

Response to Referee #1

Anonymous Referee #1

Received and published: 28 August 2017

Dear Editor,

The authors present an interesting study into the tidal dynamics of the Taiwan Strait. Particularly, they apply a so-termed ‘extended’ version of the classical ‘Taylor method’ to reproduce and explain the amphidromic pattern of the semi-diurnal tide in that region. The word ‘extended’ here refers to the treatment of the (open) boundaries and the inclusion of bottom friction. This leads to an analysis of two Kelvin waves, propagating southward and northward, the superposition of which largely determines the amphidromic pattern in the Taiwan Strait. As to the sources that may contribute to the northward Kelvin wave, the authors conduct a further analysis in which the model is extended in various ways. To be honest, I find this part a bit far-fetched, simply because the rather ‘crude’ (as the authors acknowledge themselves) geometrical choices made here clearly ignore the true geometry of the sea surrounding the Taiwan Strait, particularly regarding coastlines. This makes the conclusions of this part less convincing to me, which is actually my first concern of this study. The same applies in my opinion to statements about the “superiority” of this approach in the conclusions. Other aspects that – in my opinion – require clarification or improvement deal with (1) description of the study site, (2) literature review, (3) model formulation, (4) comparison with observations, (5) interpretation of Kelvin and Poincaré modes, and (6) phrasing. These points are detailed below. Overall, I think the topic of the paper is appropriate for OSD. The novelty of the work is apparent, but the my concerns on how this has been done are substantial. Therefore, my overall recommendation is major revision.

Reply: We sincerely thank the Referee for his careful reading of our manuscript and constructive comments and suggestions, which are of great help in improving our study. We have addressed all these comments; our responses are given below.

In this response, the Referee's comments are copied in black, our replies are shown in red, and the following abbreviations are used:

OM - original manuscript,

R1 – Revision #1 - an updated manuscript, which will be submitted as a supplement to this response.

1) Description of the study site may be extended by presenting the relative importance of other tidal constituents (S_2 , K_1 , O_1), e.g. expressed in the value of the form factor F . Why did you consider the M_2 -tide only? And what is known about the (magnitude of the) tidal currents? This helps interpretation compared to other tidal basins around the world.

Reply: In the "Introduction" of R1, we have added the following sentence to describe the magnitudes of the constituents S_2 , K_1 and O_1 relative to M_2 : "Compared to M_2 , which has a maximum amplitude of over 2.2 m, the amplitudes of the rest of the constituents are much smaller. The maximum amplitudes of S_2 , K_1 and O_1 observed at 11 coastal gauge stations reported by Jan et al. (2004b) are 0.66, 0.39 and 0.27 m,

respectively".

2) Literature review should in my opinion be improved in certain respects.

- The large number of references on tides in the Taiwan Strait makes me wonder what has been found in those studies. . .

Reply: Since this study focuses on the tidal dynamics in the strait, we describe mainly the progress in the dynamic aspects without giving a comprehensive review of the progress in the studies of tides in the strait.

- Page 2, Line 12: “was the main component” → “is the main component”.

Reply: Revised as suggested.

- Upon first introduction in Line 18, The extended Taylor method (when using “the”, please remove the “s”) requires a reference and an explanation of what ‘extended’ means here.

Reply: "the extended Taylor’s method " in the OM has been replaced with "an extended Taylor method" in R1. (Here, we replace “the” with “an” according to the comment from Referee #2).

- Roos&Velema should in fact be Roos et al (there are more co-authors). Also, unlike suggested by the authors here, the presence of the Dover strait in the south is in fact an open boundary.

Reply: “Roos and Velema” has been changed to “Roos et al.”, and the citation in the References is also revised in R1. The statement “all of the studied basins” in the OM is not accurate, and thus “all” is replaced with “most of” in R1.

- I cannot find Table 1 in the .pdf-file that for this review.

Reply: “Roos and Velema, 2011, Table 1” in the OM has been replaced with “Table 1 of Roos et al., 2011” in R1.

- Hendershott & Speranza (Deep Sea Res 1971) is worthwhile mentioning as they followed a similar approach to study the Gulf of California (two Kelvin waves)

Reply: Hendershott & Speranza’s paper has been cited in R1.

- Because of the depth-step, one may consider reference to Roos&Schuttelaars(Ocean Dyn 2011)

Reply: Roos & Schuttelaas (2011) has been cited in R1.

- Figure 2: “amphidromic chart” seems better, because it is both co-tidal and corange information that is plotted here. Also: is it Chen and Andersen or Cheng and Andersen?

Reply: We replaced “cotidal” with the more accurate term “tidal system” in R1. “Chen” has been replaced with “Cheng”. The reason for not using “amphidromic” is that there is no amphidromic point in the TS, especially in the area shown in Fig. 3.

3) Model formulation contains some inaccuracies. First of all, the title of section 2 does not really cover the content. I think “Model formulation and solution method” is more appropriate.

Reply: The section title has been revised as suggested.

- Please mention the important simplifications/approximations made here. This is a linear depth-averaged model, the validity of which is relevant. I think this should be discussed at some point.

Reply: According to this comment, we have added the following statements to the

text: “The equations in (1) are two-dimensional linearized shallow water equations on an f-plane with the momentum advection neglected. The equations are the same as those used in the work of Taylor (1922), except that the bottom friction is incorporated, as in Fang and Wang (1966) and Rienecker and Teubner (1980).”

- The pressure gradients in Eq. (1) should have spatial derivatives ($\partial/\partial x$ and $\partial/\partial y$)
Reply: Corrected as suggested.

- Page 3, Line 8: “channel” → “rectangular channel”

Reply: The word “rectangular” has been added.

- Line 11: “by introducing a collocation method” → “by applying a collocation method”

Reply: The word “introducing” has been replaced with “applying”.

- Page 4, Line 5: “for open rectangular basins” → “accounting for the finite length of the basin”

Reply: Revised as suggested.

- Line 13: please mention “depth-averaged”

Reply: Added as suggested.

- Line 15: this approximation is known as the f-plane

Reply: The word “f-plane” has been added into the next line.

- Line 16: “cosine wave” is perhaps better rephrased as “monochromatic”

Reply: The word “cosine” has been replaced with “monochromatic”.

- Line 17: please put brackets { and } after the real part: $\text{Re} \{(\zeta, u, v) \exp(i\sigma t)\}$ and please introduce (ζ, u, v) as the complex amplitudes of the quantities introduced previously.

Reply: The brackets have been added.

- Page 5, Line 4: please add “each with a different uniform depth h_A and h_B ”

Reply: Added as suggested.

- Line 9: would be nice if your radiation condition would include bottom friction. How large is μ typically?

Reply: In the present study we take $\mu = 0.15$ so that μ^2 is of the order of 0.02. If friction is considered, the expression of the radiation condition will become complicated: besides a change in the amplitude ratio, there will be a phase difference between the velocity and elevation. These changes would cause very few differences in the computed results (of the order of μ^2) and are thus ignored in this study.

- Line 23: the formula for wave speed also holds in absence of friction only. . . please reorder

Reply: The sentence has been reordered according to this comment. Here, the formulas hold only under the condition of no friction; if friction is considered, the formulas for wave speed and wave number should be modified {e.g., the wave speed

is equal to $\text{Re} \left(\frac{\sigma}{\beta} \right) = \left[\frac{2gh}{\sqrt{1+\mu^2+1}} \right]^{1/2}$, which is equal to $0.9972\sqrt{gh}$ for $\mu = 0.15$ }.

I would put the details of Eqs.(9)-(12) and Eqs. (17-24) in an appendix.

Reply: The derivation of these equations has already been made in previous works (e.g., Fang et al., 1991), so it seems unnecessary to give further details in this paper. Furthermore, these equations will be mentioned in the text that follows. Therefore, for

convenience, we wish to retain these equations as they are.

- Line 22: I think it is unnecessary to introduce Q , because you can immediately write $Q^2 = \alpha^2 - \beta^2$.

Reply: Yes, Q^2 is equal to $\alpha^2 - \beta^2$. In R1, $-Q^2$ has been replaced with $-\alpha^2 + \beta^2$ in the expression of s_n (Eq. (16)).

4) Comparison with observations is purely visual, which raises some questions.

First of all, how did you choose the basin dimensions, orientation? How do you actually project the true geometry, with curved coastlines, onto the rectangular model domain? And, as before: did you consider doing the same for other tidal constituents? Other than that, I find the title of Section 3 confusing, since the model has already been introduced in Section 2. I suggest to change the title into “Application to Taiwan Strait”, because that is what is actually done in this section. Also, please avoid if statements when you specify coefficients (Page 7, line 2) and please replace “equal to” with an equality sign =.

Reply: The main purpose of this study is to reveal the dynamics of the M_2 wave formation in the strait. We think visual comparison is capable of meeting this goal. No attempt is made to best fit the model results to observations in this study. For the same reason, the basin dimension and orientation, locations of the sidewall and open boundaries have all been chosen through visual inspection. The diurnal constituents in the strait are small and have a simple structure (please see the following tidal system chart of the largest diurnal constituent K_1). Thus, we are not interested in the study of their dynamics. The dynamics of S_2 is the same as that of M_2 , so we just pay attention to M_2 . The title of Section 2 has been changed according to this comment in R1. “If” has been replaced with “In this study”; “equal to” has been replaced with the sign “=”.

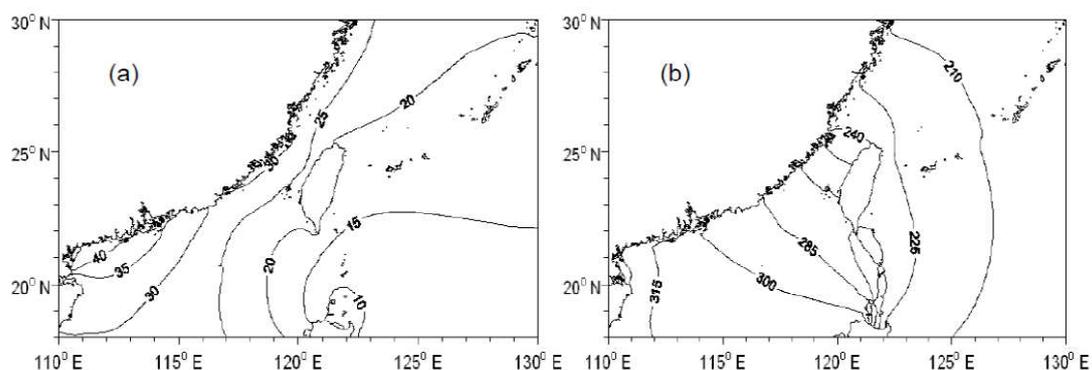


Fig. 1.1. K_1 tidal system in the Taiwan Strait and its neighbouring area, (a) amplitudes in cm (b) phase-lags in degrees (from Zhu et al., 2009). (In the figure number, the first “1” represents “Author’s Response to Referee #1”)

5) Interpretation of role of Kelvin and Poincare modes can readily be deepened by further analysis. First of all, what is the wavelength of the Kelvin waves? (I see it is mentioned later on but already here it is relevant). And are the Poincare modes free or bound (from the depth and width values I guess they are all bound), and what is the typical length scale of decay of the lowest Poincare mode? This gives insight in the extent to which these modes affect the amphidromic pattern in the (interior of the)

Taiwan Strait.

Reply: In the first paragraph of Section 3.1, we have added the following statement: "From these parameter values, we can obtain the wavelength of the M2 Kelvin wave as 1009 km. Since the basin width is smaller than half of the Kelvin wavelength, the Poincaré modes can only exist in a bound form (Godin, 1965; Fang and Wang, 1966). The e-folding length of decay of the lowest Poincaré mode is approximately 63 km, that is, the amplitude of this mode reduces to approximately 37% relative to its maximum value at a distance of 63 km away from the boundary. Equivalently, it may also reduce to approximately 20% relative to its maximum value at a distance of 100 km. The length scales of decay for higher order Poincaré modes are even shorter."

Also, I do not understand the statement that frictional force would be a major factor (as mentioned here and repeated in Section 5). I think this is not the case, in view of the mild amplitudes and large depths. Can you support this statement? I suspect you would still get a good fit if bottom friction were switched off.

Reply: Here, the words "the Coriolis and frictional forces" have been replaced with "the Coriolis force and the weaker northward wave" according to this comment.

• Page 7, Line 23: "inclusion of the Poincaré modes improves"

Reply: Revised as suggested.

• Page 8, Line 10: Also possible is that the assumption of uniform depth is too restrictive in this Taylor approach. . .

Reply: We agree with this point of view and have therefore added "at a uniform depth" following "Kelvin wave", such that the statement is now "This amplitude variation cannot be completely represented by the superposed Kelvin wave at a uniform depth".

• Page 8, Line 21: this is a basic statement about progressive waves and therefore not really insightful in my opinion.

Reply: The words "due to propagation direction" have been deleted in R1.

• Page 10, I do not understand the statement on resonance. This may hold for closed basins, but here we have a topographic step. . .

Reply: Resonance is also possible in a basin with a topographic step. This can be illustrated with one-dimensional problems as follows:

For a basin of uniform depth h and length L , if it has a closed end at $x=0$ and an opening at $x=L$, where tidal elevation is given as $\zeta(L, t) = a \cos \sigma t$, then the

elevation in the basin is $\zeta(x, t) = a \frac{\cos kx}{\cos kL} \cos \sigma t$, where $k = \sigma / \sqrt{gh}$. Resonance

will occur if $kL = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$. Please see Godin, 1993 (continental Shelf Res., 13(1), p. 103).

