear system.

688 With the restriction $S = (D - GP)^T (D - GP)$ as a minimum, GA is introduced as an
 689 optimization solution search in the model parameters space.

690 Assuming that the parameters matrix P is the population (solutions), the
 691 $S = (D - GP)^T (D - GP)$ is an objective function, $l_i = \frac{1}{S}$ is the value of individual

692 fitness, and $L = \sum_{i=1}^n l_i$ is the value of total fitness. The operating steps of GA include:

693 creation and coding of initial population (solutions), fitness calculation, the choice of
 694 male parents, crossover and variation, etc. A detailed theoretical explanation can be
 695 got from Wang (2001). The step length is 1 month during the calculation. After
 696 optimization searches and genetic operations, the target value can be rapidly
 697 converged on and each optimal parameter of the dynamical equations can be obtained.



698 Through the above approach, we can obtain parameters of a nonlinear
699 dynamical system, and reconstruct the nonlinear dynamical equations from observed
700 data.

701

702 **APPENDIX B: THE MATHEMATICAL PRINCIPLE OF** 703 **SELF-MEMORIZATION DYNAMICS OF SYSTEMS**

704 The dynamical equations of a system can be expressed as:

$$705 \quad \frac{\partial x_i}{\partial t} = F_i(x, \lambda, t) \quad i = 1, 2, \dots, J \quad (\text{B1})$$

706 where J is an integer, x_i is the i th variable of the system state, and λ is
707 the parameter. Equation (B1) represents the relationship between a source function
708 F and a local change of x . Obviously, x is a scalar function with time t and
709 space r_0 . A set of time $T = [t_p \dots t_0 \dots t_q]$ can be considered, where t_0 is an initial
710 time. A set of space $R = [r_\alpha \dots r_i \dots r_\beta]$ can be considered, where r_i is a spatial point.

711 An inner product in space $L^2 : T \times R$ is defined by:

$$712 \quad (f, g) = \int_a^b f(\xi)g(\xi)d\xi, f, g \in L^2 \quad (\text{B2})$$

713 Accordingly, a norm can be defined as:

$$714 \quad \|f\| = \left[\int_a^b (f(\xi))^2 d\xi \right]^{1/2}$$

715 For a completion L^2 , it can become a Hilbert space H . A generalized one
716 in H can be regarded as a solution of the multi-time model. By introducing a
717 memorization function $\beta(r, t)$, we can obtain:



718
$$\int_0^t \beta(\tau) \frac{\partial x}{\partial \tau} d\tau = \int_0^t \beta(\tau) F(x, \tau) d\tau \quad (\text{B3})$$

719 where r in $\beta(r, t)$ can be dropped through fixing on the spatial point r_0 . Suppose
 720 that function $\beta(r, t)$ and variable x etc. are all continuous, differentiable and
 721 integrable, an integration by the left parts of Eq. (B3) can be made as:

722
$$\int_0^t \beta(\tau) \frac{\partial x}{\partial \tau} d\tau = \beta(t)x(t) - \beta(t_0)x(t_0) - \int_0^t x(\tau)\beta'(\tau) d\tau \quad (\text{B4})$$

723 where $\beta'(t) = \partial\beta(t) / \partial t$. The mean value theorem can be introduced into the third
 724 term in Eq. (B4), the following equation can be obtained:

725
$$-\int_0^t x(\tau)\beta'(\tau) d\tau = -x^m(t_0)[\beta(t) - \beta(t_0)] \quad (\text{B5})$$

726 where $x^m(t_0) \equiv x(t_m), t_0 < t_m < t$. Substituting Eq. (B4) and Eq. (B5) in Eq. (B3) and
 727 carrying out an algebraic operation, the following equation can be obtained:

728
$$x(t) = \frac{\beta(t_0)}{\beta(t)} x(t_0) + \frac{\beta(t) - \beta(t_0)}{\beta(t)} x^m(t_0) + \frac{1}{\beta(t)} \int_0^t \beta(\tau) F(x, \tau) d\tau \quad (\text{B6})$$

729 Because the x value which is at initial time t_0 and middle time t_m , only on
 730 the fixed point r_0 itself, relates to the first term and the second term in Eq. (B6),
 731 they are be called as a self-memory term. Also, we can call the third term as an
 732 exogenous effect, i.e., which is contributed by other spatial points.

733 Similarly as Eq. (B4), for multi-time $t_i, i = -p, -p+1, \dots, t_0, t$, it gives

734
$$\int_{t-p}^{t-p+1} \beta(\tau) \frac{\partial x}{\partial \tau} d\tau + \int_{t-p+1}^{t-p+2} \beta(\tau) \frac{\partial x}{\partial \tau} d\tau + \dots + \int_0^t \beta(\tau) \frac{\partial x}{\partial \tau} d\tau = \int_{t-p}^t \beta(\tau) F(x, \tau) d\tau.$$

735 After the same term $\beta(t_i)x(t_i), i = -p+1, -p+2, \dots, 0$ was eliminated, we

736 have



$$\beta(t)x(t) - \beta(t_{-p})x(t_{-p}) - \sum_{i=-p}^0 [\beta(t_{i+1}) - \beta(t_i)]x^m(t_i) - \int_{t_{-p}}^t \beta(\tau)F(x, \tau)d\tau = 0 \quad (\text{B7})$$

As a matter of convenience, we set $\beta_t \equiv \beta(t)$, $\beta_0 \equiv \beta(t_0)$, $x_t \equiv x(t)$, $x_0 \equiv x(t_0)$; the following text uses similar notations. Then, Eq. (B7) can be expressed as:

$$\beta_t x_t - \beta_{-p} x_{-p} - \sum_{i=-p}^0 x_i^m (\beta_{i+1} - \beta_i) - \int_{t_{-p}}^t \beta(\tau)F(x, \tau)d\tau = 0 \quad (\text{B8})$$

Setting $x_{-p} \equiv x_{-p-1}^m$, $\beta_{-p-1} = 0$, the Eq. (B8) can be written as:

$$x_t = \frac{1}{\beta_t} \sum_{i=-p-1}^0 x_i^m (\beta_{i+1} - \beta_i) + \frac{1}{\beta_t} \int_{t_{-p}}^t \beta(\tau)F(x, \tau)d\tau = S_1 + S_2 \quad (\text{B9})$$

S_1 is called as a self-memory term and S_2 is called as an exogenous effect term.

