

Authors' response to OS-2017-03: "Trapped Planetary (Rossby) Waves Observed in the Indian Ocean by Satellite Borne Altimeters" by De-Leon and Paldor.

The comments of the two reviewers were helpful to us in clarifying the focus of the paper and improving its presentation. An acknowledgement of their input was added in the Acknowledgement section. We were also happy to read both reviewers' comments regarding the importance of the paper and their wish to see the paper published. We hope that both reviewers find the current version of the paper suitable for publication in OS. Our detailed response to the particular comments listed below in red and the changes that were implemented in the text following these comments are outlined (when relevant) in green.

Anonymous Referee #1

Received and published: 16 March 2017

The manuscript presents estimates of the propagation velocity of Rossby waves at the Indian Ocean, south of Australia, using different methods, and compare them with current theories. The estimates are obtained from satellite data.

In general, the manuscript is very clear and the results present a coherence between observations and the theory. However, there are some few questions that would need more explanations regarding, mainly, the estimates close to coast. This issues follow below.

- 1) In page 4, line 9, it is not clear why the mean ssh from 1993 was removed. Why not removing the average for the whole period?

The removal of the mean SSH of 1993 is done by Aviso (probably as part of their calibration process) Since this is a technical point that relates to the way Aviso generates the data they distribute we refer the reader to Aviso's web site for further details on the way data is produced instead of providing some of these details in our paper. Text will be modified accordingly

See deleted sentences in the first paragraph on (new) P. 4.

- 2) In page 6 the authors mentioned an increase of the amplitude close to the coast. Although they state the topography is not an issue for that, it is hard to assume it. First, close to the coast, satellite altimeter sampling is not very accurate. Secondly, and most important, that region seems to include the slope and the shelf, where other processes are very important, such as topographic waves and continental shelf waves. The main dynamics is not determined by the theory presented by the authors in that region.

Topographic Rossby waves as well as all other high-frequency waves are all filtered out by our 35 days averaging. The accuracy by Aviso data near the coast has been improved in the 20-year product used in this study (and in any case it should not be expected lead to an increase in the signal). The text will be modified to reflect this issue.

The requested information has been added in the first paragraph on (new) P. 7

- 3) In page 14 the authors did not consider other explanations, as mentioned before, associated with the topography. The same theory is applied for a region with very distinct characteristics and as such should be analyzed. I would recommend removing the analysis of the data between the coast and 35oS.

Done. Figure 5 will be removed in the revised version.

- 4) The authors present 3 different methods to estimate the propagation speed, and consider all of them with the same reliability. It would be a good contribution if the differences among those methods could be discussed and some suggestion about the method that could better estimate the propagation speed in the domain could be given.

The interested reader can find a detailed description of the various methods, including a comparison between them in De-Leon and Paldor, 2016 in *Acta Astronautica* (which is referenced in the manuscript) and repeating it in the present work is an unwanted digression.

- 5) It would also be a good contribution a discussion about which of the current theories could better explain the observed propagation velocities.

A detailed comparison between the applicability of harmonic and Trapped theories to observations is given in (new) Figure 5 (i.e. Figure 6 of the original manuscript) and in the discussion section. A more detailed review of all the theories that have been suggested in the last 20 years and their success in explaining these particular observations is beyond the scope of this paper in which we focus on the single case where a zonal boundary exists (over a sizeable range of longitudes) where the trapped wave theory is relevant.

Anonymous Referee #2

Received and published: 23 March 2017

The Authors use sea surface height anomaly data from satellite altimeters to identify Rossby Waves at the Great Australian Bight. They employ three different methods to estimate the phase speed of the waves, and then compare their results with both the classic harmonic theory and a more recent trapped waves theory. The study main conclusion is that the observed phenomena are Trapped Rossby Waves. The manuscript is well written, however I have some issues with the content.

Major reviews

- 1) The trapped wave theory is very recent and I'm not sure the community is well aware of it. Since it is of utmost importance for this work, I'm not satisfied by the provided explanation in page 9. I do not expect the authors to derive the full theory, but a more detailed explanation is due. Although, I'd like to see the explanation to start with equation 4 from Gildor et al. (2016), I think it would be appropriate if it started at least with equation 6. The first paragraph of page 9 was impossible to follow without Gildor et al. (2016) open beside it.

Done. Sub-section 4.1 is extended in the revised version to include more details on the trapped wave theory and the correspondence between the present work and Gildor et al (2016).

New equation 3 on P. 9 is (nearly) identical with equation 6 of Gildor et al. (2016). The discussion of the theory is now provided in the text before and after this equation.

- 2) This comment is more general about the classical linear theory phase speed used in this study. Yes, it is slower than the observed one, as was found in many other studies. However, all the theoretical advances made by Killworth and Blundel (one of their papers is cited) managed to bring the theoretical linear speed closer to the observed one. Watanabe et al. (2016, Ocean Dyn.) showed that by using an effective- β , as proposed by Herrmann and Krauss (1989, JPO), the linear theory is good enough to explain the observations at least in the tropics. Although, the tropics is not the authors' case, the main idea is that it is not a surprise that the linear theory fails to reproduce the observations, if it was calculated without considering other parameters (as in the extended theory of Killworth and Blundel, or parameterized in the effective- β as in Watanabe et al.). Also, the linear theory is for free waves, and Rossby waves can be forced by Ekman pumping. In my opinion, there should be some discussion about these matters.