For a shallow basin of uniform depth h and length L , if its mouth is at $x=L$, where tidal elevation is given as $\zeta(L, t) = a \cos \sigma t$, and it has a topography step at $x=0$, which connects with a deep basin of infinite-depth, then the elevation in the shallow

basin is $\zeta(x, t) = a \frac{\sin kx}{\sin kL} \cos \sigma t$, where $k = \sigma / \sqrt{gh}$. Resonance will occur if

$kL = \pi, 2\pi, \dots$. Please see Jan et al, 2002 (Journal of Oceanography, 58, p. 849).

6) Phrasing in general should be more precise in my opinion. For example, avoid the unnecessary and confusing use of the verb “can”. My suggestion is to consult a native speaker of the English language with knowledge of the topic to revise the text. Here I explain what I mean by giving some suggestions to improve the abstract (line 8-22).

Reply: R1 has been edited for English by a native English speaker from a language service company. Please see the following certificate issued by them

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EDITORIAL CERTIFICATE

This document certifies that the manuscript listed below was edited for proper English language, grammar, punctuation, spelling, and overall style by one or more of the highly qualified native English speaking editors at American Journal Experts.

Manuscript title:
An analytical study on M2 tidal wave in the Taiwan Strait with an extended Taylor method

Authors:
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• Page 1, Line 8: “M2” → “semidiurnal lunar (M2)”

Reply: Revised as suggested.

• Line 8, “The extended Taylor’s method”, remove “s”: and is it sufficiently clear what this means?

Reply: Revised as suggested.

• Line 10, “but” → “and” (because this does not really signify a contradiction!)

Reply: Revised as suggested.

• Line 10: “friction forces” → “bottom friction”

Reply: Revised as suggested.

• Line 16: “can further improve” is unclear. Better: “Inclusion of Poincaré modes further improves”

Reply: Revised as suggested.

• Line 18: “can be reflected” → “is reflected” (I guess this is what you mean)

Reply: Revised as suggested.

• Line 21: same with “can” as in Line 18.

Reply: Revised as suggested.

Response to Referee #2

Anonymous Referee #2

Received and published: 6 September 2017

General comments

This paper contains original contribution to analytical tide modeling using an Taylor' method. Although there are quite many thing to be clarified and improved, it is believed that authors can revise the manuscript without much difficulties. This paper is therefore recommended for the publication in OS with minor corrections.

Reply: We sincerely thank the Referee for his careful reading of our manuscript, as well as the constructive comments and suggestions which are of great help for improving our study. We have addressed all these comments; our responses are given below.

In this response, the Referee's comments are copied in black, our replies are shown in red, and the following abbreviations are used:

OM - original manuscript,

R1 – Revision #1 - an updated manuscript, which will be submitted as a supplement to this response.

Specific comments

Title

Pg.1, lines 1-2 Better to replace "the extended Taylor's method" to "an extended Taylor's method".

Reply: The expression "the extended Taylor's method" has been replaced with "an extended Taylor method" in R1. Here, we have also removed "s" according to the suggestion of Referee #1.

Abstract

Pg. 1, line 8: Again use "an extended Taylor's method" Pg. 1, lines 21-22: The sentences are a little bit unnatural. Include how much the northward KW is strengthened, that is, quantitatively, saying it is of secondary importance.

Reply: Here, the expression "the extended Taylor's method" has also been replaced with "an extended Taylor method". Since the results of Ex. 3 (with the Luzon Strait input being considered) contain some uncertainties (please see discussion in Section 5) we would not give a quantitative estimate in Abstract, but just state that "the forcing is thus of some (but lesser) importance to the M_2 tide in the TS."

1 Introduction

Pg. 2, line 4: The expression "anti-nodal band" is not familiar. Is there anyone to use the expression?

Reply: We call the area where the vertical movement of an oscillating wave is greatest the "anti-nodal band". The word "antinode" can be found, for example, in Figure 5:3 of the monograph *Tides, Surges and Mean Sea-Level*, by David T. Pugh, 1987. (In the

USA it is called a “loop”; please see page 14 of *Tide and Current Glossary* by the Center for Operational Oceanographic Products and Services, NOAA. Since OS is a European journal, we have followed Pugh’s usage).

Pg. 2, line 18: Again use "an extended Taylor’s method

Reply: The phrase "the extended Taylor’s method" here has also been replaced with “an extended Taylor method”.

Pg. 2, lines 27-28: What is the basis of "The statement "the topographic step south of the TS acts as a permeable interface which can only partially reflect the incident wave and ...by nearly 180 degree at the step". Previous studies? If not, authors already assume partial reflection of northward and southward waves at the step. More careful writing is needed.

Reply: Yes, this is a result of previous studies. In R1, we have added a citation for Dean and Dalrymple (1984, Section 5.5) as a reference to the reader.

Pg. 4, line 9: Again use "an extended Taylor’s method.

Reply: The phrase "the extended Taylor’s method" here has also been replaced with “an extended Taylor method”.

3. An analytical model for the Taiwan Strait

3.1 Model configuration and solution

Pg. 7, lines 1-2: Reference for the friction coefficient formula is required.

Reply: We have cited “e.g., Chapter 8 of Dronkers, 1964” here and added “Dronkers, J. J.: *Tidal Computations in Rivers and Coastal Waters*, North-Holland Publishing Company, Amsterdam, 518 pp, 1964” to the reference section of R1.

Pg. 7, line 5: The expression "observed harmonic constants from the global tide model" is a little bit strange. It is computed results not observed values. Recommend to change the expression.

Reply: Since this global model is not numerically simulated but is based on satellite observations, we regard the model values as observations. For clarity, we have added the following sentence to the Introduction of R1: “Figure 2 displays the distribution of the M2 tidal constituent based on the global tidal model DTU10, which is constructed on the basis of multi-mission altimeter observations. Hereafter, we shall regard the DTU10 model results as observations.” It seems that the words “observation-based” are more accurate than “observed”, but we feel the former is too lengthy.

Pg. 7, line 6: In Figure 3(a) open boundary values appear to be somewhat different with Figure 2. Describe how the global model results were adopted to the analytical model.

Reply: The open boundary values are derived from the DTU10 model values through linear interpolation. The impression of the difference mentioned in this comment may be caused by the following two issues. (1) The rectangle shown in Fig. 1 in the OM is a sketch diagram and is not accurate. Now Fig. 1 has been replaced in R1 with a new figure, in which the rectangle is plotted at its exact location. (2) The other cause might be that we use different contour intervals for Fig. 2 and Fig. 3. To eliminate this impression, we redrew Fig. 2 with a finer interval (10 degrees) for phase-lags, as shown below.

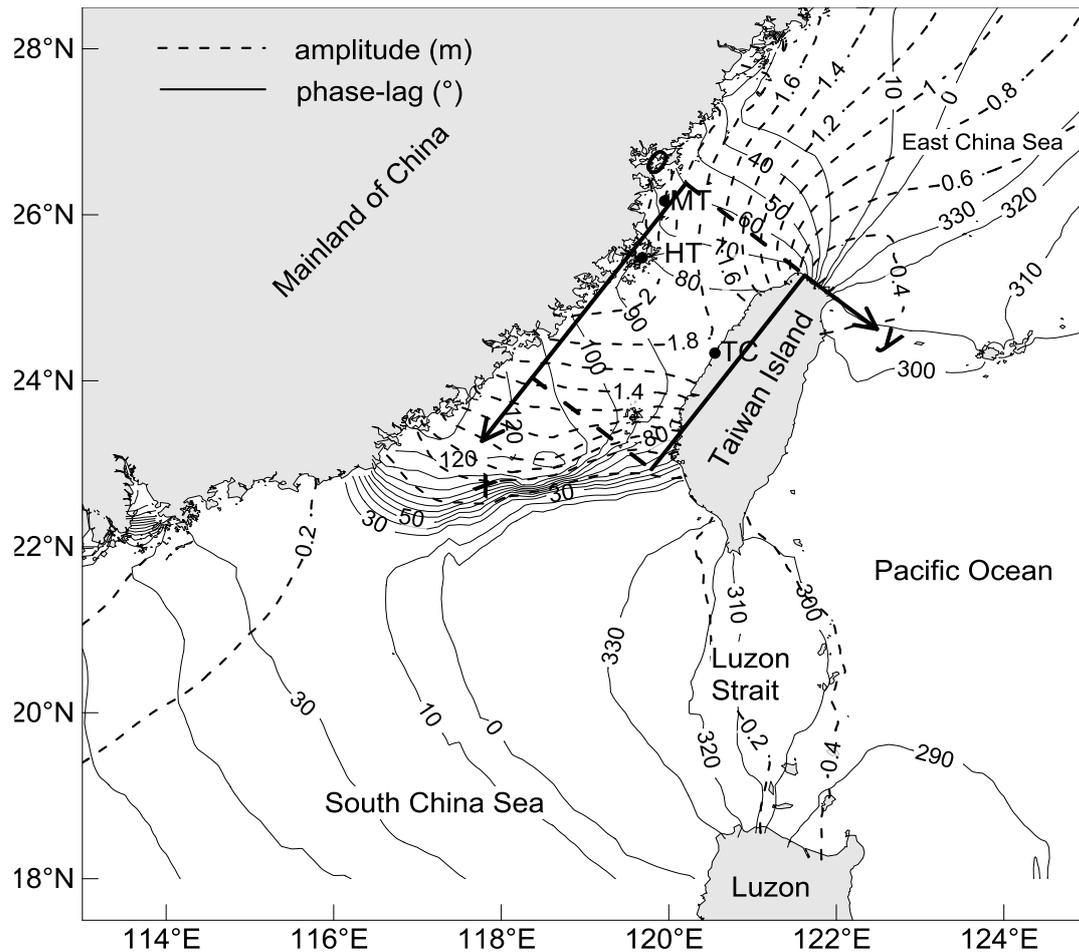


Fig. 2.1 The M_2 tidal system in the Taiwan Strait and its neighbouring area. This figure is the same as Fig. 2 of our manuscript but uses a finer contour interval (10 degrees) for phase-lags. (In the figure number, the first “2” represents the “Author’s Response to Referee #2”.)

Pg. 7, lines 15-16: The statement "that is, the wave propagates southward in the northeast area and propagate northward in the southeast area" may be incorrect in strict sense. Figure 4 shows in whole area there are southward and northward Kelvin waves. Rewriting is needed.

Reply: This statement refers to the tidal patterns given in Fig. 3. The propagation directions are shown with arrows in the following figure.

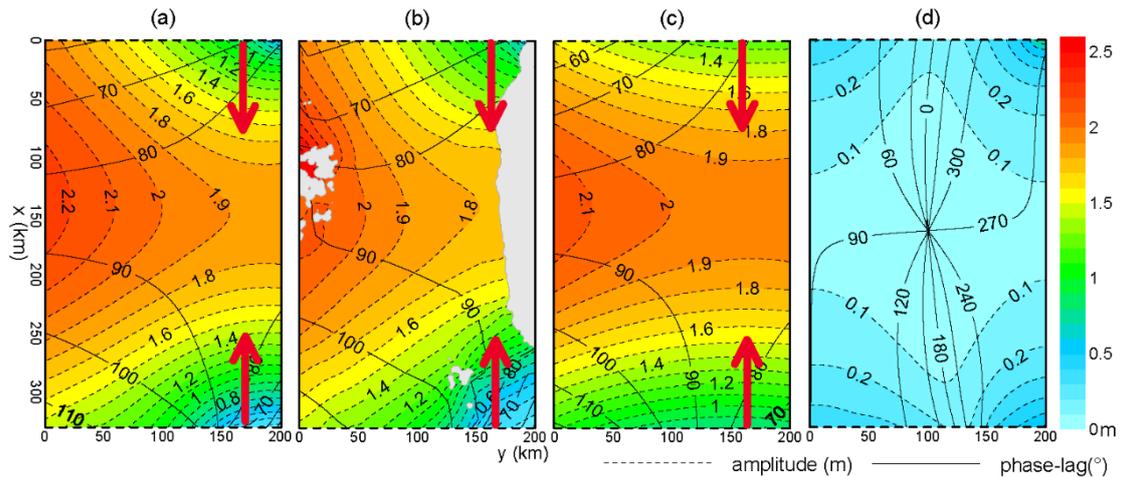


Fig. 2.2 Propagation directions of M2 tide in the north-eastern and south-eastern areas of the Taiwan Strait.

(In the figure number, the first “2” represents the “Author’s Response to Referee #2”.)

3.2 Kelvin waves and Poincare modes

Pg. 8, line 6: In Figure 3(a) open boundary values appear to be somewhat different with Figure 2. Describe how the global model results were adopted to the analytical model.

Reply: Please see the reply to the comment on Pg. 7, line 6 above.

Pg. 8, lines 9-10: The statement "the amplitude variation along the northern boundaryowing to the fact that the M2 tide is from the Pacific Ocean" is obscure. The northern boundary values of Fig. 3c and Fig.3a can be different from each other even though the M2 tide did not come from Pacific Ocean. More careful discussions are required.

Reply: To make the description clearer, the statement “This is owing to the fact that the M₂ tide is from the Pacific Ocean, its amplitude increases from the deeper outer shelf toward the shallower inner shelf” in the OM has been replaced with the statement “This shows that near the boundary, the Poincaré modes are of a certain importance. The existence of the Poincaré modes is related to the fact that the M₂ tide is from the Pacific Ocean; its amplitude increases from the deeper outer shelf toward the shallower inner shelf.” in R1.

Pg.8, lines 17-18: The statement "it is not a controlling factor" is too much definite. Obliqueness might be partly effective. Improve the statement.

Reply: We have changed “it is” to “it seems”.