For the convenience of calculations, the above self-memorization equation can be discretized. The differential by difference and the summation can replace the integration in Eq. (B9), and the mean of two values which are at adjoining times; i.e., $x_i^m \approx \frac{1}{2}(x_{i+1} + x_i) \equiv y_i$ can simply replace x_i^m .

Taking an equal time interval $\Delta t_i = t_{i+1} - t_i = 1$ and incorporating β_i and β_t , we can obtain a discretized self-memorization equation as follows:

$$x_t = \sum_{i=-p-1}^{-1} \alpha_i y_i + \sum_{i=-p}^0 \theta_i F(x, i) \quad (\text{B10})$$

where F is the dynamic kernel of the self-memorization equation, $\alpha_i = \frac{(\beta_{i+1} - \beta_i)}{\beta_t}$;

$$\theta_i = \frac{\beta_i}{\beta_t}.$$

Based on Eq. (B10), the above technique performed computations and the forecast can be called as a self-memorization principle.

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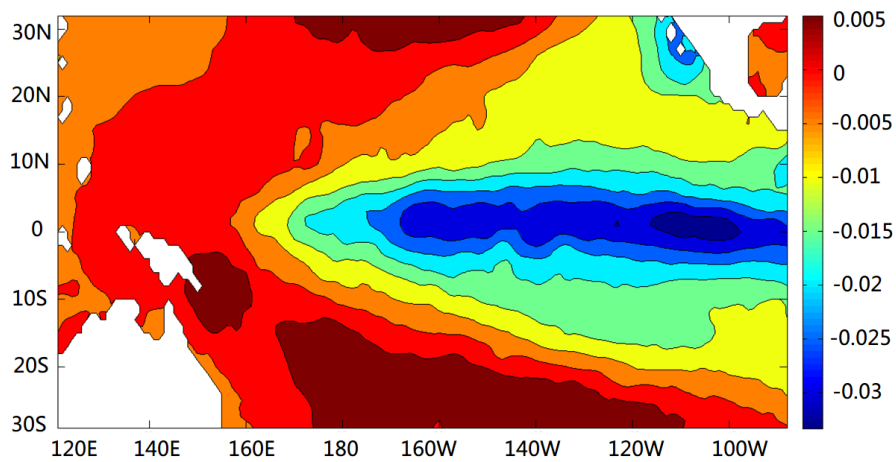
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947 summer.

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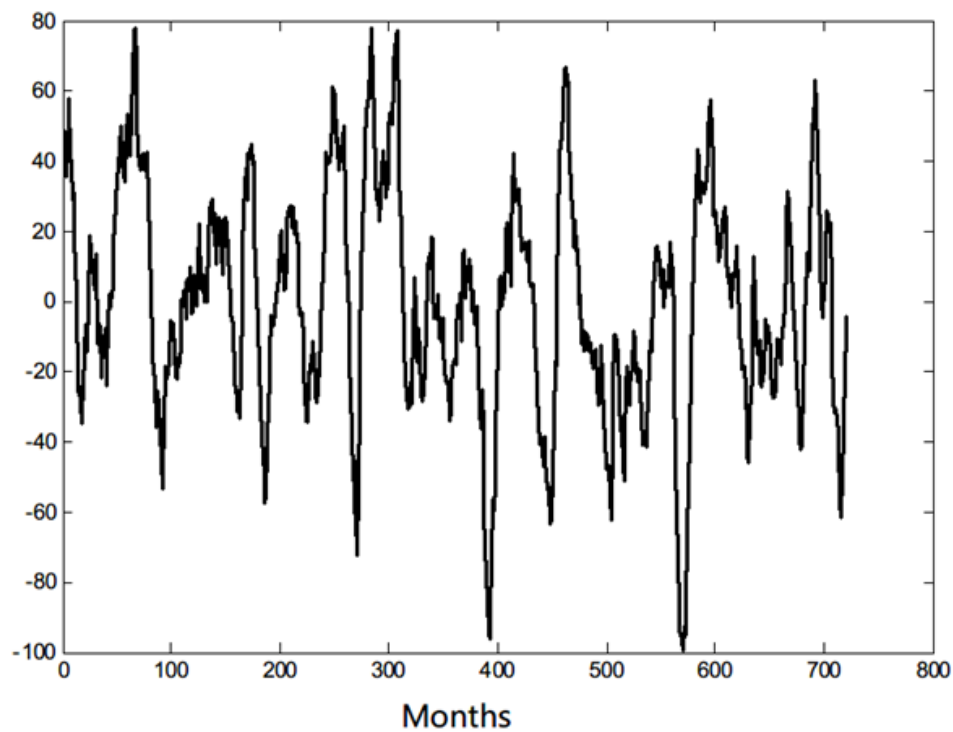
950 **Figure:**