Done. Requested references and discussions added in the revised version.

See discussion in 3rd paragraph on new P. 2 and the revised list of references on P. 19

Minor reviews

- 1) Since it is a study about Rossby waves, I missed some more recent references, such as O'Brien et al. (2013, Remote Sens. Environ.) and Polito and Sato (2015, J. Geophys. Res.), among others. There is also quite a few works from Dr. Angela Maharaj that could be useful as reference. However, the reference I missed the most is Potemra (2001, J. Geophys. Res.), since it is one of the first studies of Rossby waves in the Indian Ocean.

Done. Additional references and discussions added in the revised version.

See first paragraph on P. 2, paragraph above Figure 2 on P. 7 and new list of references

- 2) The described method based on variance (page 9) is very similar to the one used by Polito and Liu (2003, J. Geophys. Res.), the only difference is that they used in filtered data, however that does not change the method, so I think they should be cited. Also, they showed how this method is superior to the traditional Radon transform, so the relevant comparison is between the Radon transform based on variance and the 2DFFT one.

Done. Reference added in the revised version. Indeed the superiority of the variance-based Radon transform over the traditional (sum-based) counterpart in our AA paper agrees with the conclusions of Polito and Liu (2003).

See first paragraph on P. 5 and list of references

- 3) The explanation about the boundaries of the trapped wave theory is made in the introduction (page 3, lines 11-14). I believe it would be more appropriated if it is moved to section 4 or at least repeated there.

Done. In the revised version of the paper the explanation appears in sub-section 4.1

See first paragraph on P. 9

- 4) Figure 1 looks like from Google Maps. There should be some referencing, right? Ignore this comment if it was indeed rendered by Authors.

Done.

See caption of Figure 1

- 5) In page 8, the explanation of harmonic theory (line 15 until end of page) is more detailed than needed (unlike the trapped wave theory). I recommend the Authors to just present the general phase speed and the one for the long waves. In this case, to just cite the basic literature is sufficient.

Done. Explanation shortened in the revised version (for the most part it now includes explanations of the symbols only).

See changes in section 4.1 on P. 8

- 6) I did not like the math notation. Since the authors are using the β plane, there is no reason to not use f_0 and β . The full expression of both are cumbersome. Also, it is easier to realize what is the last term in the denominator of equation 3 (and whenever it is shown again), if it is shown as a multiplication of 2β and $f_0/(g \theta_0)$, instead of the full expression $(2\Omega)^2 \sin(2\phi_w)/(g \theta_0)$. So please change accordingly.

Done. The notation of f_0 and β has been used (but we also added the explicit expressions of these parameters since $f(y)$ is expanded about different mean latitudes (i.e. ϕ_m in the harmonic theory and ϕ_w in the trapped wave theory).

See changes in section 4.1 on P. 8

- 7) Lines 11-12 of page 18: add a "in mid latitudes" after "[...] the harmonic theory is valid only in domains narrower than a few hundred kilometres [...]". In the tropics, it works for a few thousand kilometers, as shown by Watanabe et. al (2016, Ocean Dyn.).

Done.

See second paragraph on P. 17

- 8) In the abstract, in the penultimate sentence, please add something like "as was observed by previous studies" after the "[...] 140% to 200%". This result is not exactly unexpected, and the abstract will be better if it acknowledges this.

Done.

See new Abstract

Other

The jet colormap is not very good. A sequential colormap for spectral amplitude and a divergent one for SSHA would be substantially better. However, since the colormap jet is well accepted by the community (no idea why), I leave it to the Authors' discretion.

Since this is the accepted style in the community we've chosen to adhere to the existing colormap.

Final remarks

I would like to add that I enjoyed reading the manuscript, and I think it is very interesting. I'm sure the Author's will have no difficult in addressing the proposed reviews, and I'll be glad to recommend it to publishing then.

Thank you for the compliment.

Trapped Planetary (Rossby) Waves Observed in the Indian Ocean by Satellite Borne Altimeters

Yair De-Leon¹, Nathan Paldor¹

¹Fredy and Nadine Herrmann Institute of Earth Sciences, The Hebrew University of Jerusalem, Edmond J. Safra Campus, Givat Ram, Jerusalem, 9190401, Israel

Correspondence to: Nathan Paldor (nathan.paldor@huji.ac.il)

Abstract. Using 20 years of accurately calibrated, high resolution, observations of Sea Surface Height Anomalies (SSHA) by satellite borne altimeters we show that in the Indian Ocean south of the Australian coast the low frequency variations of SSHA are dominated by westward propagating, trapped, i.e. non-harmonic, Rossby (Planetary) waves. Our results demonstrate that the meridional-dependent amplitudes of the SSHA are large only within a few degrees of latitude next to the South-Australian coast while farther in the ocean they are uniformly small. This meridional variation of the SSHA signal is typical of the amplitude structure in the trapped wave theory. The westward propagation speed of the SSHA signals is analyzed by employing three different methods of estimation. Each one of these methods yields speed estimates that can vary widely between adjacent latitudes but the combination of at least two of the three methods yields much smoother variation. The estimates obtained in this manner show that the observed phase speeds at different latitudes exceed the phase speeds of harmonic Rossby (Planetary) waves by 140 % to 200 % (which was also reported in previous studies). In contrast, the theory of trapped Rossby (Planetary) waves in a domain bounded by a wall on its equatorward side yields phase speeds that approximate more closely the observed phase speeds in the study area.