4 Formation mechanism of the northward Kelvin wave in the Taiwan Strait

4.1 Reflection of the incident wave from the East China Sea at the topographic step

Pg. 9, line 17: "observation taken from the global tide model" needs to be changed as mentioned earlier.

Reply: The design of experiments 1-3 has been changed according to your comment on Pg. 14, line 1-2; these words have been deleted in R1. Please see the response to your comment on Pg. 14, line 1-2 below.

Pg. 10. 4: Include reference for "the resonant period of the ECS(13.7h)".

Reply: We have added “, obtained by Cui et al., 2015” after “13.7 h”.

5. Summary and discussion

Pg. 14, lines 1-2: Regarding the statement "the reflected wave is slightly weaker", authors implied that there is a partial reflection of southward KW at the step. It is noted that at the northern open boundary southward KW, northward KW and Poincare waves are all specified in Ex.1. If there is northward KW at the northern open boundary, there should be northward KW at the southern open boundary, regardless of topography. It is curious to know what will happen if only southward KW is imposed at the northern boundary in Ex.1. Additional experiment is recommended to clarify the partial reflection at the step. You may include results in appropriate place.

Reply: This is an important suggestion. According this comment, we have redesigned our experiments and replaced Figs. 5, 6, 7 and the related discussion with these new results. In particular, the statement describing the original experiments “As in the single basin solution, the open boundary condition (7) is used at the northern opening with values of $\hat{\zeta}$ equal to those taken from the global tidal model DTU10” in the OM has been replaced with the statement describing the redesigned experiments “The experimental design for area A is similar to that of Roos and Schuttelaars (2011): a southward Kelvin wave is specified to be identical to the single basin solution, as shown in Fig. 4a in the preceding section. The Poincaré modes trapped at the cross section $x = 0$ are neglected, while those trapped at the cross section $x = 400$ km are retained.” in R1.

Technical corrections

Pg. 1, line 29: Better to replace "M2 amplitudes" to "M2 tide".

Reply: We already use “tide” for the subject of this sentence, so it does not seem appropriate to use “tide” again.

Pg. 3, lines 5-6: Definition of phase lag needs to be added.

Reply: In R1, we have added “Solid lines represent Greenwich phase-lag (in degrees)” into this figure caption.

Pg. 11, line 2: In Figure caption, there is no (d).

Reply: Amended: “(d)” has been added.

Modification list

(According to Revision#1-Updated manuscript)

Page 1.

Line 3. The title has been replaced with “An analytical study of M_2 tidal waves in the Taiwan Strait using an extended Taylor method”.

Line 10. “are featured by” has been changed to “feature”, and “ M_2 ” has been changed to “semidiurnal lunar (M_2)”.

Line 10. (and other place: Page2. Line30) “The extended Taylor’s method” has been changed to “An extended Taylor method”.

Line 12. “but the Coriolis and bottom friction force” has been changed to “and the Coriolis force and bottom friction forces”.

Line 16. (and several other places: Lines 18, 21, 24; Page10. Lines 6, 9, 17; Page17. Lines 17, 18; Page18 Line 6) The word “can” has been deleted.

Line 18. “The superposition of Poincaré modes can further improve” has been replaced with “Inclusion of Poincaré modes further improves”.

Line 19. (and other place: Page7. Lines 13) “In order to” has been replaced with “To”.

Line 23. “The forcing at the Luzon Strait” has been changed to “Inclusion of the forcing”

Line 24. “and thus is of secondary importance to the M_2 tide in the TS” has been changed to “and the forcing is thus of some (but lesser) importance to the M_2 tide in the TS”

Line 28. (and several other places: Page2. Lines 1, 6; Page 10. Lines 8, 9, 12; Page 11. Lines 3, 4; Page 12. Lines 24, 27; Page 13. Line 4; Page 16. Line 5) The word “about” has been replaced with “approximately”.

Line 28. “mostly located” has been changed by “located mostly”.

Page 2.

Line 4. “The greatest amplitude by tidal gauge observation...” has been changed to “The greatest amplitude based on tidal gauge observations...”.

Line 5. “... is 1.73 m at Taichung, while that along the mainland coast is 2.10 m at Matsu” has been changed to “... is 1.73 m at Taichung and is 2.10 m at Matsu near the mainland coast”.

Lines 6-8. “... 20 km away from the coast, the satellite observation indicate that the greatest amplitude, exceeding 2.2 m, appears near Haitan Island, located south of Matsu (Fig. 2)” has been changed to “... 20 km away from the coast. Satellite observations indicate that the greatest amplitude appears near Haitan Island, located south of Matsu (Fig. 2), and exceeds 2.2 m”.

Lines 10-17. “which is called the asymmetry by Yu et al. (2015). The tides in the TS have attracted a great number of studies since 1980s” has been deleted, and “and this feature is called asymmetry by Yu et al. (2015). Compared to M_2 , which has maximum amplitude over 2.2 m, the amplitudes of the rest of the constituents are much smaller: the maximum amplitudes of S_2 , K_1 and O_1 observed at 11 coastal gauge stations reported by Jan et al. (2004b) are 0.66, 0.39 and 0.27 m, respectively. Figure 2 displays the distribution of the M_2 tidal constituent based on the global tidal model DTU10, which is constructed on the basis of multi-mission altimeter observations. Hereafter, we shall regard the DTU10 model results as observations. The tides in the TS have attracted a great number of studies since the 1980s. Most studies have attempted to establish accurate numerical models and thus to give accurate spatial structures of the tides and tidal currents in the strait.” has been added.

Line 19. “Most investigators have” has been changed to “It has been well”.

Lines 24, 25. The word “was” has been changed to “is”.

Line 24. The word “that” has been added.

Line 25. The word “made” has been replaced with “completed”, and the word “on” has been replaced with “of”.

Line 26. The word “a” has been added before “special focus on ...”.

Line 27. The word “modeling” has been replaced with “modelling”.

Line 28. The word “in” before “Jan et al. (2002)” has been replaced with “in”.

Line 30. “(see Section 2 for details)” has been added after “an extended Taylor method”.

Line 31. “... in the natural basin, and thus enable.” has been changed to “... in the natural basin. This enables”.

Page 3.

Line 4. (and several other places: Page 5. Lines 4, 8) “Taylor’s problem” has been replaced by “The Taylor problem”

Line 4. “(Hendershott and Speranza, 1971)” has been added after “...tidal dynamic problem”.

Lines 5-6. “(e. g., Roos and Velema , 2011, Table 1)” has been changed to “(e. g., Table 1 of Roos and Velema, et al., 2011)”.

Line 6. “all the studied” has been changed to “, most of the studied”.

Line 7. “, thus” has been added after “...the incident tidal wave”.

Line 8. The word “which” has been changed to “that”.

Line 9. “(see Section 5.5 of Dean and Dalrymple, 1984)” has been added.

Line 10. “the Taylor’s method” has been replaced by “Taylor’s method”.

Line 11. Figure 1. has been replaced with a new figure.

Page 4.

Line 5. (and other places: Page 9. Line 5) “Cotidal chart of M2 constituent” has been changed to “The M2 tidal system”, and “and its neighbouring area” has been added after “Taiwan Strait”.

Lines 5-6. “Chen and Andersen, 2011” has been corrected by “Cheng and Andersen, 2011”, and the sentence “Solid lines represent the Greenwich phase-lag (in degrees), and dashed lines represent amplitude (in metres).” has been added.

Line 8. The title “Solution Method” has been changed to “Model formulation and solution method”.

Line 9. The word “rectangular” has been added before “channel of uniform depth”.

Page 5.

Line 1. “Defant in 1925” has been changed to “In 1925, Defant”, and the word “introducing” has been replaced with “applying”.

Line 2. “version of” has been added before “Taylor’s problem”, and “Defant’ approach” has been corrected to “Defant’s approach”.

Line 4. “in the governing equations” has been added.

Line 6. “towards” has been changed to “towards”.

Lines 7. The sentence “The mechanism of the shift of the amphidromic point was also explained by Hendershott and Speranza (1971), in which the dissipation was assumed to occur at the closed end of the basin rather than during the wave propagation.” has been added.

Line 9. "...enabling solutions for open rectangular basins" has been changed to "enabling solutions accounting for the finite length of the basin".

Line 10. (and other place: Line 14) "Roos and Velema (2011)" has been corrected to "Roos et al. (2011)", and ", Roos and Schuttelaars (2011)" has been added.

Line 17. The equations have been corrected to:

$$\begin{cases} (\mu + i)u - \nu v = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial x} \\ (\mu + i)v + \nu u = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial y} \\ \zeta = \frac{ih}{\sigma} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \end{cases}$$

Lines 18-21. The sentence has been changed to "where t represents time; (x, y) are the Cartesian coordinates; (\tilde{u}, \tilde{v}) are the depth-averaged velocity components in the (x, y) directions; $\tilde{\zeta}$ is the tidal elevation; h is the water depth, assumed uniform; γ is the frictional coefficient, taken as a constant; $g = 9.8 \text{ ms}^{-2}$ is the acceleration due to gravity; and f is the Coriolis parameter, also taken as a constant due to the smallness of the study area."

Lines 21-24. The sentence "The equations in (1) are two-dimensional linearized shallow water equations on an f -plane with the momentum advection neglected. The equations are the same as those used in the work of Taylor (1922), except that the bottom friction is incorporated, as in Fang and Wang (1966) and Rienecker and Teubner (1980). When a monochromatic ..." has been added.

Line 24. (and several other places: Line 27; Page 6 Line5; Page 7 Line 14) "as follows:" has been added.

Line 25. The equation $(\tilde{\zeta}, \tilde{u}, \tilde{v}) = \text{Re}(\zeta, u, v)e^{i\sigma t}$ has been corrected to $(\tilde{\zeta}, \tilde{u}, \tilde{v}) = \text{Re}\{(\zeta, u, v)e^{i\sigma t}\}$.

Lines 26-27. The sentence has been changed to "where (ζ, u, v) are complex amplitudes of $(\tilde{\zeta}, \tilde{u}, \tilde{v})$, respectively, and σ is the angular frequency of the wave, $i \equiv \sqrt{-1}$. For this wave, the equations in (1) reduce as follows:" has been added.

Page 6.

Line 1. The equations (3) have been corrected to :

$$\begin{cases} (\mu + i)u - \nu v = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial x} \\ (\mu + i)v + \nu u = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial y} \\ \zeta = \frac{ih}{\sigma} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \end{cases}$$

Line 4. "Consider" has been changed to "Considering", the word "of" before B has been deleted, and "put" has been changed to "placed".

Line 5. The word "another" has been changed to "the other".

Line 14. ", each with a different uniform depth of h_A and h_B " has been added.

Line 16. "Eqs. (8) are ..." has been replaced with "The equations in (8) show the ...".

Line 17. The word "on" has been changed to "for".

Lines 16-19. Some ", " has been added.

Line 20. The word "of" has been deleted.

Line 22. The word "in" has been added before "(3)", and the word "of" has been added before "(4)".

Line 23. The typeface of " β " has been change to Italic.

Lines 23-24. The sentence "Note: the error occurred during their preparation of the manuscript and the correct expression was used in their computations." has been added.

Page 7.

Line 10. The equation (16) has been changed to :

$$s_n = (r_n^2 - \alpha^2 + \beta^2)^{\frac{1}{2}}$$

Line 11. “ $c = \sqrt{gh}$ ” has been changed to “with $c = \sqrt{gh}$ being the wave speed”, and “the” has been added before “absence”.

Line 12. The equation about Q^2 has been deleted, and “for each n ” has been added.

Line 17. The equation (18) has been changed to:

$$B_n = \frac{\nu(\mu+i)(\alpha^2-\beta^2)}{(\mu+i)^2 r_n^2 + \nu^2 s_n^2}$$

Line 24 “(a, b, κ_n , λ_n) are Coefficients related” has been changed to “Coefficients (a, b, κ_n , λ_n) are related”.

Page 8.

Line 2. The word “in” has been changed to “when”.

Line 4. “, and” has been added before “the total number of ...”.

Line 9. The word “fast” has been changed to “quickly”.

Line 11. The title “An analytical model for the Taiwan Strait” Application to the Taiwan Strait”.

Line 16. The word “put” has been changed to “place”, and the word “the” has been added before “x axes ...” and “y axes ...”.

Line 17. “an” has been added before “offshore direction”, and “by” has been changed to “to”.

Line 18. “But for short” has been changed to “However, to keep it short,”, and “by” has been changed to “to”.

Line 19. (and other place: Page 12. Line 13) “is taken” has been changed to “is taken as”.

Lines 20, 2. The typeface of “ γ ” has been change to Italic.

Lines 24-29. The sentence “In this study, we take $C_D = 0.0026$ and $U = 0.5$ m/s based on the numerical results of Fang et al. (1984), and then, $\mu = \gamma/\sigma$ is approximately equal to 0.15. From these parameter values, we can obtain the wavelength of the M_2 Kelvin wave as 1009 km. Since the basin width is smaller than half of the Kelvin wavelength, the Poincaré modes can only exist in a bound form (Godin, 1965; Fang and Wang, 1966). The e-folding length of decay of the lowest Poincaré mode is approximately 63 km, that is, the amplitude of this mode reduces to approximately 37% relative to its maximum value at a distance of 63 km away from the boundary. Equivalently, it may also reduce to approximately 20% relative to its maximum value at a distance of 100 km. The length scales of decay for higher order Poincaré modes are even shorter.” has been added.

Line 30 “In this study, the” has been added before “families of Poincaré modes...”, and “are” has been deleted.

Page 9.

Line 3. Figure 3. has been replaced with a new figure.

Line 5. (and other places: Line 8; Page 10. Line 4) The word “cotidal” has been changed to “tidal system”.