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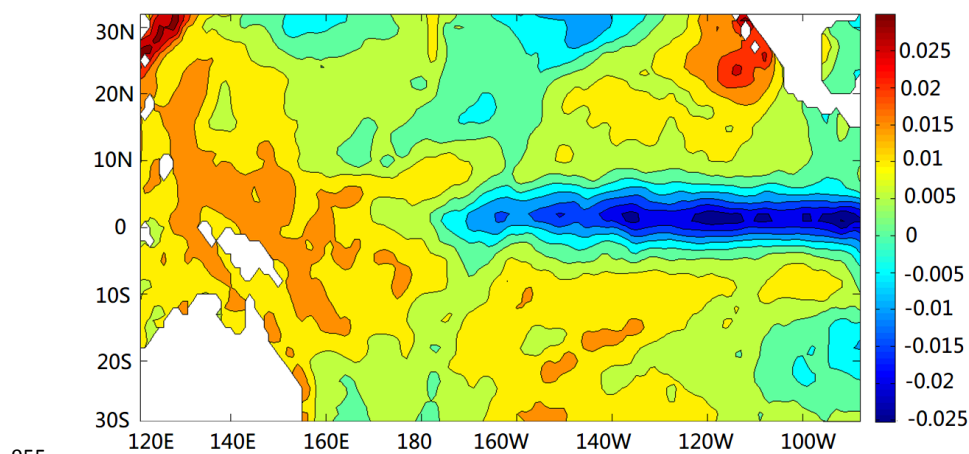
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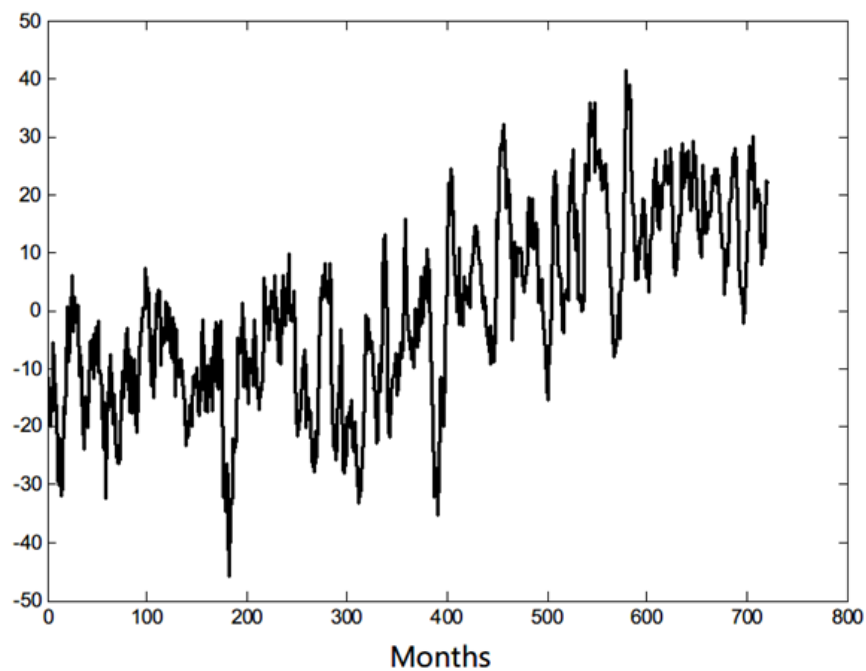
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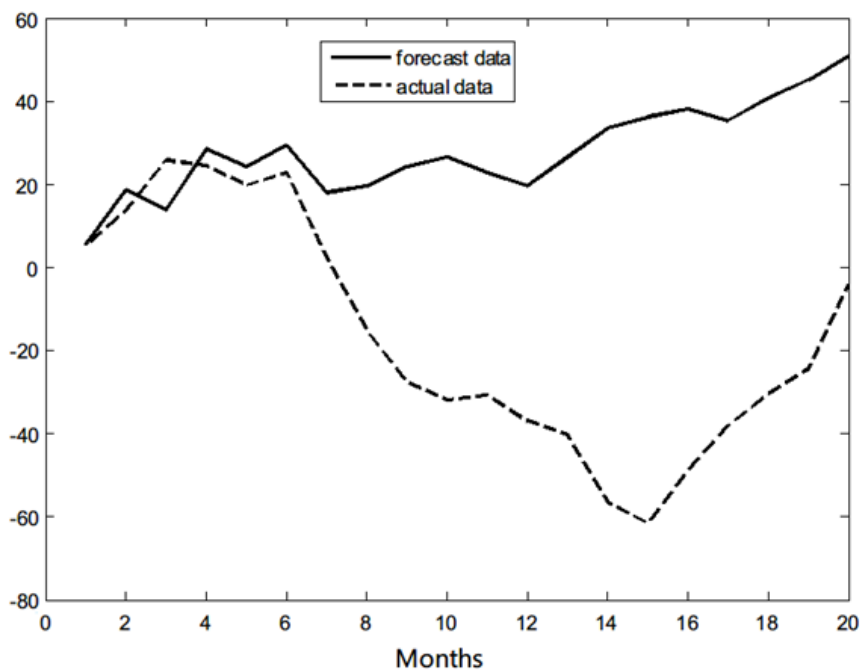
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959 **Fig. 1** (a, c) First and second modes of the EOF deconstruction of the SSTA field, and (b, d) the

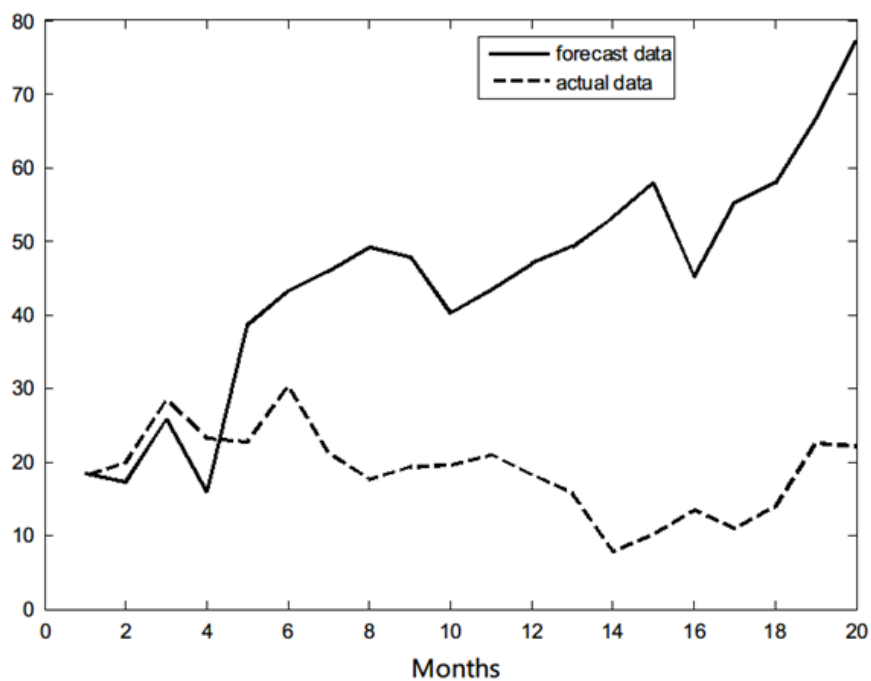
960 corresponding PC time series.