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1 Introduction

The analysis of observations of Sea Surface Height Anomalies (SSHA), i.e., the deviation of the Sea Surface Height from its mean value at any given point in the ocean, was carried out since the 1990s in various parts of the world ocean by various satellite borne altimeters. Chelton and Schlax (1996), for example, analyzed the first three years of altimetry data collected by the TOPEX/Poseidon satellite in the world ocean, Zang and Wunsch (1999) analyzed five years of TOPEX/Poseidon data in the North Pacific Ocean and Osychny and Cornillon (2004) analyzed six years of modified TOPEX/Poseidon data in the North Atlantic Ocean. Additional observational studies are summarized in Barron et al. (2009) and references therein.

In most parts of the ocean the satellite observations showed a ubiquitous and pronounced westward migration of SSHA with amplitude of a few centimetres. This westward, rather than eastward, propagation led to the interpretation of these

observations as a surface manifestation of the first baroclinic mode of planetary (also known as Rossby) waves that propagate westward (i.e. their phase speed is negative) in the ocean thermocline. Recent studies (e.g., Chelton et al., 2007; Chelton et al., 2011), however, argue that the [observed](#) SSHA features belong to mesoscale eddies and [are](#) not surface manifestations of planetary waves in the thermocline but this change of view has no effect on the estimate of the westward propagation speed since these eddies propagate westward at the [same](#) phase speed [as](#) of long Rossby waves (Chelton et al., 2011; [O'Brien et al., 2013](#); [Polito and Sato, 2015](#); see also Nof, 1981 for theoretical estimate of eddy migration rate on the β -plane).

The quantification of the rate of westward propagation of the observed SSHA features is based on the construction of time-longitude (also known as Hovmöller) diagrams at a given latitude. The slopes of contours on these diagrams are proportional to the propagation speed of the SSHA features. These slopes can be calculated using methods that are commonly employed in image processing such as the Radon transform (or its more recent alternative – the variance method) and the Two Dimensional Fast Fourier Transform (2D FFT) which are described in details [in Sect. 2.2](#) below.

Previous studies of the westward propagation of [observed](#) SSHA in mid-latitudes have all yielded rates of westward propagation that are faster than the phase speeds predicted by the harmonic planetary wave theory (see below for details). Explanations for these underestimates by the harmonic theory were proposed which are based on considerations that involve either the addition of mean zonal flows in the equations (Killworth et al., 1997 and see also Colin de Verdière and Tailleux, 2005, who emphasized the curvature effect of the mean flow rather than the mean flow itself) or the influence of the bottom topography (Tailleux and McWilliams, 2001) while Killworth and Blundell (2005) applied a combination of these two effects. [Watanabe et al. \(2016\) showed that the standard linear wave theory can be tailored to fit the observations in the tropics by considering parameters such as effective- \$\beta\$ \(that includes the meridional gradient of the background potential vorticity\) and forcing by Ekman pumping.](#) LaCasce and Pedlosky (2004) argued that due to baroclinic instability the wave structure is changed and becomes more barotropic so it propagates faster and no mean flow is required. Along similar lines, Hochet et al. (2015) suggested that the assumption that observations are of the first baroclinic mode cannot be made a-priori, but the vertical structure is predicted from the altimetry data. Thus, they found that in some regions the vertical structure is more barotropic than baroclinic so the theoretical phase speed is larger and no discrepancy exists between theory and observations. By incorporating physical elements that are not included in the simple linear wave theory of the Shallow Water Equations (e.g. velocity shear, non-linear terms, topography, mean flows and juxtaposing barotropic and baroclinic modes) these (and other) past studies were successful in bridging some of the discrepancies found between the observed SSHA propagation speeds and the phase speeds of harmonic wave theory.

In contrast to the phase speed, other wave characteristics such as the meridional variations of the SSHA amplitudes (which are predicted by the harmonic theory to be sinusoidal) have never been verified in these past studies. The reason is

that in the framework of the harmonic theory (see more details below) the central latitude, ϕ_0 , which determines the origin of the y - (meridional) coordinate, is determined by the latitude of observation. Thus, observations of SSHA at adjacent latitudes cannot be compared to one another since their y -dependencies are determined by the same equations but with different origins so the same y -coordinate denotes different points in the two sets of equations.

5 The interpretation of these SSHA observations has employed the harmonic theory of westward propagating, low frequency, waves that assumes the existence of a zonal channel that bounds the north-south extent on the β -plane. Under these assumptions zonally propagating wave solutions of the Shallow Water Equations can be constructed and explicit expressions can be derived for both the zonal phase speed of the waves and the spatial structure of their amplitudes. The emerging spatial structure of the waves is oscillatory (harmonic) in both the zonal and meridional coordinates i.e., the waves simply oscillate with wavenumber k in the zonal direction and wavenumber l in the meridional direction (Pedlosky, 1982; Cushman-Roisin, 1994; Vallis, 2006).