Line 5. “(in degrees)” has been added.

Line 8. (and other places: Page 12. Lines 24, 27; Page 17. Line 4) “the” has been added before “ M_2 ”.

Line 9. “It can be seen that although” has been changed to “Although”.

Line 11. "...has the features that the amplitudes are ..." has been changed to "...features ...", and "amplitudes" has been added after "greater".

Line 14. "..., wave nature, that ..." has been changed to "... wave nature,. That ...".

Line 15. "A highest amplitude band is present ..." has been changed to "The highest amplitude band present ...", and "..., appearing ..." has been changed to "..., and appears ...".

Page 10.

Line 3. "the" has been added before "Poincaré modes", and "It can be observed that the contribution of the Poincaré modes is much smaller than that of Kelvin waves" has been changed to "The contribution of the Poincaré modes is observed to be much smaller than that of the Kelvin waves".

Line 6. "inclusion of the" has been added before "Poincaré modes".

Line 10. "... with southward one stronger than the northward one" has been changed to "..., with the southward one being stronger than the one moving northward".

Lines 10-11. "Here both the Coriolis and frictional forces ..." has been changed to "Here, both the Coriolis force and the weaker northward wave ...".

Line 14. "to enhance" has been changed to "enhances".

Line 18. "This is owing ..." has been replaced with "This shows that near the boundary, the Poincaré modes are of a certain importance. The existence of the Poincaré modes is related ...".

Line 21. "at a uniform depth" has been added, and "for" has been added before "their difference".

Line 27. "but it is not a controlling factor" has been changed to "but it seems not to be a controlling factor".

Line 28. "the" has been added before "southward and northward Kelvin waves".

Line 31. "due to propagation direction" has been deleted, and the word "the" has been added before.

Page 11.

Line 1. The word "the" has been added before "opposite".

Line 4. (and other place: Line 9) . The word "them" has been changed to "the waves".

Lines 17-19. The sentence "In the preceding section, we have shown that the northward Kelvin wave is weaker than the northward wave on average, but they have a similar magnitude along the Taiwan coast. In this section, we will examine the formation mechanism of the northward Kelvin wave." has been added.

Line 19. "the" has been added before "reflection".

Line 20. "and" has been added before "another is an incident wave ...".

Page 12.

Lines 6-10. "As in the single basin solution, the open boundary condition (7) is used at the northern opening with values of equal to the observations taken from the global tidal model DTU10." has been changed to "The experimental design for area A is similar to that of Roos and Schuttelaars (2011): a southward Kelvin wave is specified to be identical to the single basin solution, as shown in Fig. 4a in the preceding section. The Poincaré modes trapped at the cross section $x = 0$ are neglected, while those trapped at the cross section $x = 400$ km are retained.".

Lines 13, 14. "Fig. 3a" has been changed to "Fig. 3c".

Line 20. The word "made" has been changed to "performed".

Line 21. "..., meaning ..." has been changed to "..., obtained by Cui et al., 2015). This means ...".

Line 29. “south” has been changed to “southern”, and “the” has been added before “single area case”.

Line 32. “and the” has been added before “reflected and transmitted Kelvin waves”.

Page 13.

Line 2. The number “0.63” has been changed to “0.64”

Line 3. The numbers “0.60” and “0.38” have been changed to “0.61” and “0.37”, respectively.

Lines 4-6. The typeface of variables have been corrected to Italic.

Line 7. “being” has been added before “taken into account...” and “ignored”.

Line 8. Figure 5. has been replaced with a new figure.

Page 14.

Line 2. “(d)” and “(in degrees)” has been add.

Line 6. “carry out” has been changed to “performed”.

Line 6. “the deep basin is” has been changed to “, the deep basin has”.

Line 7. “... radiate freely southward ...” has been changed to “... to freely radiate southward ...”.

Line 9. “... the deep basin is ...” has been changed to “... the deep basin has ...”.

Page 15.

Line 1. Figure 6. has been replaced with a new figure.

Line 5. The word “making” has been changed to “making”.

Lines 6, 7. “the Taylor’s model” has been changed to “Taylor’s mode”.

Line 7. “... not allow to open any part of sidewalls” has been changed to “... not allow any part of the sidewalls to open”.

Line 8. The word “simulate” has been changed to “solve”, and “still” has been deleted.

Page 16.

Lines 3-5. Some “,” have been added.

Line 4. “about” after “roughly” has been deleted.

Line 5. “Since a significant part of incident wave ...” has been changed to “Since a significant portion of the incident wave ...”.

Line 9. The word “southeastern” has been changed to “south-eastern”.

Line 10. Figure 7. has been replaced with a new figure.

Page 17.

Line 4. “In the present study we first establish an analytical model ...” has been changed to “In the present study, we first stablished an analytical model ...”.

Line 8. “carry” has been changed to “carried”.

Line 10. “From this study we obtain the following results” has been changed to “From this study, we have obtained the following results”.

Line 11. “by” has been added after “basically represented”.

Line 12. “being” has been added before “stronger than the latter”.

Line 15. The word “composition” has been changed to “superposition”.

Line 16. “the Coriolis force and friction ...” has been changed to “the Coriolis force and the weaker northward wave ...”.

Line 16. “Inclusion of the Poincaré nodes into the analytical model can improve the model result: ...” has been changed to “Inclusion of the Poincaré nodes in the analytical model improves the model results: ...”.

Page 18.

Line 1. The word “have” has been changed to “make”.

Line 4. “...in the strait, though ...” has been changed to “... in the strait. However, ...”.

Line 5. “the” has been added before “observations”.

Line 7. “TS tides” has been changed to “M2 tide in the TS”.

Line 11. “the water depths change from ...” has been changed to “the water depth changes from ...”.

Line 12. “... of continental slope there” has been changed to “... of the continental slope at that location”.

Lines 13-14. “...in the result for the magnitude of reflected wave” has been changed to “...in the results for the magnitude of the reflected wave”.

Line 17. “...,the National Natural Science Foundation of China (Grant No. 41706031)” has been added.

Lines 19-20. “The authors sincerely thank Dr. Huthnance and two 20 anonymous Referees for their constructive comments and suggestions, which are of great help in improving our study” has been added.

Lines 29-30. “Dronkers, J. J.: Tidal Computations in Rivers and Coastal Waters, North-Holland Publishing Company, Amsterdam, 518 pp, 1964.” has been added.

Page 19.

Line 1. “... , 8, 60-77” has been added.

Line 8. “Bruce, P” has been changed to “Paker, B”, and “ins.” has been corrected to “inc.”.

Lines 12-13. “Hendershott, M. C., and Speranza, A.: Co-oscillating tides in long, narrow bays; the Taylor problem revisited. Deep-Sea Research, 18, 959–980, 1971.” has been added.

Line 15. “212502” has been changed to “21”.

Line 31. “Roos P. C., and Schuttelaars H. M.: Influence of topography on tide propagation and amplification in semi-enclosed basins, Ocean Dynamics, 61, 21-38, 2011.” has been added.

Page 20.

Line 8. “Roos, P. C., Velema, J. J.” has been changed to “Roos, P. C., Velema, J. J., Hulscher, S. J. M. H., and Stolk, A.”.

Line 8. “... 39-450, 1985 (in Chinese with English abstract).” has been changed to “... 39-450 (in Chinese with English abstract), 1985.”.

An analytical study ~~onof~~ M_2 tidal ~~wavewaves~~ in the Taiwan Strait with ~~theusing an~~ extended Taylor's method

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10 **Abstract.** The tides in the Taiwan Strait (TS) ~~are featured by~~ feature large semidiurnal lunar (M_2) amplitudes. ~~The An~~ extended Taylor's method is employed in this study to provide an analytical model for the M_2 tide in the TS. The strait is idealized as a rectangular basin with a uniform depth, ~~but~~ and the Coriolis force and bottom friction ~~forces~~ are retained in the governing equations. The observed tides at the northern and southern openings are used as open boundary conditions. The obtained analytical solution, which consists of a stronger southward propagating Kelvin wave, a weaker northward

15 propagating Kelvin wave, and two families of Poincaré modes trapped at the northern and southern openings, agrees well with the observations in the strait. The superposition of two Kelvin waves ~~can~~ basically represent represents the observed tidal pattern, including an anti-nodal band in the central strait, and the cross-strait asymmetry (greater amplitudes in the west and smaller in the east) of the anti-nodal band. ~~The superposition~~ Inclusion of Poincaré modes ~~can~~ further improve improves the model result in that the cross-strait asymmetry can be better reproduced. ~~In order to~~ To explore the formation mechanism

20 of the northward propagating wave in the TS, three experiments are carried out, including the deep basin south of the strait. The results show that the southward incident wave ~~can be~~ reflected to form a northward wave by the abruptly deepened topography south of the strait, but the reflected wave is slightly weaker than the northward wave obtained from the above analytical solution, in which the southern open boundary condition is specified with observations. ~~The~~ Inclusion of the forcing at the Luzon Strait ~~can strengthen~~ strengthens the northward Kelvin wave in the TS, and the forcing is thus ~~is~~ of

25 secondary some (but lesser) importance to the M_2 tide in the TS.

1 Introduction

The Taiwan Strait (TS) is the sole passage connecting the East China Sea (ECS) and the South China Sea (SCS). The strait is ~~about~~ approximately 350 km long, 200 km wide and ~~mostly~~ mostly located mostly on the continental shelf with a mean depth of

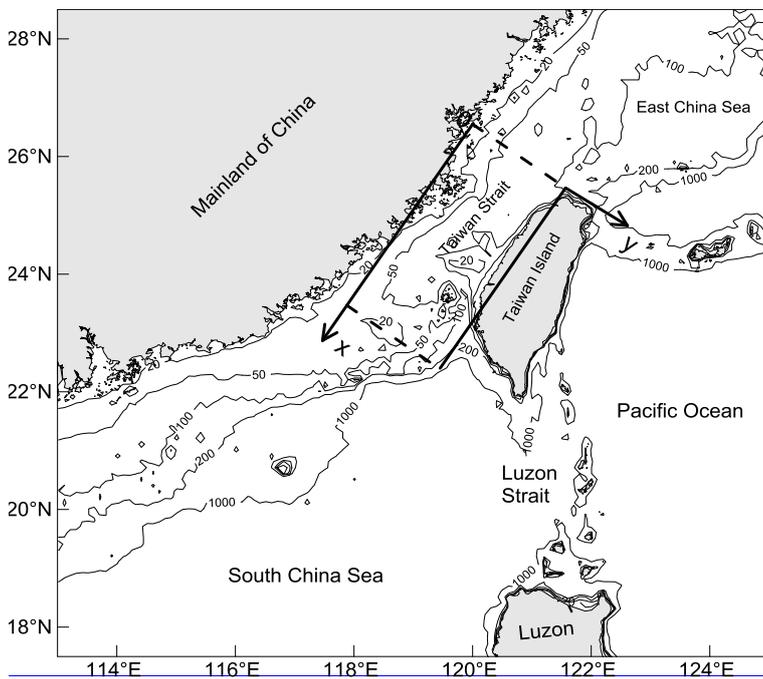
~~about~~approximately 50 metres. The bottom topography of the TS can be viewed as the extension of the ECS shelf in the north and becomes irregular in the south. The SCS deep basin is located south of the strait and is connected to the Pacific Ocean through the Luzon Strait (LS). An abrupt depth change is present between the TS and the SCS deep basin (Fig. 1).

The tides in the strait feature large M_2 amplitudes. The greatest amplitude ~~by~~based on tidal gauge ~~observation~~observations along the western Taiwan coast, reported by Jan et al. (2004b)), is 1.73 m at Taichung, ~~while that along the mainland coast~~ and is 2.10 m at Matsu ~~near the mainland coast~~. Matsu is an island located ~~about~~approximately 20 km away from the coast, ~~the satellite observation indicates~~. Satellite observations indicate that the greatest amplitude, ~~exceeding 2.2 m~~, appears near Haitan Island, ~~which is~~ located south of Matsu (Fig. 2-), ~~and exceeds 2.2 m~~. Thus, the tidal regime of the M_2 constituent has an anti-nodal band near the cross-strait line from Haitan to Taichung, with greater amplitudes in the west and smaller in the east, ~~which is called the asymmetry by Yu et al. (2015)~~. The tides in the TS have attracted a great number of studies since ~~1980s~~ and this feature is called asymmetry by Yu et al. (2015). Compared to M_2 , which has maximum amplitude over 2.2 m, the amplitudes of the rest of the constituents are much smaller: the maximum amplitudes of S_2 , K_1 and O_1 observed at 11 coastal gauge stations reported by Jan et al. (2004b) are 0.66, 0.39 and 0.27 m, respectively. Figure 2 displays the distribution of the M_2 tidal constituent based on the global tidal model DTU10, which is constructed on the basis of multi-mission altimeter observations. Hereafter, we shall regard the DTU10 model results as observations. The tides in the TS have attracted a great number of studies since the 1980s. Most studies have attempted to establish accurate numerical models and thus to give accurate spatial structures of the tides and tidal currents in the strait (Yin and Chen, 1982; Fang et al., 1984; Ye et al., 1985; Lü et al., 1999; Lin et al., 2000; Lin et al., 2001; Jan et al., 2002; Jan et al., 2004a; Jan et al., 2004b; Zhu, et al., 2009; Hu et al., 2010; Zeng et al., 2012; Yu et al., 2015; Yu et al., 2017). ~~Most investigators have~~It has been well recognized that the semidiurnal tides in the TS ~~mainly~~consist ~~mainly~~ of two oppositely propagating waves, one from north to south and another from ~~the~~ south to north. In particular, Fang et al. (1984, 1999) suggested that the semidiurnal tidal motion in the TS was maintained mainly by the energy flux from the ECS and partly by that from the SCS. Jan et al. (2002, 2004) further noticed that the southward propagating wave could be reflected when encountering the sharply deepened bottom topography south of the strait, and suggested that the reflected wave ~~was~~is the main component of the northward propagating wave, and ~~that~~ the contribution of the SCS ~~was~~is negligible. Yu et al. (2015) ~~made~~completed an extensive numerical study ~~on~~of the formation of the M_2 tide in the strait with ~~a~~ special focus on the asymmetric nature in the cross-strait direction.