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(a)



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(b)

965 Fig.2 Forecast results of the first time coefficient series T_1 (a) and the second time coefficient series

966 T_2 (b) of the SSTA field by the original model

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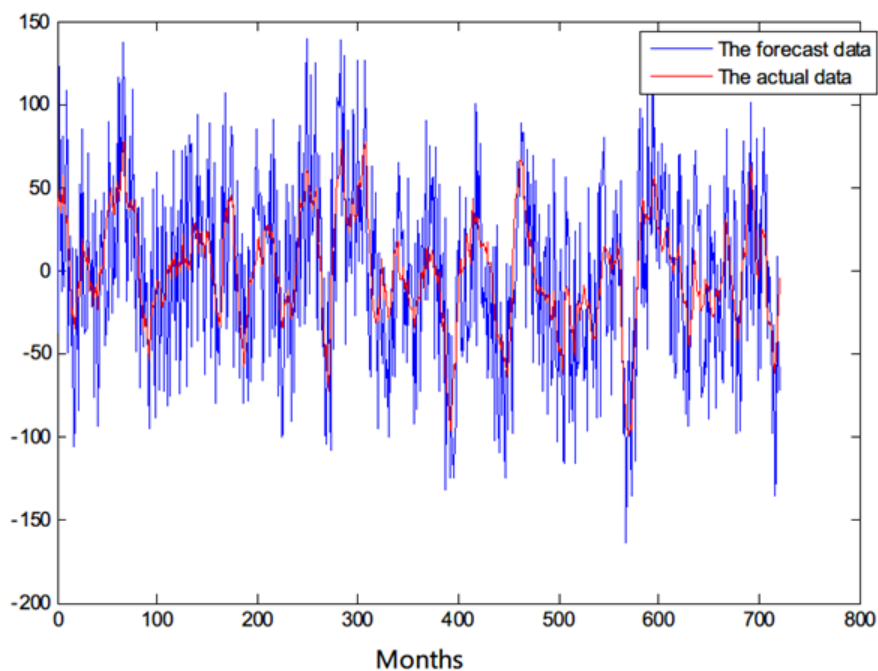
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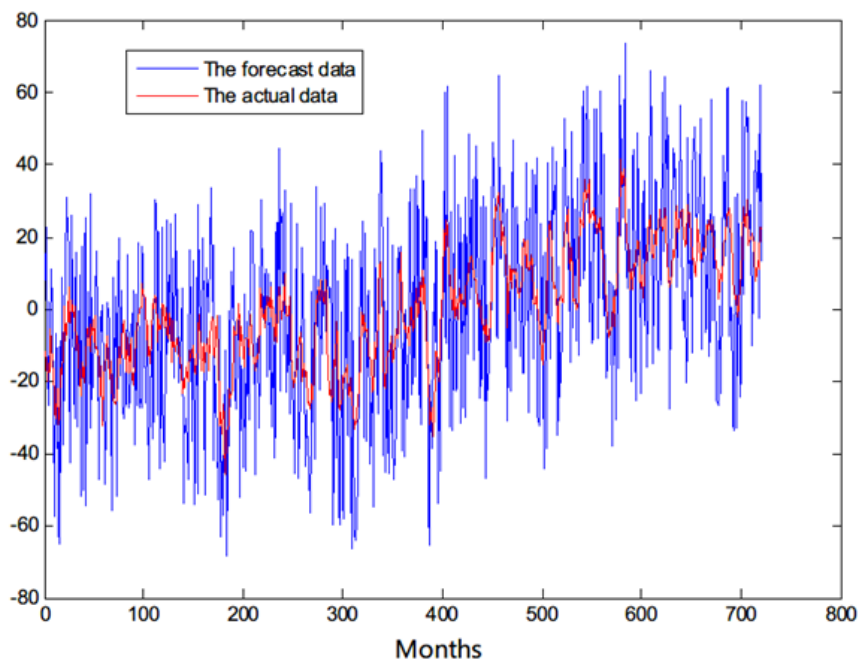
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(a)



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(b)

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Fig.3 The cross-validated retroactive hindcast results of the first time coefficient series T_1 (a) and the

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second time coefficient series T_2 (b) of the SSTA field by the original model

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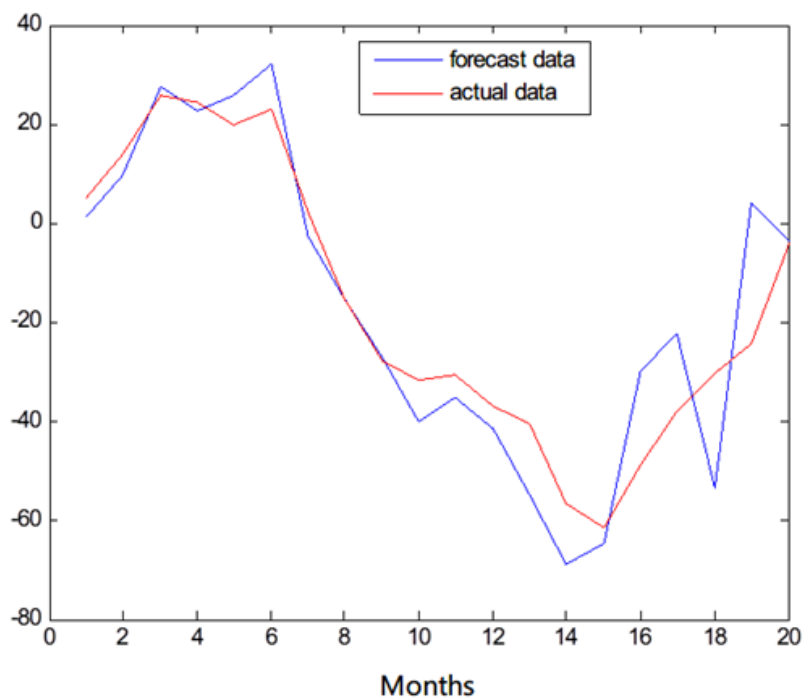
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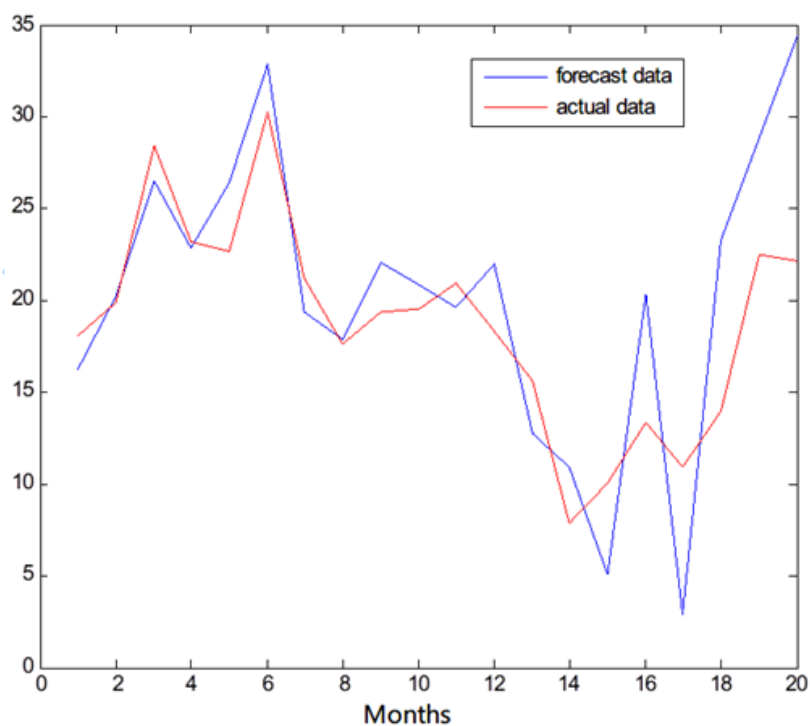
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(a)