15 An alternative to the traditional harmonic theory is the trapped wave theory which was developed on the mid-latitude β -plane by Paldor et al. (2007) and Paldor and Sigalov (2008). In this theory the meridional variation of the wave's amplitude is not harmonic ~~but~~ is given instead by the Airy function, (see details in Sect. 4.1 below) ~~and~~ the requirement of two channel walls of the harmonic theory is replaced in ~~this~~ trapped wave theory by a single wall that marks the equatorward boundary of the domain. In sufficiently wide meridional ranges the phase speed of the trapped waves is higher than that of the corresponding harmonic waves by a factor of 2 to 4.

The current study employs the available series of SSHA observations sampled on a $1/4^\circ$ spatial grid which are compared to the theoretical phase speeds and meridional structures of the height field using the trapped, and harmonic, wave theories. 20 The comparisons provide a measure of the relevance of the trapped and harmonic wave theories to the observed SSHA fields in the Indian Ocean.

This paper is organized as follows: Section 2 provides details of the observations and methods used for estimating the observed phase speed and in Sect. 3 we compare the theoretical and observational meridional variation of the height field in the Indian Ocean south of the Australian coast (which includes the Great Australian Bight). Section 4 describes theoretical expressions for the phase speed and ~~the~~ meridional structure of the height field of the harmonic and trapped wave theories that are compared with SSHA observations in the region of interest in Sect .5. The paper ends in Sect. 6 with summary and discussion of the findings.

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2 Data and Methods

2.1 SSHA Data

The altimetry products used for a comparison with theory were produced by Ssalto/*Duacs* and distributed by *Aviso*, with support from *CNES*. The data we used are the multi-mission (i.e., up to four satellites at a given time, e.g., TOPEX/Poseidon, Jason 1, Jason 2, Envisat) gridded Sea Surface Heights, sampled on a $1/4^\circ \times 1/4^\circ$ Cartesian grid once a week from 1/1/1993 to 31/12/2012. These data are improved compared to those used in previous studies since the combination of data from several, present-day, satellites enables high precision altimetry in both time and space at finer resolutions. [More details on the way the SSHA data are produced by Aviso can be found at http://www.aviso.altimetry.fr/duacs/](http://www.aviso.altimetry.fr/duacs/). The SSHA time-series of each grid-point in this region were low-pass filtered [in the present study](#) by performing a 5-week running-average to eliminate short-term variability such as storms, tides (including the fortnightly component) and other variations of periods less than one month. Though this filtering leaves parts of the high frequency signals in the averaged signal these parts are minute since the window contains many cycles of the high amplitude signals such as the M2 tides. Calculations with wider windows of 27 and 53 weeks (done to examine the possible contribution of longer term variability such as seasonal winds) yielded qualitatively identical results (see details in Sect. 3 below).

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Deleted: The SSHA data distributed by *Aviso* are based on a twenty-year reference period, 1993-2012, which integrates the evolution of the sea level in terms of trends, annual and inter-annual signals at small and large scales (e.g., El Niño, La Niña) in that period (and the possible bias is removed by *Aviso* in the calibration of the data by subtracting the mean SSHA of 1993 from all data).

2.2 Methods of estimating observed phase speed of SSHA

The basis for estimating the speed of westward propagation of SSHA is time-longitude (Hovmöller) diagrams of the SSHA field at fixed latitude. In this diagram the westward propagation is evident from the left-upward tilt of constant SSHA values i.e., same color contours and the angle between this tilt and the ordinate is directly proportional to the speed of westward propagation. The diagram provides a time-series of the SSHA changes at fixed longitude and a longitude variation series at any particular time so Fast Fourier Transforms can be easily calculated in time and longitude to yield the frequency and zonal wavenumber spectra of observed SSHA.

Three objective methods are employed in the literature for calculating the phase speed of waves from time-longitude diagrams. The first method is the frequently used (e.g., Chelton and Schlax, 1996; Chelton et al., 2003; Tulloch et al., 2009) Radon transform used in image processing for detecting structures on any digital image (see details in e.g., Jain, 1989). The Radon transform of a two dimensional function $f(x, y)$ that describes the intensity of an image at (x, y) , such as SSHA values in a given (longitude, time) domain, is the integral of $f(x, y)$ along a line L inclined at an angle θ relative to the ordinate (i.e., $\theta \pm 90^\circ$ relative to the abscissa) and displaced a distance s from the origin. For each angle θ we sum the squares of the values of the integrals along all lines having the same θ (i.e., having different distance s). The angle at which this sum-of-squares attains its maximum is the most accurate estimate for the orientation of structures with the same SSHA value on the