The existing studies almost all employed data analysis and numerical ~~modeling~~modelling, except that some simple dynamical analyses were performed using one-dimensional solutions to explain the model results ~~in~~by Jan et al. (2002) and Yu et al. (2015). The purpose of the present study is to establish two-dimensional analytical models using ~~the~~an extended Taylor's method. (see Section 2 for details). In the analytical models, the classical Kelvin waves and Poincaré modes in idealized basins are used to approximately represent the tides in the natural basin, ~~and thus enable~~. This enables us to estimate the strengths of the southward and the northward waves, to reveal the role of each classical wave in the formation of

the tides in the strait, and to clarify how the waves are generated. In particular, we can roughly estimate the relative importance of the reflected wave at steep topography versus the incident wave from the LS in the formation of the northward Kelvin wave in the TS.

The Taylor's problem is a classical tidal dynamic problem: (Hendershott and Speranza, 1971). Since his pioneering work, Taylor's method has been subsequently developed and applied to many sea areas (e.g., Table 1 of Roos and Velema, et al., 2011, Table 1). In the previous applications all, most of the studied basins have a closed end that can almost perfectly reflect the incident tidal wave, thus closely retaining the phase of the tidal elevation. In contrast, the topographic step south of the TS acts as a permeable interface which that can only partially reflect the incident wave, and furthermore, the elevation phase of the reflected wave is changed by nearly 180° at the step: (see Section 5.5 of Dean and Dalrymple, 1984). Therefore, the strait is also a locality of particular interest for the application of the Taylor's method.



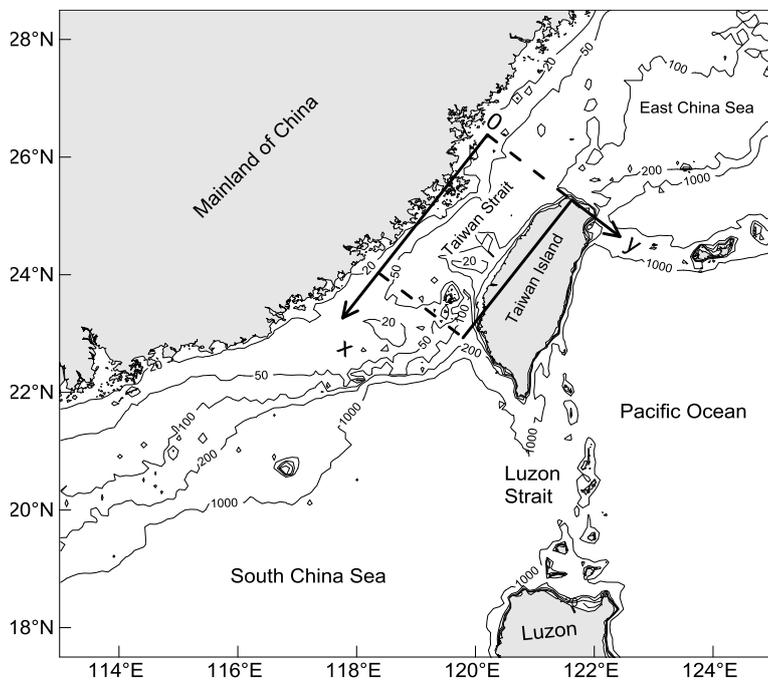
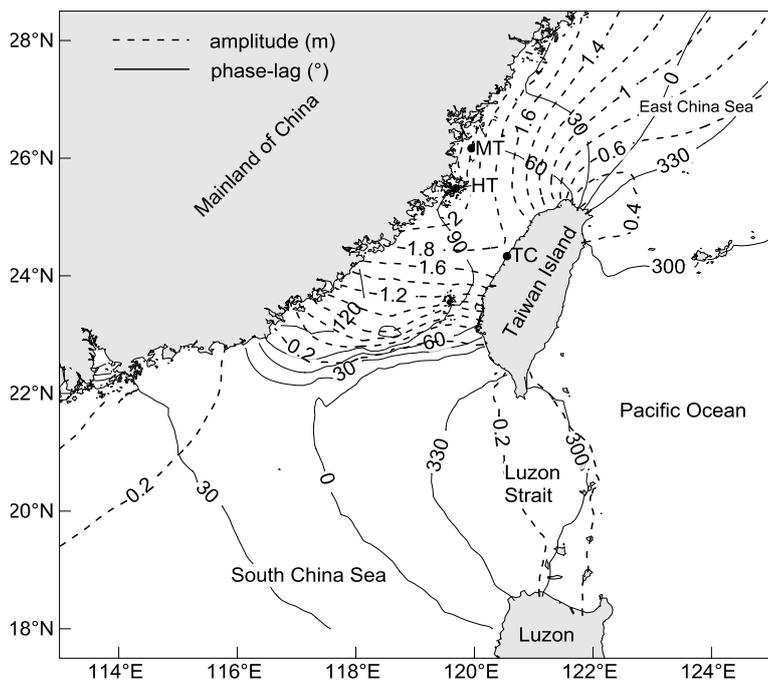


Figure 1. Bathymetric chart of the Taiwan Strait and its neighbouring area; the rectangle indicates the idealized model basin representing the Taiwan Strait. Isobaths are in metres (based on ETOPO1 from the US National Geophysical Center).



5 **Figure 2.** Cotidal chart of the M_2 constituent tidal system in the Taiwan Strait and its neighbouring area (based on DTU10, see [Chen Cheng](#) and Andersen, 2011). Solid lines represent the Greenwich phase-lag (in degrees), and dashed lines represent amplitude (in metres). (MT-Matsu, HT-Haitan, TC-Taichung)

2 Solution Method Model formulation and solution method

Taylor (1922) first presented an analytical solution for tides in a semi-infinite rotating rectangular channel of uniform depth
 10 to explain the existence of amphidromic systems in gulfs. His solution showed that the tide in such a channel can be represented by the superposition of an incident Kelvin wave, a reflected Kelvin wave and a family of Poincaré modes

trapped near the closed end. [In 1925](#), Defant ~~in 1925~~ simplified Taylor's solution approach by [introducing applying](#) the collocation method (see Defant, 1961, pp. 213-215). In the original [version of](#) Taylor's problem, as well as the Defant's approach, the friction and open boundary condition were left out of consideration. Fang and Wang (1966) and Rienecker and Teubner (1980) extended the Taylor's problem by taking friction into consideration [in the governing equations](#). The introduction of friction can explain why the amphidromic point in the northern hemisphere shifts from the central axis [towardstoward](#) the right, as seen from the closed end and looking seawards. [The mechanism of the shift of the amphidromic point was also explained by Hendershott and Speranza \(1971\), in which the dissipation was assumed to occur at the closed end of the basin rather than during the wave propagation](#). Fang et al. (1991) further extended the Taylor's problem by introducing the open boundary condition, enabling solutions [for open rectangular basins accounting for the finite length of the basin](#). Jung et al. (2005) ~~and~~, Roos and [VelemaSchuttelaars \(2011\)](#), and [Roos et al. \(2011\)](#) further extended the Taylor's method to model tides [within](#) multiple rectangular basins. ~~Solution~~[The solution](#) method used in the present study is basically the same as Fang et al. (1991), but with minor correction and generalization, as [was](#) done in studies of Jung et al. (2005) ~~and~~, Roos and [VelemaSchuttelaars \(2011\)](#), and [Roos et al. \(2011\)](#). The analytical method initiated by Taylor and developed afterward is called [thean](#) extended Taylor's method in this paper.

15 2.1 Governing equations and boundary conditions

The governing equations used in this study are as follows:

$$\begin{cases} \frac{\partial \tilde{u}}{\partial t} - f\tilde{v} = -g\frac{\partial \tilde{\zeta}}{\partial x} - \gamma\tilde{u} \\ \frac{\partial \tilde{v}}{\partial t} + f\tilde{u} = -g\frac{\partial \tilde{\zeta}}{\partial y} - \gamma\tilde{v} \\ \frac{\partial \tilde{\zeta}}{\partial t} = -h\left[\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y}\right] \end{cases} \begin{cases} \frac{\partial \tilde{u}}{\partial t} - f\tilde{v} = -g\frac{\partial \tilde{\zeta}}{\partial x} - \gamma\tilde{u} \\ \frac{\partial \tilde{v}}{\partial t} + f\tilde{u} = -g\frac{\partial \tilde{\zeta}}{\partial y} - \gamma\tilde{v} \\ \frac{\partial \tilde{\zeta}}{\partial t} = -h\left[\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y}\right] \end{cases} \quad (1)$$

where t represents time; (x, y) [are](#) the Cartesian coordinates; (\tilde{u}, \tilde{v}) [are](#) the [depth-averaged](#) velocity components in [the](#) (x, y) directions; $\tilde{\zeta}$ [is](#) the tidal elevation; h [is](#) the water depth, assumed uniform; γ [is](#) the frictional coefficient, taken [as a](#) constant; $g = 9.8 \text{ ms}^{-2}$ is the acceleration due to gravity; [and](#) f is the Coriolis parameter, also taken as a constant due to [the smallness of the study area](#). [The equations in \(1\) are two-dimensional linearized shallow water equations on an \$f\$ -plane with the momentum advection neglected. The equations are the same as those used in the work of Taylor \(1922\), except that the bottom friction is incorporated, as in Fang and Wang \(1966\) and Rienecker and Teubner \(1980\).](#) When ~~one cosine~~ [a monochromatic](#) wave is considered, $(\tilde{\zeta}, \tilde{u}, \tilde{v})$ can be expressed as [follows](#):

$$25 \quad (\tilde{\zeta}, \tilde{u}, \tilde{v}) = \text{Re}(\zeta, u, v)e^{i\sigma t} \{(\zeta, u, v)e^{i\sigma t}\} \quad (2)$$

where (ζ, u, v) [are complex amplitudes of](#) $(\tilde{\zeta}, \tilde{u}, \tilde{v})$, respectively, and σ is the angular frequency of the wave, $i \equiv \sqrt{-1}$.

For this wave ~~Eqs.~~ [the equations in](#) (1) reduce ~~to~~ [as follows](#):

$$\begin{cases} (\mu + i)u - \nu v = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial x} \\ (\mu + i)u + \nu u = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial y} \\ \zeta = \frac{i\hbar}{\sigma} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \end{cases} \quad (3)$$

$$\begin{cases} (\mu + i)u - \nu v = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial x} \\ (\mu + i)v + \nu u = -\frac{g}{\sigma} \frac{\partial \zeta}{\partial y} \\ \zeta = \frac{i\hbar}{\sigma} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \end{cases} \quad (3)$$

in which $\mu = \frac{\gamma}{\sigma}$, $\nu = \frac{f}{\sigma}$.

Considering a rectangular basin with two parallel sidewalls of length L and with a width B , we placed the x axis along a sidewall and the y axis perpendicular to the x axis and pointing to the other sidewall. Thus, the basin is confined by $x = 0, L$ and $y = 0, B$, respectively. The boundary conditions along the sidewalls are taken as follows:

$$v = 0 \text{ at } y = 0 \text{ and } y = B \quad (4)$$

within $x \in (0, L)$. Along the cross sections, $x = 0$ and $x = L$, various choices of boundary conditions are applicable depending on the problem concerned:

$$u = 0, \text{ if the cross section is a closed boundary;} \quad (5)$$

$$u = \pm \sqrt{\frac{g}{h}} \zeta, \text{ if the free radiation in the positive/negative } x \text{ direction occurs on the cross section;} \quad (6)$$

$$\zeta = \hat{\zeta}, \text{ if the tidal elevation is specified as } \hat{\zeta} \text{ along the cross section;} \quad (7)$$

and/or;

$$\zeta_A = \zeta_B \text{ and } u_A h_A = u_B h_B, \text{ if the cross section is a connecting boundary of two basins } A \text{ and } B, \text{ each with a different uniform depth of } h_A \text{ and } h_B. \quad (8)$$

The equations in (8) show the matching conditions accounting for sea level continuity and volume transport continuity, respectively. The individual equations (5) to (8), or their combination, may be used as boundary conditions on the cross sections. Strictly speaking, Eq. (6) is valid only in the frictionless case; if friction is considered, it contains an error of the order of μ^2 , and there is a phase difference between u and ζ (Fang and Wang, 1966). In the present study we still use the form of Eq. (6) due to smallness of the value of μ .