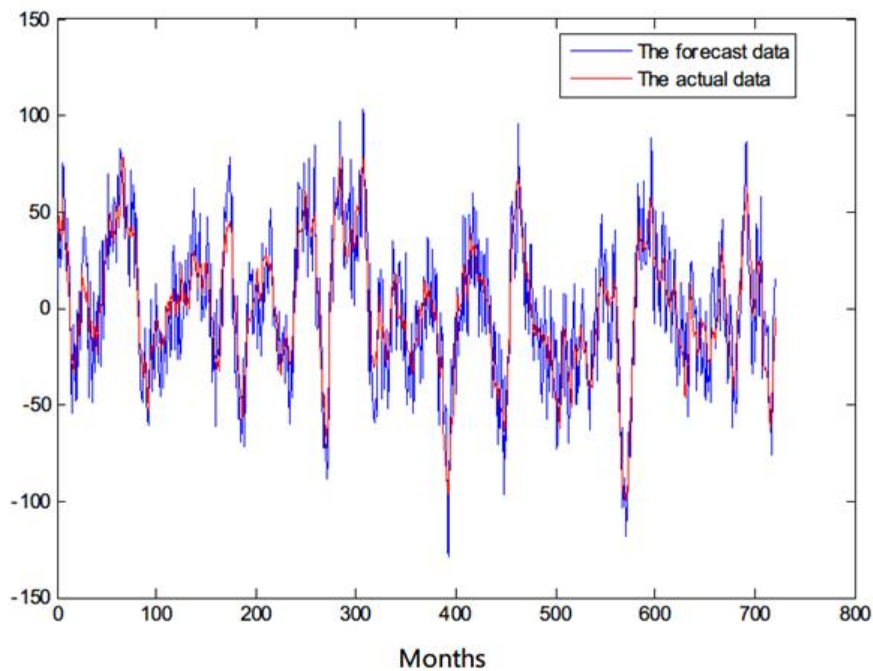
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(b)

997 Fig. 4. Long-term step-by-step forecast results of the first time coefficient series T_1 (a) and the second
998 time coefficient series T_2 (b) of the SSTA field by the improved model

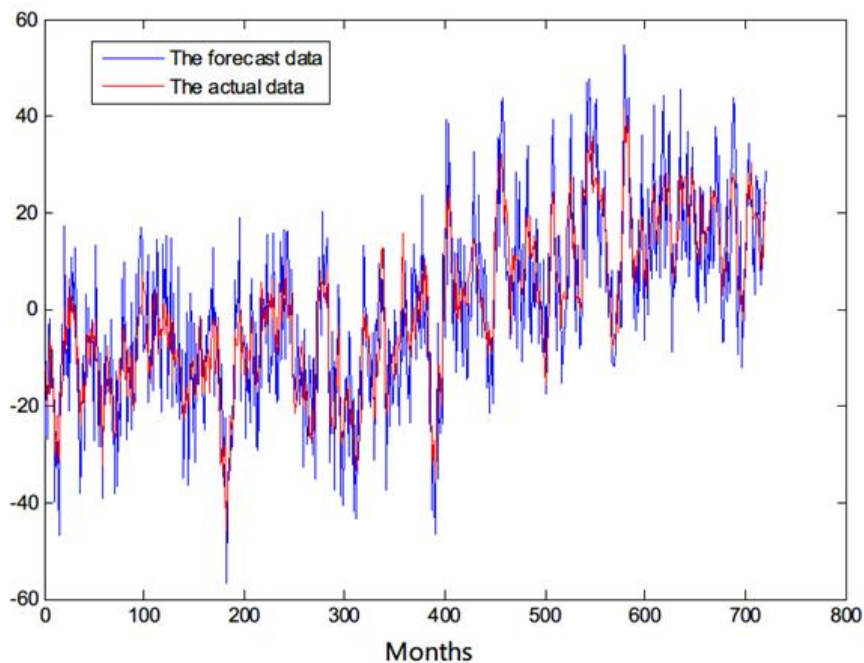
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(a)



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(b)

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Fig. 5. The cross-validated retroactive hindcast results of the first time coefficient series T_1 (a) and the

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second time coefficient series T_2 (b) of the SSTA field by the improved model