time-longitude diagram. The tangent of this preferred θ is proportional to the sought westward propagation speed. Note that in order to minimize the effect of few very high entries on the sum-of-squares we apply the Radon transform to a modified time-longitude diagram where the signal is scaled on the [0,1] interval and the mean of the scaled signal is subtracted. The second method is a relatively new algorithm (Polito and Liu, 2003; Barron et al., 2009) that constitutes an adaptation of Radon transform to a propagating wave. In this method the **variance** of amplitude values is calculated along the same lines. For each angle θ we average the variances along all lines at allowed distances s and the westward propagation speed is then determined by the tangent of the angle θ at which the mean of variances is minimal. The third method commonly used (e.g., Zang and Wunsch, 1999; Osychny and Cornillon, 2004) to obtain the observed phase speed is the application of the 2D FFT to the time-longitude diagram to get a frequency-wavenumber (i.e., ω, k) diagram of the signal's amplitude. The phase speed is obtained by locating the values of ω and k where the amplitude is maximal (i.e., maximum spectral coefficient) and calculating $C=\omega/k$ at this point of maximum spectral coefficient. Alternatively, the directionality of the spectral coefficients in the (ω, k) diagram can be found by sweeping over all lines that pass through the origin and inclined at angles ranging from 0° to 180° relative to the abscissa (i.e., ω/k lines). The value of C is then determined as the slope of the line of maximal sum-of-squares of spectral coefficients ("total energy").

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A comparison between the three methods was made using synthetic signals (De-Leon and Paldor, 2016). Based on the insight gained from the study of synthetic signals, an estimation of the observed phase speed is accepted here only when an isolated peak (characterized by the point at which the derivative changes sign at a clearly defined sharp peak and maintains the same sign in bands that are at least 3° wide on either side of the peak) is evident in at least two of the three methods and the phase speeds that correspond to these peaks agree by better than 10 %.

Note that the observed phase speed is obtained from the Hovmöller diagrams in units of one-quarter degree longitude per week which is converted to units of one centimetre per second by multiplying the observed phase speed by $4.6 \cos \phi_m$ (where ϕ_m is the latitude of observation).

2.3 The study domain in the Indian Ocean

The trapped wave theory in mid-latitudes applies without any modification to domains of large meridional extent (so the β -plane approximation applies) that are bounded on their equatorward side by a wide zonal boundary. As shown in Fig. 1 such a nearly zonal boundary exists in the Indian Ocean south of Australia. The domain of study extends from the south coast of Australia at about 31.5° S to only about 45° S since south of this latitude the SSHA field is strongly affected by the nearly 2000 km wide, fast, and strongly meandering, Antarctic Circumpolar Current (ACC).

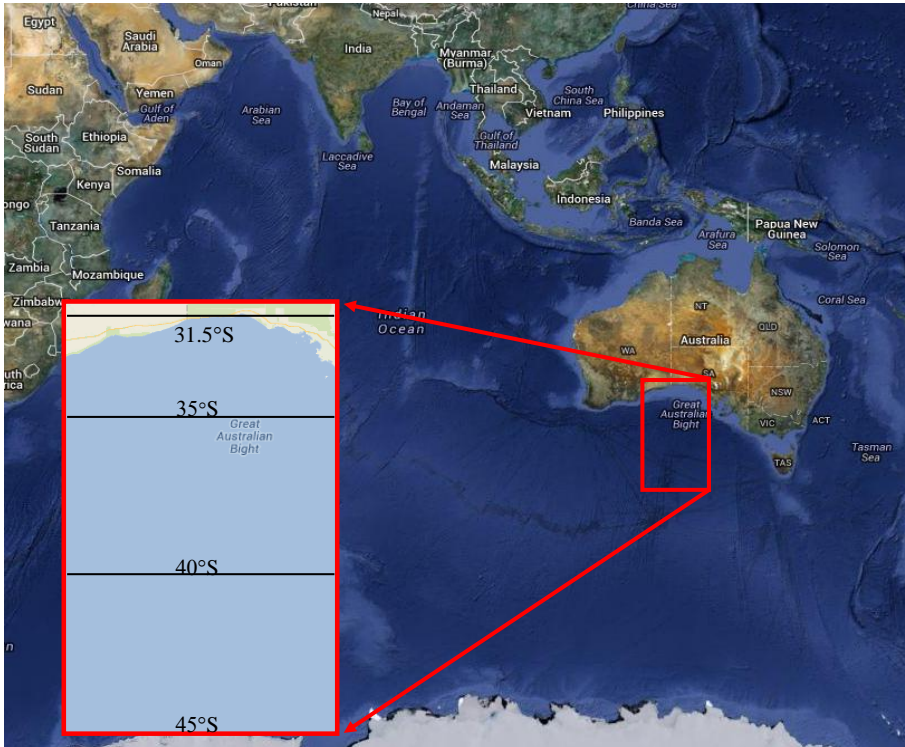


Figure 1: The domain under study in the Indian Ocean with a zoom in on the longitude band of 124.5°E - 134.5°E where altimetry data are analyzed ([reproduced from Google Maps](#)).