2.2 General solution and collocation method

The governing equations in (3) have only the following four forms satisfying the sidewall boundary condition of (4) (see Fang et al., 1991, an error in the equation for $\beta\beta$ in their paper has been corrected here. Note: the error occurred during their preparation of the manuscript and the correct expression was used in their computations):

$$\begin{cases} v_1 = 0 \\ u_1 = -a \exp(\alpha y + i\beta x) \\ \zeta_1 = \frac{\beta}{\sigma} h a \exp(\alpha y + i\beta x) \end{cases} \quad (9)$$

$$\begin{cases} v_2 = 0 \\ u_2 = b \exp[-(\alpha y + i\beta x)] \\ \zeta_2 = \frac{\beta}{\sigma} hb \exp[-(\alpha y + i\beta x)] \end{cases} \quad (10)$$

$$\begin{cases} v_2 = 0 \\ u_2 = b \exp[-(\alpha y + i\beta x)] \\ \zeta_2 = \frac{\beta}{\sigma} hb \exp[-(\alpha y + i\beta x)] \end{cases} \quad (10)$$

$$\begin{cases} v_3 = \sum_{n=1}^{\infty} \kappa_n \sin r_n y \exp(-s_n x) \\ u_3 = \sum_{n=1}^{\infty} \kappa_n (A_n \cos r_n y + B_n \sin r_n y) \exp(-s_n x) \\ \zeta_3 = \frac{ih}{\sigma} \sum_{n=1}^{\infty} \kappa_n (C_n \cos r_n y + D_{1,n} \sin r_n y) \exp(-s_n x) \end{cases} \quad (11)$$

and

$$5 \begin{cases} v_4 = \sum_{n=1}^{\infty} \lambda_n \sin r_n y \exp[-s_n(L-x)] \\ u_4 = \sum_{n=1}^{\infty} \lambda_n (A'_n \cos r_n y + B'_n \sin r_n y) \exp[-s_n(L-x)] \\ \zeta_4 = \frac{ih}{\sigma} \sum_{n=1}^{\infty} \lambda_n (C'_n \cos r_n y + D'_n \sin r_n y) \exp[-s_n(L-x)] \end{cases} \quad (12)$$

where

$$\alpha = \frac{v}{(1-i\mu)^{\frac{1}{2}}} k \quad (13)$$

$$\beta = (1-i\mu)^{\frac{1}{2}} k \quad (14)$$

$$r_n = \frac{n\pi}{B} \quad (15)$$

$$10 \quad s_n = (r_n^2 - Q^2)^{\frac{1}{2}} (r_n^2 - \alpha^2 + \beta^2)^{\frac{1}{2}} \quad (16)$$

in which $k = \sigma/c$ is the wave number, with $c = \sqrt{gh}$ being the wave speed of the Kelvin wave in the absence of friction

and $c = \sqrt{gh}$ is the wave speed. In Eq. (16), $Q^2 = \frac{(1-i\mu)^2 - v^2}{4-i\mu} k^2$; s_n has two complex values for each n , and here, we

choose the one that has a positive real part. In order to satisfy Eqs. equations in (3), (A_n, B_n, C_n, D_n) and (A'_n, B'_n, C'_n, D'_n)

should be as follows:

$$15 \quad A_n = \frac{[(\mu+i)^2 + v^2] r_n s_n}{(\mu+i)^2 r_n^2 + v^2 s_n^2} \quad (17)$$

$$B_n = \frac{v(\mu+i)Q^2}{(\mu+i)^2 r_n^2 + v^2 s_n^2} \frac{v(\mu+i)(\alpha^2 - \beta^2)}{(\mu+i)^2 r_n^2 + v^2 s_n^2} \quad (18)$$

$$C_n = r_n - s_n A_n \quad (19)$$

$$D_n = -s_n B_n \quad (20)$$

$$A'_n = -A_n \quad (21)$$

$$20 \quad B'_n = B_n \quad (22)$$

$$C'_n = C_n \quad (23)$$

$$D'_n = -D_n \quad (24)$$

Eqs. (9) and (10) represent Kelvin waves propagating in the $-x$ and x directions, respectively; Eqs. (11) and (12)

represent two families of Poincaré modes trapped at the cross sections $x = 0, 0, L$, respectively. Coefficients $(a, b, \kappa_n, \lambda_n)$

25 are coefficients related to amplitudes and phases of Kelvin waves and Poincaré modes. These coefficients are to be

determined by boundary conditions.

The collocation method is convenient in when determining the coefficients $(a, b, \kappa_n, \lambda_n)$. The calculation procedure can be as follows. First, we truncate the family of Poincaré modes, Eqs. (11) and (12), at the N -th order, so that the number of undetermined coefficients for Poincaré modes is $2N$; and the total number of undetermined coefficients (plus those for Kelvin waves) is thus $2N + 2$. To determine these unknowns, we take $N + 1$ equally spaced $N + 1$ dots, called collocation points, located at $y = \frac{B}{2(N+1)}, \frac{3B}{2(N+1)}, \dots, \frac{(2N+1)B}{2(N+1)}, \dots, \frac{(2N+1)B}{2(N+1)}$ on both the cross sections $x = 0$ and L . At these points, one of the boundary conditions given by Eqs. (5)-(8) should be satisfied. This yields $2N + 2$ equations. By solving this system of equations, we can obtain $2N + 2$ coefficients $(a, b, \kappa_n, \lambda_n)$. Since the high-order Poincaré modes decay from the boundary very fast quickly (e.g., Godin, 1965), it is generally necessary to retain only a few lower order terms.

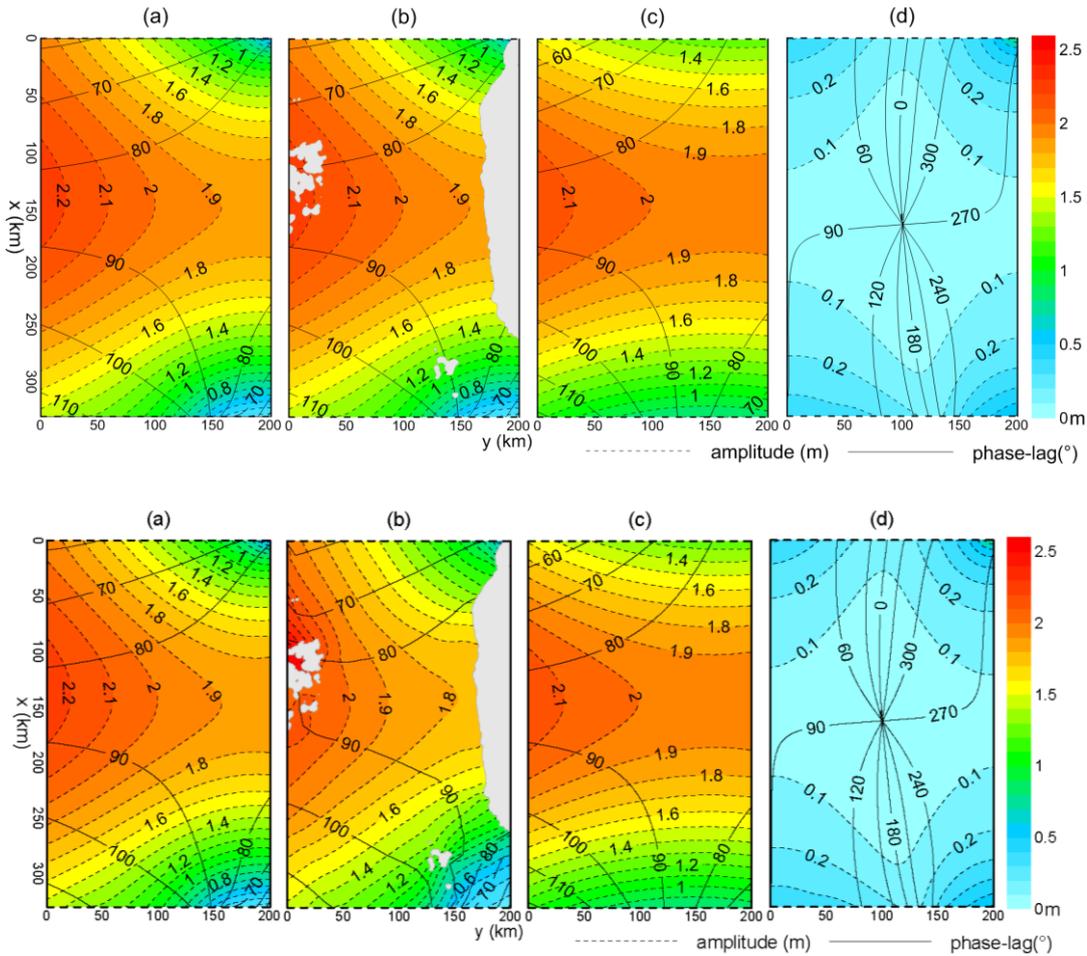
3 An analytical model for Application to the Taiwan Strait

3.1 Model configuration and solution

In this section, we will first establish an idealized analytical model for the TS. The strait is idealized as a rectangular basin with two sidewalls roughly along the China mainland and Taiwan coastlines, as shown in Fig. 1. The width and length of the model domain are taken as $B=200$ km and $L=330$ km, respectively. The depth is taken as $h = 52-52$ m, a mean depth calculated based on ETOPO1. We put place the origin of the coordinates at the northernmost corner of the rectangle, the x axis along the mainland coast, and the y axis in an offshore direction. The axis of the strait is toward the south by to southwest. But for However, to keep it short, we will hereafter simply use “south” to refer “south by to southwest”, and similarly for other directions. The Coriolis parameter f is taken 0.594 as $0.594 \times 10^{-4} \text{s}^{-1}$, corresponding to a latitude of $\varphi = 24^\circ \text{N}$. The angular frequency of the M_2 tide is $1.4052 \times 10^{-4} \text{s}^{-1}$. The friction coefficient γ can be estimated from the relation $\gamma = C_D \left(\frac{8}{3\pi} \right) \frac{U}{h}$, in which C_D and U represent the drag coefficient and amplitude of the M_2 tidal current, respectively. If (e.g., Chapter 8 of Dronkers, 1964). In this study, we take C_D equal to $=0.0026$ and U equal to $=0.5$ m/s, based on the numerical results of Fang et al. (1984), and then $\mu = \gamma/\sigma$, $\mu = \gamma/\sigma$ is approximately equal to 0.15. The From these parameter values, we can obtain the wavelength of the M_2 Kelvin wave as 1009 km. Since the basin width is smaller than half of the Kelvin wavelength, the Poincaré modes can only exist in a bound form (Godin, 1965; Fang and Wang, 1966). The e-folding length of decay of the lowest Poincaré mode is approximately 63 km, that is, the amplitude of this mode reduces to approximately 37% relative to its maximum value at a distance of 63 km away from the boundary. Equivalently, it may also reduce to approximately 20% relative to its maximum value at a distance of 100 km. The length scales of decay for higher order Poincaré modes are even shorter.

In this study, the families of Poincaré modes are truncated at $N = 19, 19$ and 20 collocation points are set along both the

northern and southern open boundaries. The boundary condition (7) is employed with the values of $\hat{\zeta}$ equal to the observed harmonic constants from the global tide model DTU10 (Cheng and Anderson, 2011).



5 **Figure 3.** ~~Cotidal~~Tidal system charts for the M_2 constituent: (a) Present analytical model; (b) Observed distribution based on DTU10; (c) Contribution of Kelvin waves; (d) ~~contribution~~Contribution of Poincaré modes. Solid lines represent Greenwich phase-lag; (in degrees); dashed lines represent amplitude (in metres).

The obtained analytical solution of the M_2 constituents is shown in Fig. 3a. For comparison, the observed M_2 ~~cotidal~~tidal system chart based on DTU10 is also shown in Fig. 3b. ~~It can be seen that although~~ Although the complicated bottom topography and the irregular coastlines are greatly simplified, the analytical model still agrees well with the observation. The observed tidal regime ~~has the features that the amplitudes are~~ significantly greater amplitudes along the mainland coast than along the Taiwan coast, showing the cross-strait asymmetry. The phase-lags near the mainland coast increase from north to south, showing a progressive wave nature, while those near the middle Taiwan coast have only small changes, showing a standing wave nature, ~~that. That~~ is, the wave propagates southward in the northeast area and propagates northward in the southeast area. ~~The highest amplitude band is present~~ presents roughly along the cross section of $x \approx 150$ km, ~~appearing and appears~~ as an anti-nodal band. The phase-lags in this band range from 80° to 90° . These features have all been reproduced in the analytical model.

3.2 Kelvin waves and Poincaré modes

To reveal the relative importance of the Kelvin waves and Poincaré modes in the model, the superposition of two Kelvin waves is given in Fig. 3c, and that of ~~the~~ Poincaré modes is given in Fig. 3d. ~~It can be observed that the~~ ~~The~~ contribution of the Poincaré modes is ~~observed to be~~ much smaller than that of ~~the~~ Kelvin waves. The ~~oortidal system~~ chart constructed using ~~a~~ superposed Kelvin wave alone (Fig. 3c) resembles the complete model (Fig. 3a) and the observation (Fig. 3b) quite well, though the ~~inclusion of the~~ Poincaré modes ~~can improve~~ ~~improves~~ the model to a certain degree. From Fig. 3a we can see that the difference between ~~the~~ highest amplitude on the west sidewall and that on the east sidewall in the anti-nodal band is ~~about~~ ~~approximately~~ 0.4 m, while the corresponding difference shown in Fig. 3c is ~~about~~ ~~approximately~~ 0.2 m. Thus ~~about~~ ~~a~~, ~~approximately~~ half of the cross-strait asymmetry ~~can be~~ ~~is~~ explained by the superposition of two oppositely propagating Kelvin waves, with ~~the~~ southward one ~~being~~ stronger ~~than~~ ~~the~~ ~~one moving~~ northward ~~one~~. Here, both the Coriolis ~~force~~ and ~~frictional force~~ ~~the weaker northward wave~~ are the major factors. The superposition of Poincaré modes in this band has an amplitude of ~~about~~ ~~approximately~~ 0.1 m on both sides, and has ~~a~~ nearly the same phase-lag as the superposed Kelvin wave on the west and a nearly opposite phase-lag to the superposed Kelvin wave on the east. Therefore, the superposed Poincaré mode plays a role to increase ~~the~~ amplitudes in the west and reduce the amplitudes in the east and hence ~~to enhance~~ ~~enhances~~ the asymmetry. The superposed Poincaré mode has nearly the same contribution to the cross-strait asymmetry as the superposed Kelvin wave.