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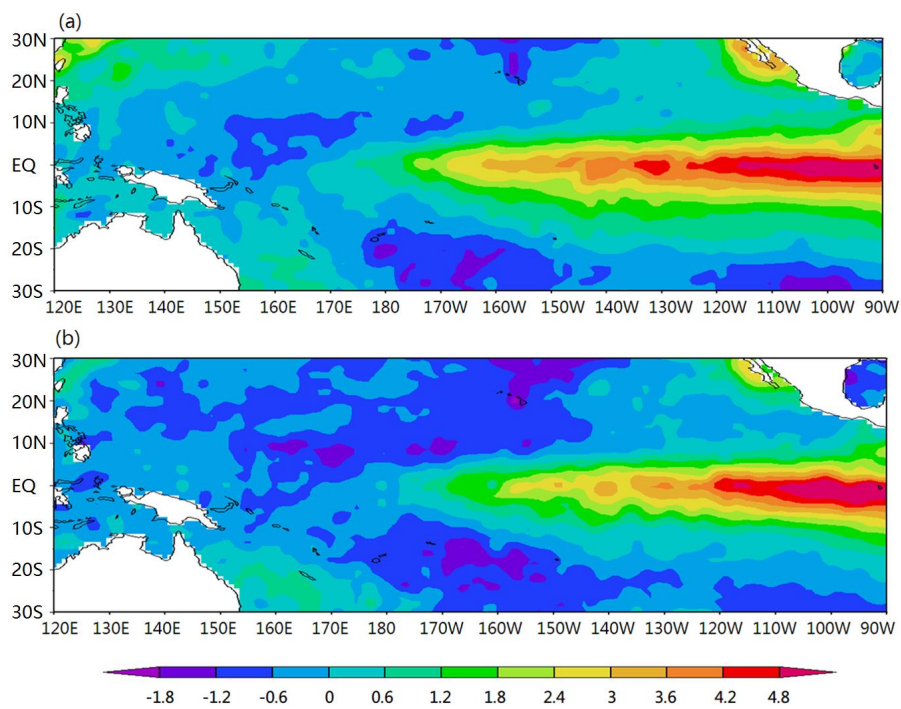
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1015 Fig.6. The forecast SSTA field(a) and the actual SSTA field (b)of an El Niño event (Dec.1997)

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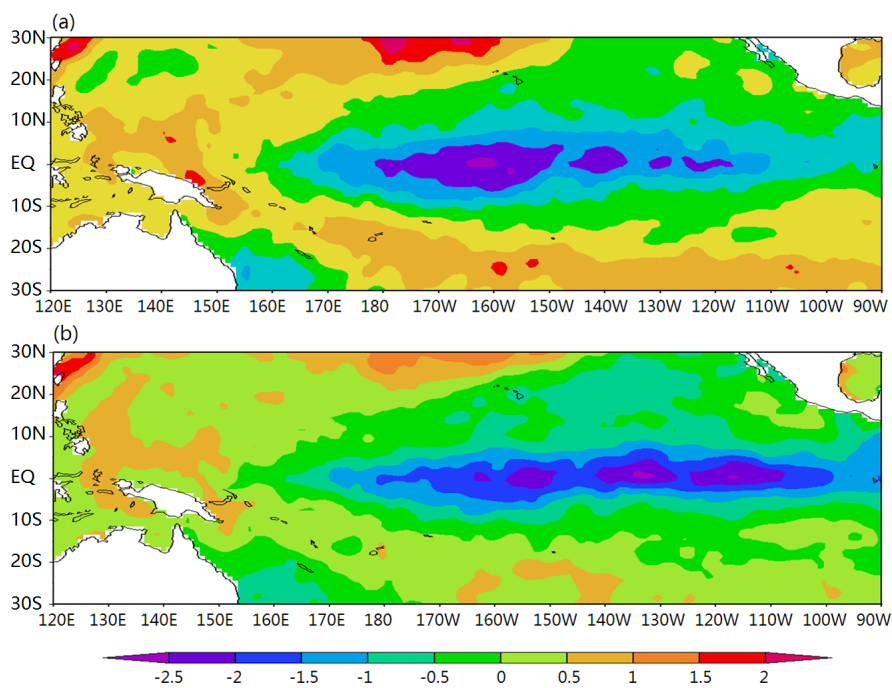
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1027 Fig. 7. The forecast SSTA field(a) and the actual SSTA field (b)of a La Niña event (Dec.1999)

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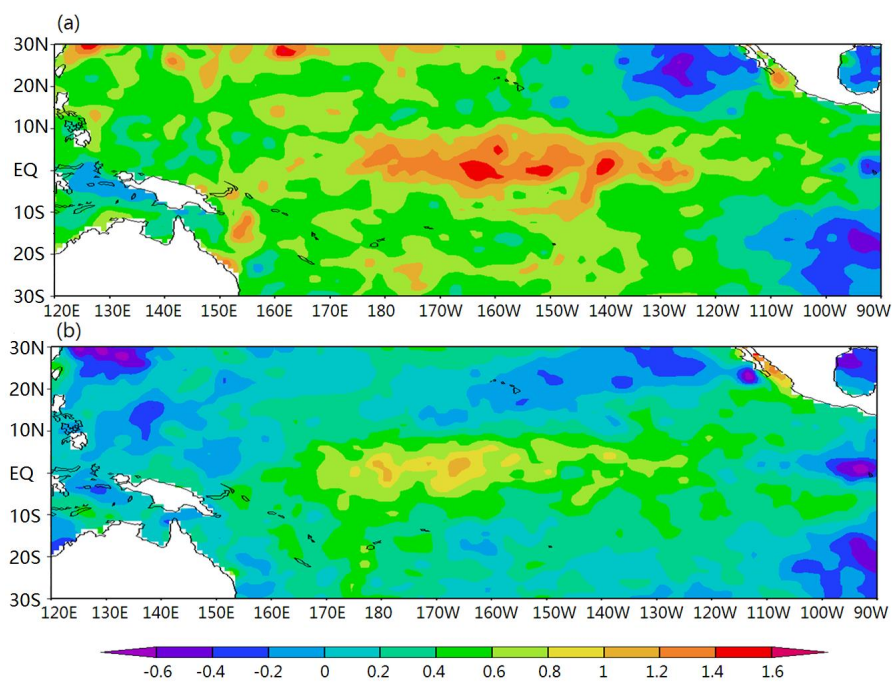
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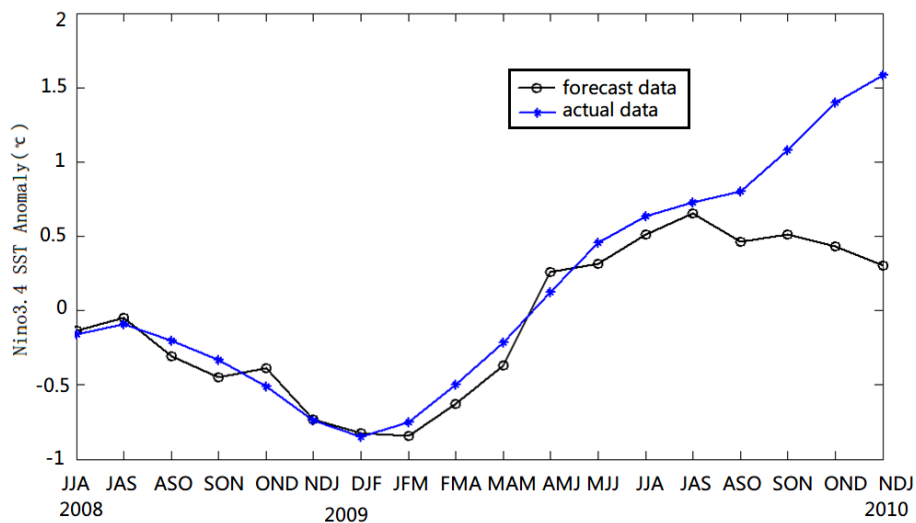
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1038 Fig.8. The forecast SSTA field(a) and the actual SSTA field (b)of neutral event (Nov.2002)