3 The Meridional structure of SSHA

- 5 | The standard deviation of the temporal changes of SSHA [observations](#) in each point of this domain over the entire 20 years is shown in Fig. 2(a), which clearly demonstrates an increase in the SSHA signal from 4 cm to 9 cm in the band of width 2° - 3° right next to the Australian coast. The meridional structure of the observed SSHA is clearly non-uniform, while in the harmonic (oscillatory) theory the height field is uniform (i.e., constant) for $l=0$ and sinusoidal for $l>0$. Although the ocean depth decreases towards the shore, the observed variability of SSHA signal there cannot be attributed to topography since
- 10 | steady winds affect only the average displacement of the sea surface which is subtracted from the SSHA signal when the

standard deviation is calculated, while the effect of winds of periods shorter than 5 weeks are filtered out by our low-pass filter. In order to examine the possible effect of longer term winds (seasonal to annual) the calculations were repeated with windows of 27 weeks and 53 weeks. These calculations yielded very similar results to those obtained with the 5 weeks window but with slight decrease in the amplitude of main signal near the coast and minute changes in the structure far from it. For the same reason this coastal peak cannot be associated with a mean long-coast current since such a current will not show up on a map of temporal standard-deviation. The 5 week filter which we applied to the data also eliminates high-frequency waves such as Kelvin waves and topographic Rossby waves (or continental shelf waves) since in the Great Australian Bight where the slope is 0.01 the period of these waves is $O(1 \text{ day})$ (see e.g. Cushman-Roisin, 1994 for harmonic waves and Cohen et al., 2010 for non-harmonic waves).

Deleted: Also, this variability cannot be associated with a coastal-trapped Kelvin waves since the observed phase speeds of 0.02 to 0.03 m s^{-1} (see Sect. 5.2 below) are two orders of magnitude smaller than the 2.5 to 2.8 m s^{-1} phase speed of Kelvin waves in this region.

The mean over all longitudes (Fig. 2(b), thick blue line) of the 41 individual latitudinal cross-sections (thin light-blue lines) is compared with analytical expressions (described in the next section) for the meridional structure of the height field of both the trapped wave theory (dashed red line) and the harmonic theory (dotted green line). The decay rates with latitude of both observed and trapped wave theoretical curves are similar in contrast to the flat curve of the harmonic theory. An unexplained minor secondary peak is found near 36° on the observed curve (also evident near 36° S , 125° E in panel (a)) and upon examining a larger SSHA map it turns out that this secondary peak is an eastward extension of the Leeuwin current that flows poleward along the west coast of Australia between March and July (Godfrey and Ridgway, 1985). Alternatively, this peak can be interpreted as a poleward propagation (into the Indian Ocean) of energy generated in the equatorial Pacific Ocean by the wind and by Ekman pumping which forms Rossby waves in the study area (Potemra, 2001).

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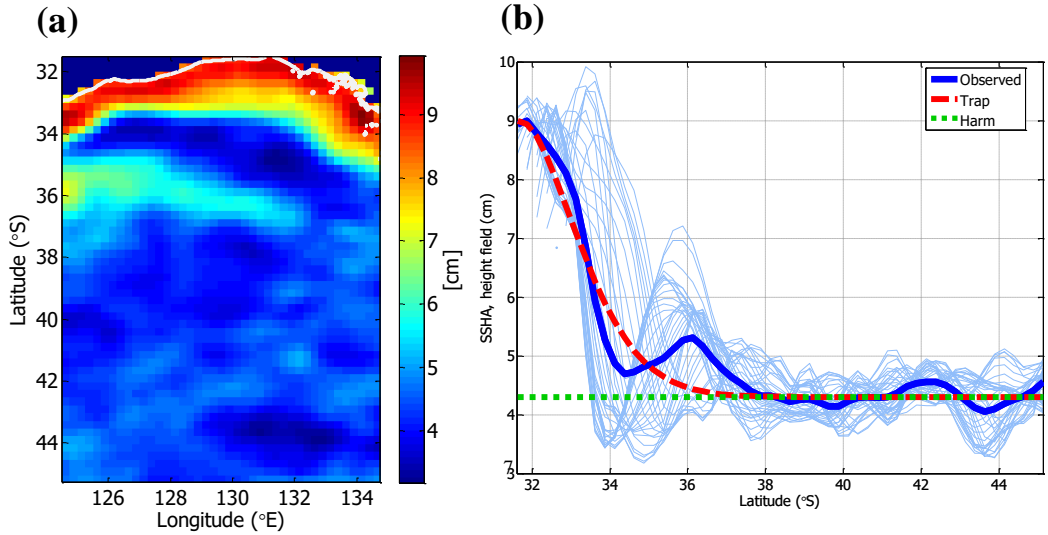


Figure 2. (a) The temporal standard deviation of satellite derived SSHA over the entire 20 year period poleward of the Great Australian Bight. The coastline is plotted in white. (b) Latitudinal cross-sections of the data of 2(a) every 0.25 degree longitude (thin light-blue lines), the mean over all longitudes of the latitudinal cross-sections (thick blue line) and the analytical expression for the meridional structure of the height field of trapped waves for zero zonal wavenumber and zero meridional mode number (dashed red line, see Eq. 5 below). The maximal trapped wave amplitude is set to match that of the mean observed cross-section where the off-shore minimum is about 4 cm since the temporal mean of *Aviso*'s original data is not zero. The analytical expression for the meridional structure of the height field of harmonic waves for $k=0$ and $l=0$ is constant i.e. described by a straight line parallel to the abscissa at arbitrary value of the ordinate, here it is set to match the off-shore minimum of about 4 cm (dotted green line).