From ~~the~~ comparison, we ~~can~~ find that the amplitude variation along the northern boundary in Fig. 3c is less than that in Fig. 3a. This ~~shows that near the boundary, the Poincaré modes are of a certain importance. The existence of the Poincaré modes is~~ ~~owing~~ ~~related~~ to the fact that the M_2 tide is from the Pacific Ocean; its amplitude increases from the deeper outer shelf toward the shallower inner shelf. This amplitude variation cannot be completely represented by the superposed Kelvin wave ~~at a uniform depth~~, and a superposed Poincaré mode is necessary to compensate ~~for~~ their difference. The situation at the southern boundary is similar. The distribution of the superposed Poincaré mode in the anti-nodal band is clearly related to those at the northern and southern openings (Fig. 3d). Yu et al. (2015) suggested that the orientation of the topographic step south of the strait was not perpendicular to the strait axis, but had an angle. This might cause the reflected wave to propagate toward the mainland coast and thus amplify the tides there. The present solution indicates that the obliqueness of the topographic step south of the TS may also play a role in the formation of the cross-strait asymmetry, as suggested by Yu et al. (2015), but it ~~is~~ ~~seems~~ not ~~to be~~ a controlling factor.

The obtained analytical solution enables us to see the magnitudes and characteristics of both ~~the~~ southward and northward Kelvin waves. These two oppositely propagating waves, which correspond to Eqs. (9) and (10) respectively, are displayed separately in Figs. 4a and 4b. From Fig. 4a, we see that the phase-lag of the southward wave increases from north to south ~~due to propagation direction~~. The amplitude decreases from north to south due to friction and from west to east due to ~~the~~

Coriolis effect. The characteristics of the northward wave are the opposite. The area mean amplitude of the southward wave is 1.18 m, while that of the northward wave is 0.84 m, smaller than the former by 0.34 m. Along the western sidewall, the amplitudes of the southward wave range from aboutapproximately 1.4 m to 1.6 m, while those of the northward wave range from aboutapproximately 0.6 to 0.7 m, so that, thus, the superposition of themthe waves is dominated by the former and appears as a southward progressive wave. Around the cross section $x \approx 150$ km, the phase-lags of the southward and northward waves are nearly equal, between 80° and 90° ; thus, Thus, the superposed tides have the greatest amplitudes, equal to the sum of the amplitudes of these two waves, which exceeds 2.1 m, as already seen in Fig. 3c. Along the eastern sidewall, however, the differences of in amplitudes of the southward and northward waves are much smaller, so that and thus, the superposition of themthe waves tends to appear as a standing wave. Around the point $x \approx 150$ km, the phase-lags of the southward and northward waves are also nearly equal, thus, Thus, the amplitude of the combined tide is also relatively large, equal to the sum of the amplitudes of these two waves, but now it is only slightly greater than 1.9 m, which is smaller than the corresponding value at the western sidewall.

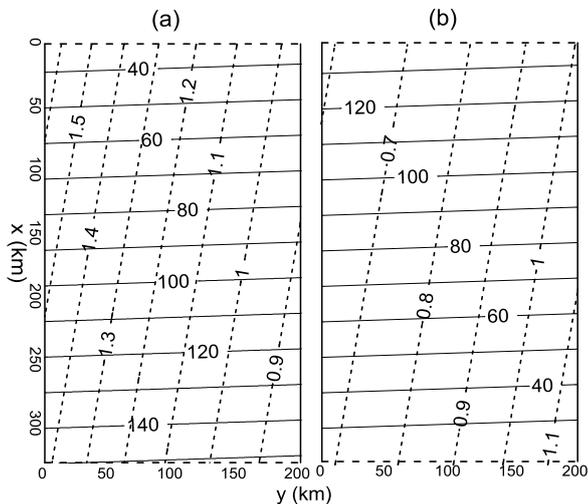


Figure 4. Southward (a) and northward (b) propagating Kelvin waves. Solid lines represent Greenwich phase-lag; dashed lines represent amplitude (in metres).

4 Formation mechanism of the northward Kelvin wave in the Taiwan Strait

In the preceding section, we have shown that the northward Kelvin wave is weaker than the northward wave on average, but they have a similar magnitude along the Taiwan coast. In this section, we will examine the formation mechanism of the northward Kelvin wave. There are two possible origins for the northward Kelvin wave in the TS. One is the reflection of the southward wave at sharply deepened topography; and another is an incident wave from the Luzon Strait propagating toward the TS. In the following, we examine their respective contributions by using the extended Taylor's models.

4.1 Reflection of the incident wave from the East China Sea at the topographic step

Three experiments have been carried out to explore the formation mechanism of the northward Kelvin wave in the TS. The

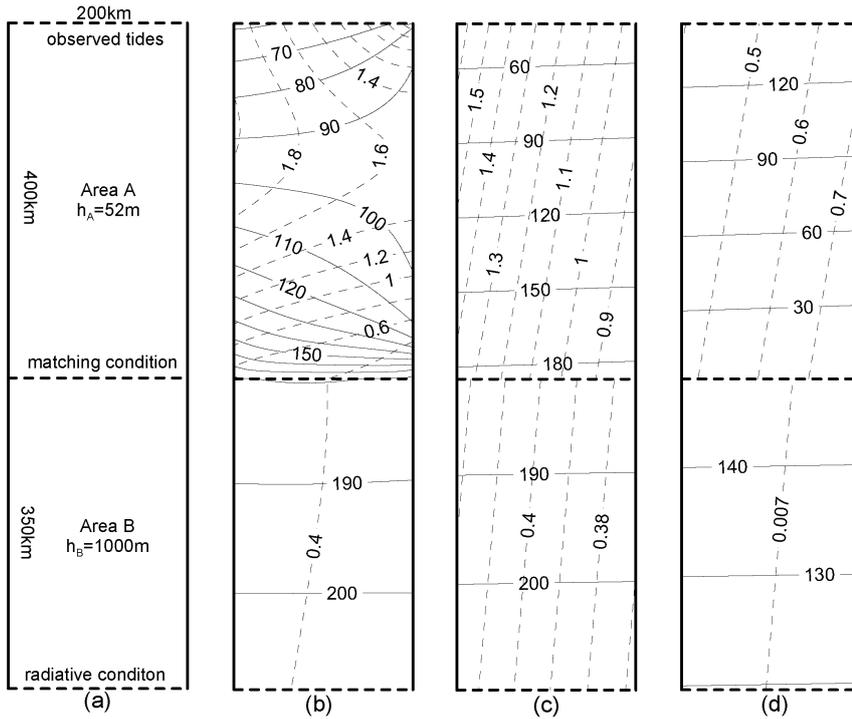
first experiment (denoted as Ex. 1) has [the](#) model geometry shown in Fig. 5a. The TS is represented by area A, with the width and depth equal to the above single area model. Since the topographic step is located away from the southern boundary of the single area model domain (Fig. 1), we extend the length of the area to 400 km. [The area Area B](#) represents the deep basin south of the topographic step, and the water depth of the deep basin is taken [as](#) 1000 m, [as was](#) done in Jan et al. (2002 and 2004). The purpose of this experiment is to examine the effect of the topographic step in reflecting the incident wave from the ECS. ~~As in the single basin solution, the open boundary condition (7) is used at the northern opening with values of $\hat{\zeta}$ equal to the observations taken from the global tidal model DTU10.~~ [The experimental design for area A is similar to that of Roos and Schuttelaars \(2011\): a southward Kelvin wave is specified to be identical to the single basin solution, as shown in Fig. 4a in the preceding section. The Poincaré modes trapped at the cross section \$x = 0\$ are neglected, while those trapped at the cross section \$x = 400\$ km are retained.](#) The matching condition (8) is applied at the connecting boundary of areas A and B, and the radiative condition (6) is used at the southernmost opening.

Figure 5b displays the solution of Ex.1. It can be seen that the basic pattern of the tidal regime is similar to that of the single area model solution shown in Fig. [3a3c](#). In particular, there is also an anti-nodal band near ~~$x = -x = 150$ km~~, though the amplitudes in this band produced by this experiment are smaller than those given in Fig. [3a3c](#). The smallest amplitudes appear along the connecting cross section, showing [that](#) a nodal band exists there. Therefore, the anti-node is located ~~about~~[approximately](#) 250 km away from the topographic step. The wavelength of [the](#) M_2 tide in a channel of [a](#) uniform depth of 52 m is equal to 1009 km, [and](#) so, the distance between the anti-node and the topographic step is equal to one quarter of the wavelength. This result further implies that if the channel were 500 km long, resonance would occur. However, Taiwan Island is ~~about~~[approximately](#) 380 km long, and is not able to support a resonance for [the](#) M_2 constituent. In fact, the resonant period of the TS is 13.5 h, according to the experiments ~~made~~[performed](#) by Cui et al. (2015), which is almost the same as one of the resonant periods of the ECS (13.7 h), ~~meaning, obtained by Cui et al., 2015~~. [This means](#) that the tidal response in the TS is not independent, but rather closely related to the tides in the ECS.

The southward and northward Kelvin waves obtained from Ex. 1 are shown in Figs. 5c and 5d, respectively. Comparison of these figures with Figs. 4a and 4b indicates that in area A, the southward wave is identical, but the northward wave from Ex. 1 is weaker. For the area ~~$x = -x = 0$ to 330 km, $y = -$~~ [and](#) $y = 0$ to 200 km, the area mean amplitude of the northward Kelvin wave is 0.57 m, which is smaller than the single area model value by 32%. In area B, the amplitudes of the transmitted southward Kelvin wave are ~~about~~[approximately](#) 0.4 m, and those of [the](#) northward wave are negligible. An important difference in [the](#) co-phase-lag distributions is that Figs. 3a-c show a northward propagation along the ~~south~~[southern](#) part of the eastern sidewall, while Fig. 5b does not have such a feature. This is because in [the](#) single area case, the amplitudes of the northward Kelvin wave are greater than those of the southward Kelvin wave in this area (Figs. 4a, 4b), while in Ex. 1, this situation does not occur (Figs. 5c, 5d).

The relative magnitudes of the incident, [and the](#) reflected and transmitted Kelvin waves can be evaluated by comparing

their amplitudes along the connecting cross section at $x = -x = 400$ km. The sectional mean amplitudes for the incident, reflected and transmitted waves, H_i, H_r, H_t, H_i, H_r and H_t, H_t are 1.06, 0.6364 and 0.40 m_i respectively (Figs. 5c, 5d). Thus, the ratios $H_r/H_i, H_r/H_i$ and H_t/H_t are equal to 0.6061 and 0.3837 respectively. The corresponding values based on the theory ignoring the earth's rotation can be calculated from $\frac{H_r}{H_i} = \frac{1-\rho}{1+\rho}$ and $\frac{H_t}{H_t} = \frac{2\rho}{1+\rho}$ with $\rho = \sqrt{h_A/h_B} \sqrt{h_A/h_B}$ (e.g., Dean and Dalrymple, 1984, p. 144). Substitution of the present model depths into these equations yields $H_r/H_i = H_r/H_i = 0.63$ and $H_t/H_t = H_t/H_t = 0.37$. This indicates that the magnitude of the reflected waves in the two-dimensional case with the earth's rotation being taken into account is smaller than that based on the theory with the earth's rotation being ignored.



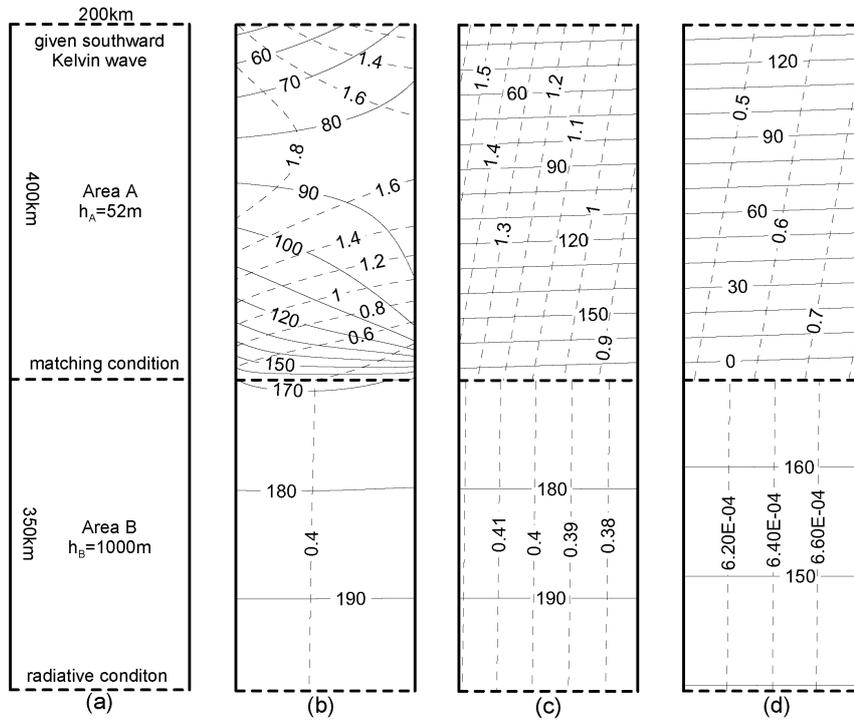


Figure 5. Model domain and boundary conditions of Ex. 1 (a); solution of Ex. 1 (b); southward Kelvin waves (c); northward Kelvin waves (d). Solid lines represent Greenwich phase-lag; (in degrees); dashed lines represent amplitude (in metres).