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Fig.9. The improved dynamical-statistical model prediction of the ENSO index

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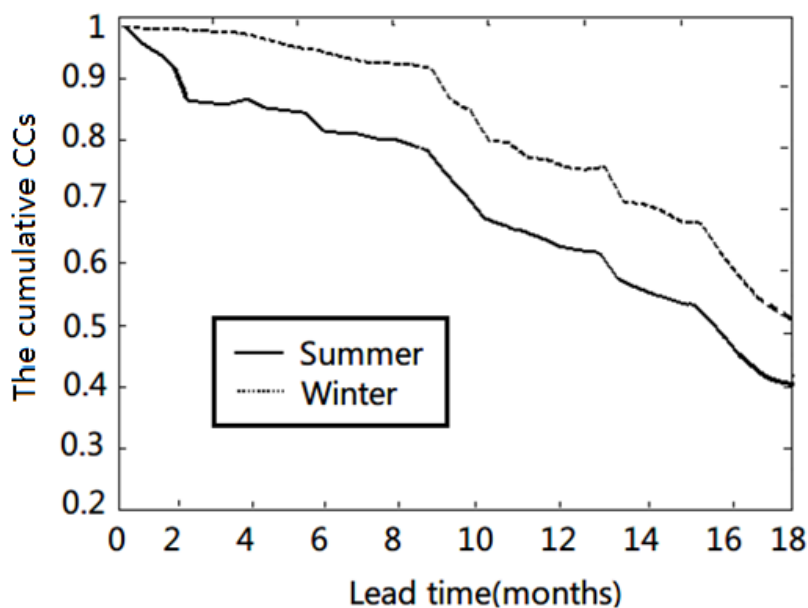
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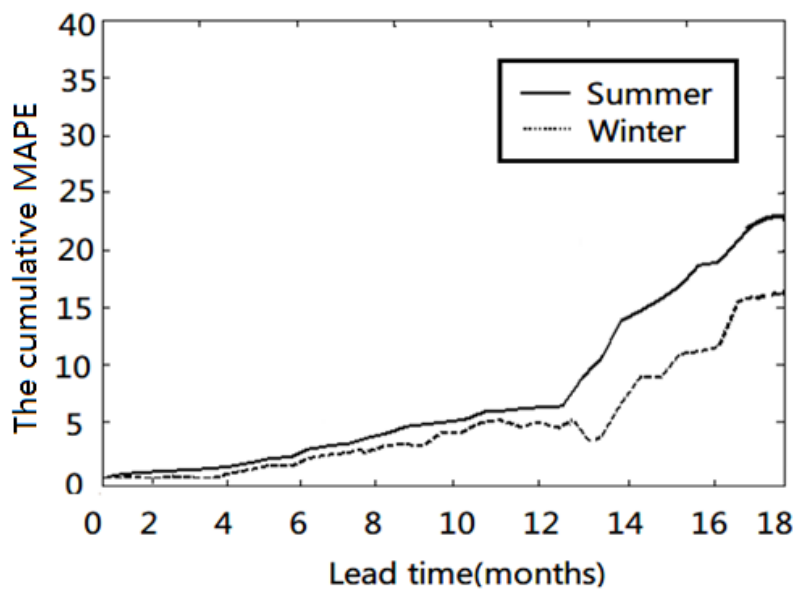
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(a)



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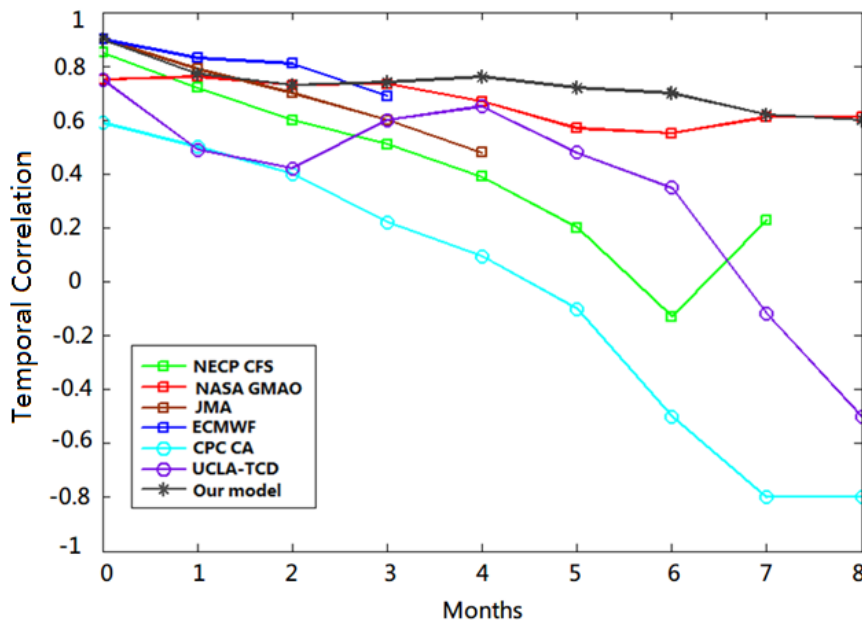
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(b)

1052 Fig.10. The cumulative correlation coefficients(CCs) (a) and cumulative mean absolute percentage
1053 error(MAPE) (b) changing with time of different lead times



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1056 Fig. 11. Temporal correlation between model forecasts and observations for all seasons combined, as a
1057 function of lead time. Each line highlights one model.