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4 Application of wave theories to observations

The relevance of the trapped wave theory to observations can be best assessed by comparing the theoretical phase speed and meridional structure of the waves with observations such as those described above. In addition, it is also natural to compare the observations with phase speed and meridional structure of the harmonic planar theory and use the observations to assess the applicability of each of these theories.

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4.1 Explicit expressions for the phase speeds of the two wave types

The Coriolis frequency on the β -plane, expanded linearly about some latitude, ϕ_0 , is given by $f(y) = f_0 + \beta y = 2\Omega \sin \phi_0 + \frac{2\Omega}{a} \cos \phi_0 \cdot y$, where Ω is the frequency of Earth's rotation about its polar axis, a is Earth's radius and $y = (\phi - \phi_0) \cdot a$ (where ϕ is the latitude) is the north coordinate.

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In a channel on the mid-latitudes β -plane where the Coriolis frequency is expanded near $\phi_0 = \phi_m$, the latitude of observation, the fastest baroclinic phase speed (in units of metre per second) of harmonic Rossby waves is (see Cushman-Roisin, 1994; Vallis, 2006):

$$C^{\text{harm}} = \frac{-\beta}{k^2 + l^2 + \frac{f_0^2}{g'H'}} = \frac{-\frac{2\Omega}{a} \cos \phi_m}{k^2 + l^2 + \frac{(2\Omega)^2 \sin^2 \phi_m}{g'H'}}, \quad (1)$$

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where k and l (the latter is denoted in other studies by n) are the zonal and meridional wavenumbers of the Cartesian coordinates, respectively, g' is the reduced gravity and H' is the weighted depth of the two (or more) layers that make up the baroclinic ocean so $(g'H')^{1/2}$ is the speed of gravity waves. For sufficiently long waves when both k and l can be neglected this phase speed reduces to:

$$C^{\text{harm}} = \frac{-\beta}{\frac{f_0^2}{g'H'}} = \frac{-g'H' \cos \phi_m}{2\Omega \sin^2 \phi_m}. \quad (2)$$

Deleted: In 3D models where the density varies continuously with depth, $(g'H')^{1/2}$ is the separation constant between the horizontal and vertical equations (see Chapman and Lindzen, 1970).

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In contrast to the harmonic wave theory which is fully described in many textbooks the application of the trapped wave theory requires some more detailed explanation. In this theory, the waves are trapped next to a single wall that marks the equatorward boundary of the domain and the meridional variation of the wave's amplitude is given by the regular (at infinity) Airy function, Ai , that oscillates (but is not periodic in contrast to harmonic/sinusoidal oscillations) in the $(-\infty, 0)$ interval and decays to zero faster than exponential in the $(0, +\infty)$ interval (see e.g., Abramowitz and Stegun, 1972). The phase speed of trapped waves in a mid-latitude channel is (see Eq. (6) of Gildor et al. (2016)):

$$C^{\text{trap}} = \frac{-\beta}{k^2 + \frac{f_0^2}{g'H'} + \zeta_n \cdot \left(\frac{2f_0\beta}{g'H'}\right)^{2/3} - \frac{2f_0\beta}{g'H'} \cdot y_w}, \quad (3)$$

where ζ_n is the absolute value of the n th zero of Ai and y_w is the location of the equatorward wall. Following the studies of Paldor and Sigalov (2008) and De-Leon and Paldor (2009), we expand here the Coriolis frequency near $\phi_0 = \phi_w$, where ϕ_w is the latitude of the equatorward boundary of the domain so $y_w = 0$ there in which case the last term in the denominator of Eq. (3) vanishes (in contrast to Gildor et al. (2016) where the wall was placed at $y_w = -L/2$ where L is the channel width). In addition, the boundary condition at ϕ_w in Gildor et al. (2016) is the vanishing of the meridional velocity while in the present application Fig. 2(b) implies that the meridional derivative of the height field vanishes at ϕ_w . Thus, ζ_n in Eq. (3) should be replaced in the present application by ξ_n - the absolute value of the n th zero of the derivative of Ai (see the discussion following Eq. (5) below). The resulting expression for the phase speed of the first baroclinic mode of sufficiently long trapped waves (i.e., for zonal wavenumber $k=0$ and meridional mode number $n=0$ for which $\xi_0=1.0188$, see P. 478 of Abramowitz and Stegun, 1972) is:

$$C^{\text{trap}} = \frac{-\beta}{\frac{f_0^2}{g'H'} + 1.0188 \cdot \left(\frac{2f_0\beta}{g'H'}\right)^{2/3}} = \frac{-\frac{2\Omega}{a} \cos \phi_w}{\frac{(2\Omega)^2 \sin^2 \phi_w}{g'H'} + 1.0188 \cdot \left(\frac{(2\Omega)^2}{ag'H'} \sin 2\phi_w\right)^{2/3}}. \quad (4)$$

Note that in contrast to the planar harmonic theory where l is a meridional wavenumber (measured in units of m^{-1}) which cannot be determined when no channel exists (and the same is true for the zonal wavenumber, k), in the trapped wave theory n is a non-dimensional mode number that counts the number of zeros of the eigenfunction inside the meridional domain.