4.2 Influence of the shelf region southwest of the Taiwan Strait

5 From Fig. 1, we can see that there is a narrow shelf along the mainland coast. To simulate the effect of the narrow shelf on the tides in the TS, we carry-outperformed the second experiment, numbered Ex. 2. In this experiment, the deep basin ishas moved 60 km eastward, allowing the tides in the shallow basin to freely radiate freely-southward as shown in Fig. 6a. The radiative condition (6) is retained along the southernmost opening. The results of Ex. 2 are given in Fig. 6. It can be seen that the tides in area A have only small changes, though the deep basin ishas moved 60 km eastward. Observable changes can

10 only be found in area B where the tidal amplitudes are slightly reduced.

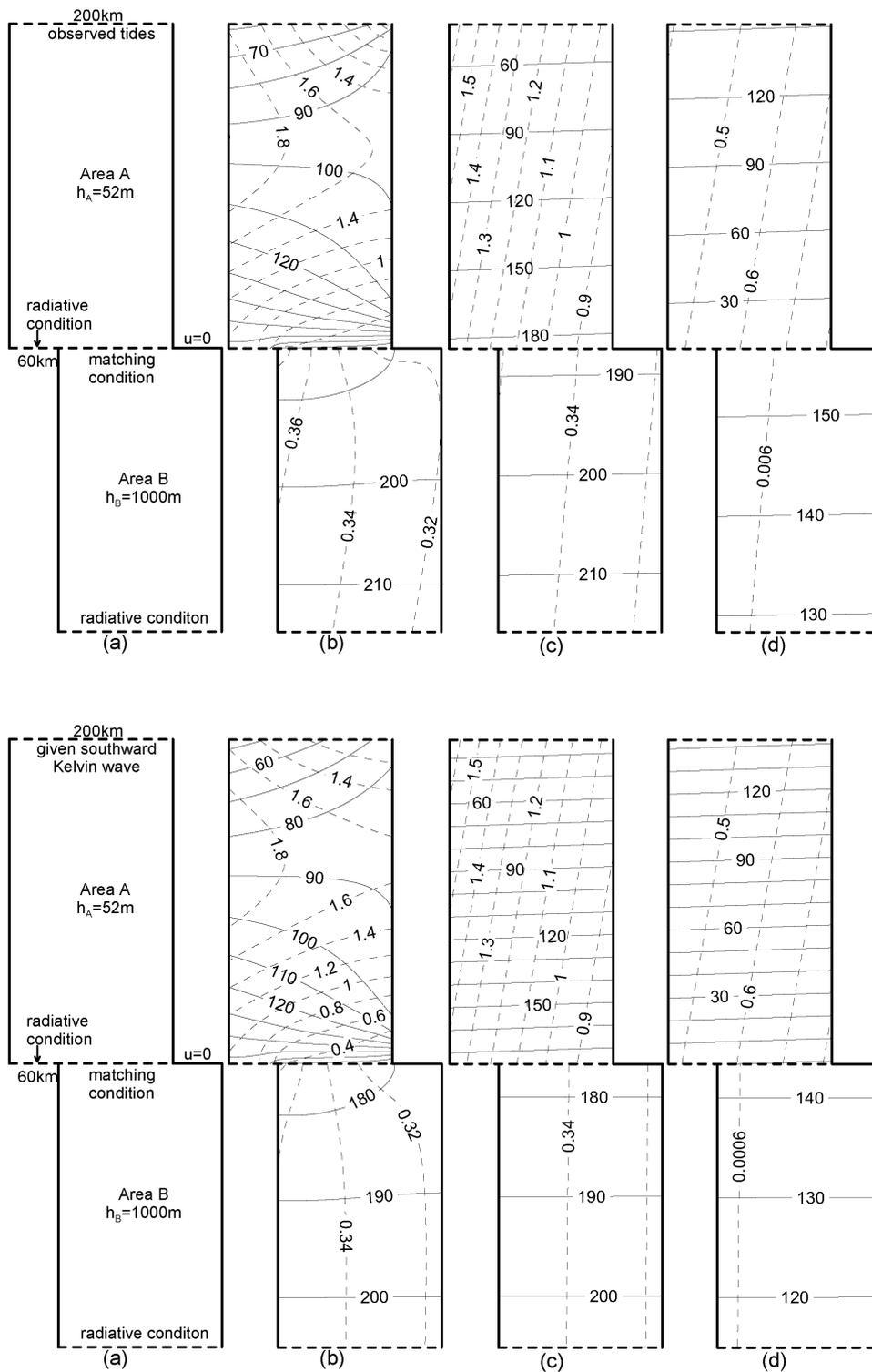
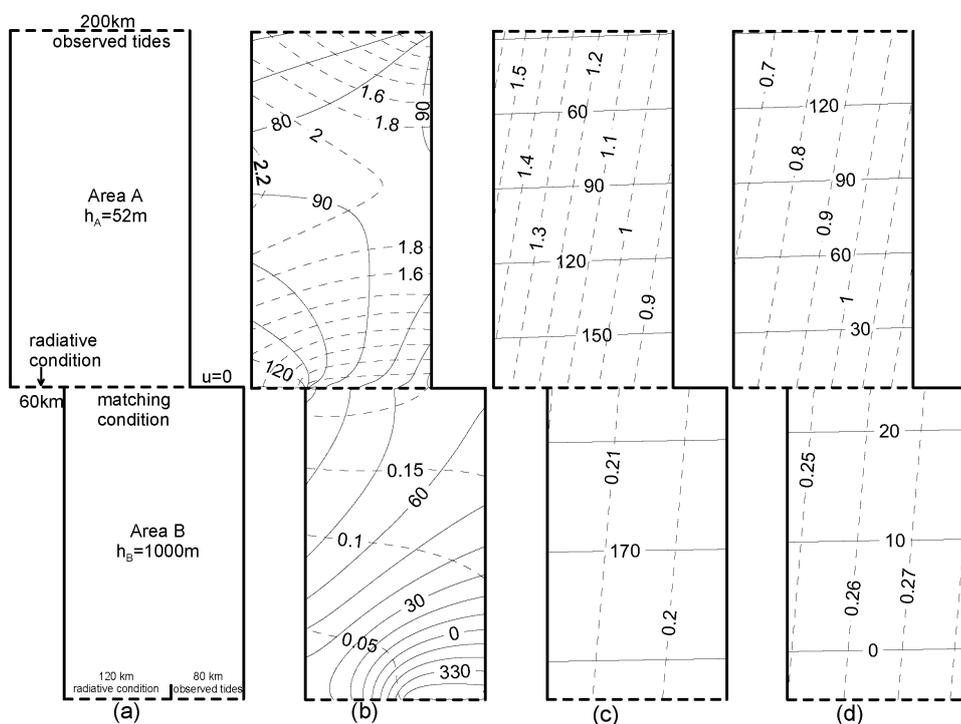


Figure 6. Same as Figure 5, but for Ex. 2.

4.3 Influence of the Luzon Strait forcing

5 The purpose of ~~makingperforming~~ the third experiment, numbered Ex. 3, is to consider ~~the~~ tidal input from the LS. The major difficulty in including the LS input in ~~the~~ Taylor's model for the TS is that the LS has a meridional orientation, while ~~the~~ Taylor's model does not allow ~~to open~~ any part of ~~the~~ sidewalls ~~to open~~. Here, we will use a rather crude model to ~~simulatesolve~~ this issue. We ~~still~~ use the same model domain as Ex. 2, but the radiative boundary condition (6) is retained

only for the west segment of the southernmost opening, and the boundary condition (7) is applied to the remaining east segment of the opening. From Fig. 1, we can see that the cross section from the mainland shelf to the LS is much longer than the width of the LS, ~~thus~~. Thus, in our model, we take the lengths of the west and east segments to be 120 km and 80 km, respectively, as shown in Fig. 7a. In addition, from Fig. 2, we observe that the tidal amplitude along the LS is roughly about 0.2 m, and the phase-lag is ~~about~~ approximately 310° . Since a significant ~~part~~ portion of the incident wave from the LS propagates toward the SCS deep basin (e.g., Fang et al., 1999; Yu. et al., 2015), we use a 0.1 m amplitude and 310° phase-lag as an open boundary condition for the east segment of the southernmost opening in Ex. 3. The model results are given in Figs. 7b to 7d. From Fig. 7b, we can see that the amplitudes of the tide in area A now become greater than the results of Ex. 2 (Fig. 6b), and a northward propagating character can be seen in the ~~southeastern~~ south-eastern portion of area A. These improvements can be attributed to the increased amplitudes of the northward Kelvin wave (Figs. 6d, 7d).



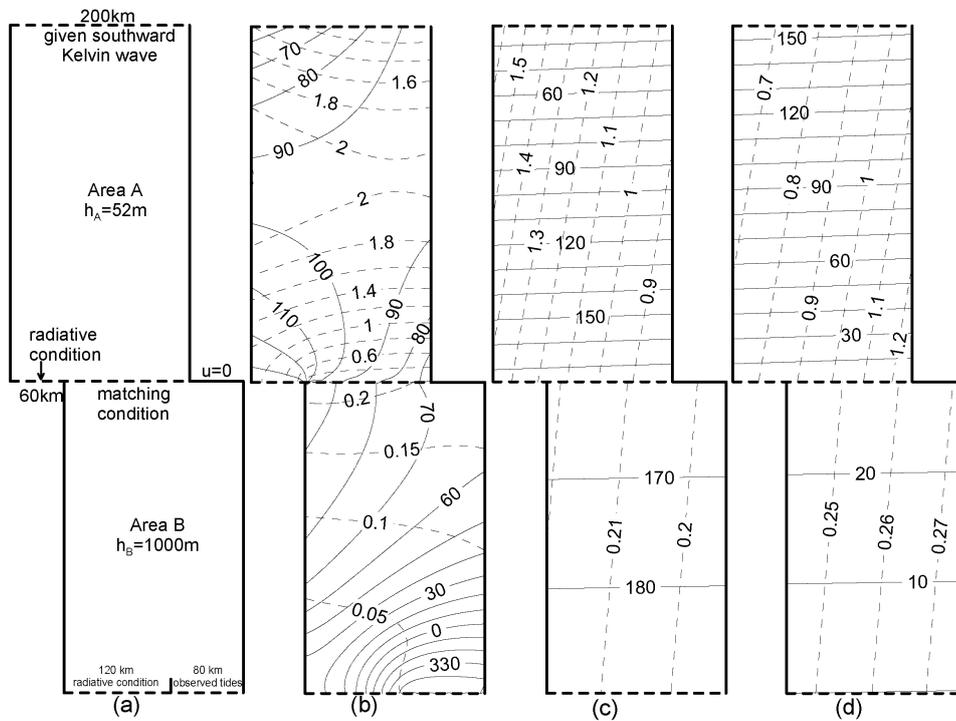


Figure 7. Same as Figure 5, but for Ex. 3.

5 Summary and discussion

In the present study, we first ~~establish~~ established an analytical model for the M_2 tide in the TS using the extended Taylor² method. The superiority of the analytical solution is that the tides can be decomposed into a southward Kelvin wave, a northward Kelvin wave, and two families of Poincaré modes, providing a deeper insight into the dynamics of the tides in the area. Though the coastlines and bottom topography are greatly simplified, the model-produced pattern resembles the observed tidal regime quite well. We then ~~carry~~ carried out several experiments to examine the formation mechanism of the northward propagating wave, especially the roles of the abruptly deepened bottom topography south of the TS and the tidal forcing in the LS in the formation of the northward wave. From this study, we ~~obtain~~ have obtained the following results.

The M_2 tide in the TS can be basically represented by the superposition of a southward propagating and a northward propagating Kelvin waves, with the former being stronger than the latter. The superposed Kelvin waves give an anti-nodal band near the cross-strait transection, roughly from Haitan Island to Taichung. The maximum amplitude on the mainland side is greater than that on the Taiwan side, showing the cross-strait asymmetry. Therefore, the observed features can be reproduced by the ~~composition~~ superposition of a stronger southward propagating and a weaker northward propagating Kelvin wave. In this regard, the Coriolis force and ~~friction~~ the weaker northward wave play essential roles.

Inclusion of the Poincaré modes ~~into~~ in the analytical model ~~can improve~~ improves the model ~~result~~ results: the east to west increase in amplitudes along the northern and southern openings ~~can be~~ is better reproduced; and in particular, the Poincaré

modes ~~havemake~~ approximately the same contribution as the Kelvin waves to the cross-strait asymmetry in the anti-nodal band.

The reflection of the southward wave at the abruptly deepened topography south of the TS is a major contribution to the formation of the northward propagating wave in the strait, ~~though. However,~~ the reflected wave is slightly weaker than that obtained from the analytical solution with open boundary conditions determined by ~~the~~ observations. Inclusion of the tidal forcing at the LS ~~can strengthenstrengthens~~ the northward Kelvin wave in the TS, and thus improves the model result. This indicates that the LS forcing is of some (but lesser) importance to the M₂ tide in the TS tides.

The analytical solutions can help us to understand the dynamics of tidal motion in the TS, but there are some limitations. For example, the LS is located on the east side of the study area, while ~~the~~ Taylor's model does not allow ~~for~~ a forcing on the sidewalls, ~~and~~ thus, we are bound to let a part of southern opening represent the LS (Fig. 7a). In addition, we have assumed that the water ~~depths changedepth changes~~ from 52 m to 1000 m immediately at the connecting cross section without considering the existence of ~~the~~ continental slope ~~thereat that location~~. The obliqueness of the orientation of the topography step relative to the cross-strait direction is also ignored. These approximations will induce uncertainty in the ~~resultresults~~ for the magnitude of ~~the~~ reflected wave.

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