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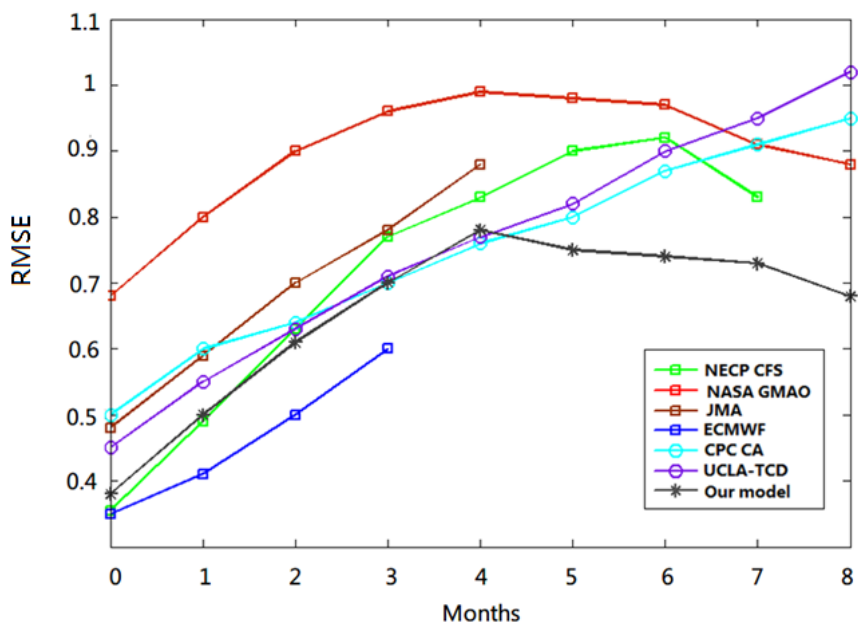
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1074 Fig. 12. RMSE in standardized units, as a function of lead time for all seasons combined. Each line

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1090 **Table:**

1091 **Table1.** The forecast results of the models of different variables

The model	The forecast skill of 60 cross-validated retroactive hindcasts experiments of the ENSO index for all seasons combined at lead times of 8 months	
	the temporal correlation	the root mean square error
One variable (T_1)	0.5051	0.8075
Two variables (T_1, T_2)	0.5613	0.7679
Three variables (T_1, T_2, SOI)	0.6027	0.7275
Four variables ($T_1, T_2, SOI, EAWMI$)	0.6344	0.6728
Five variables ($T_1, T_2, SOI, EAWMI, u_1$)	0.5923	0.7344
Six variables ($T_1, T_2, SOI, EAWMI, u_1, PNA$)	0.5528	0.7806

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1119 **Table2.**The correlation coefficient(CC) and Mean absolute percentage error(MAPE) of long-term
 1120 fitting test when the retrospective order p is different

p		4	5	6	7	8	9	10
The forecast results of long-term fitting test	CC	0.75	0.73	0.81	0.74	0.70	0.72	0.68
	MAPE	18.42%	19.36%	14.56%	20.39%	25.31%	24.18%	27.33%
p		11	12	13	14	15	16	
The forecast results of long-term fitting test	CC	0.68	0.70	0.65	0.62	0.60	0.62	
	MAPE	28.10%	26.58%	30.91%	33.14%	34.97%	33.56%	

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1135 **Table3.** The forecast results of T_1 and T_2 in different examples within 6 and 12 months

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Forecast events	The results within 6-months		The results within 12-months	
	CC	MAPE	CC	MAPE
The average of 18 El Niño examples of T_1	0.824	8.45%	0.719	12.67%
The average of 22 La Niña examples of T_1	0.846	7.68%	0.740	11.28%
The average of 20 Neutral examples of T_1	0.885	6.23%	0.789	9.85%
The average of total 60 examples of T_1	0.850	7.41%	0.748	10.95%
The average of 18 El Niño examples of T_2	0.811	8.79%	0.703	13.28%
The average of 22 La Niña examples of T_2	0.833	7.35%	0.731	11.96%
The average of 20 Neutral examples of T_2	0.896	6.68%	0.795	10.08%
The average of total 60 examples of T_2	0.842	7.64%	0.740	11.71%

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1146 **Table 4.** Temporal correlation(CC) and the mean absolute percentage error (MAPE) between
 1147 model forecasts and observations within 12 months for Nov–Jan, Dec–Feb, and Jan–Mar as lead
 1148 time.of winter and for May–July, June–August and July–Sep. as lead time of summer.

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Forecast events	Lead time of all seasons combined		Lead time of summer (MJJ-JJA-JAS)		Lead time of winter (NDJ-DJF-JFM)	
	CC	MAPE	CC	MAPE	CC	MAPE
The average of 18 El Niño examples	0.604	9.70%	0.569	10.33%	0.677	8.02%
The average of 22 La Niña examples	0.625	8.97%	0.581	9.82%	0.695	7.83%
The average of 20 Neutral examples	0.798	5.96%	0.752	6.86%	0.844	4.60%
The average of total 60 examples	0.712	7.62%	0.633	8.51%	0.776	6.52%

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1162 **Table5.** The correlation coefficients among four factors

Correlation coefficients	T_1	T_2	SOI	EAWMI
T_1		0.419	0.401	0.337
T_2	0.419		0.424	0.356
SOI	0.401	0.424		0.408
EAWMI	0.337	0.356	0.408	

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