The trapped wave theory is valid when the meridional range is larger than $(2 + \zeta_n) \left(\frac{ag'H'}{4\Omega^2 \sin 2\phi_w}\right)^{1/3}$ (see Eq. (7) of Gildor et al., 2016). For $n=0$, typical values of $(g'H')^{1/2}$ of 2 to 3 m s^{-1} and $\phi_w=30^\circ$ this condition is satisfied when the domain is wider than about 500 kilometres. Accordingly, the harmonic theory applies only in unrealistically narrow channels that are only a few hundred kilometres wide (see also Fig. 3 in Paldor and Sigalov, 2008).

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4.2 Explicit expressions for the meridional structure of the **two wave types**

The meridional structure of the height field of harmonic waves in mid-latitudes varies with y , the meridional coordinate, as $\cos(l y)$ (with arbitrary amplitude and possible phase) which for $l=0$ yields height and velocity fields that do not vary with y .

The meridional structure of the height field of trapped waves is (see Eq. (5) in Gildor et al., 2016 with the modifications outlined in Sect. 4.1):

$$\eta(y) = \frac{H' a v_0}{g' H' - C^2} \left\{ C \cdot \left(\frac{4\Omega^2}{a g' H'} \sin 2\phi_w \right)^{1/3} \cdot Ai' \left(\left(\frac{4\Omega^2}{a g' H'} \sin 2\phi_w \right)^{1/3} \cdot y - \xi_n \right) - f(y) Ai \left(\left(\frac{4\Omega^2}{a g' H'} \sin 2\phi_w \right)^{1/3} \cdot y - \xi_n \right) \right\}, \quad (5)$$

where v_0 is an arbitrary amplitude and the phase speed, C , is given by C^{trap} of Eq. (3) **with the modifications outlined in Sect. 4.1**. Note that this theoretical expression for $\eta(y)$ consists of two terms: $Ai(y)$ and $Ai'(y)$, so $d\eta/dy$ contains terms proportional to $Ai(y)$, $Ai'(y)$ and $Ai''(y)$. The Airy differential equations relates $Ai''(y)$ to $y Ai(y)$ so at $y=0$ the $Ai''(y)$ term vanishes and the coefficient of $Ai(y)$ is negligible compared to that of $Ai'(y)$ which clarifies why the extremum of η occurs at $y \approx 0$.

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Deleted: Note that the theoretical expression for $\eta(y)$ in Eq. (5) consists of two terms: $Ai(y)$ and $Ai'(y)$, so its derivative contains two terms as well: $Ai'(y)$ and $Ai''(y)$, but when we apply the boundary condition, $\eta'(y(\phi_w))=0$, (i.e. maximal amplitude of $\eta(y)$ at the wall) we consider only the $Ai'(y)$ term (whose zeros are well known) since the coefficient of $Ai''(y)$ is negligible compared to that of $Ai'(y)$ (the ratio between them is of order 10^{-3}).

5 Results and comparison between observations and theories

5.1 Meridional structure of the height field

The meridional structure of the height field of the trapped waves curve **in the area of study** is computed from $\eta(y)$ of Eq. (5) with $C = C^{\text{trap}}$ of Eq. (4). In the calculation **of these expressions of C^{trap} and $\eta(y)$** , the value of ϕ_w was set to 31.5° S, $k=0=n$ (so $\xi_n = \xi_0 = 1.0188$), and $(g'H')^{1/2}$, the speed of gravity waves, was set to 2.8 m s^{-1} following Fig. 2 in Chelton et al. (1998) (see also http://www-po.coas.oregonstate.edu/research/po/research/rossby_radius/). The analytical expression for the meridional structure of the height field of harmonic waves for $k=0$ and $l=0$ is constant i.e. described by a straight line parallel to the abscissa at arbitrary value of the ordinate. As shown in Fig. 2(b) the curve of the trapped wave theory (dashed red line) fits the observed one (solid blue line) much better than that of the harmonic theory (dotted green line).

5.2 Phase speeds

An estimation of the speed of westward propagation of observed SSHA is obtained by analyzing time-longitude (Hovmöller) diagrams of the SSHA field at fixed latitude as explained in Sect. 2.2. Figure 3 shows two examples of such diagrams calculated at 36° S (panel a) and at 45° S (panel b); both are sufficiently far from any major current or continent and

sufficiently far from the equatorward boundary so that the condition for the validity of the trapped wave theory derived in the paragraph following Eq. (4) is satisfied (and sufficiently far (at least 200km) from the ACC). Also plotted on these diagrams are the two lines corresponding to the theoretical phase speeds for $k=0$ and $n=0$ of trapped wave theory (Eq. (4), dashed) and the harmonic wave theory (Eq. (2), dotted). A casual visual inspection shows that the line of trapped wave theory fits the

5 | observed tilt of SSHA features more closely than the harmonic one (especially at 45° S (panel b)).

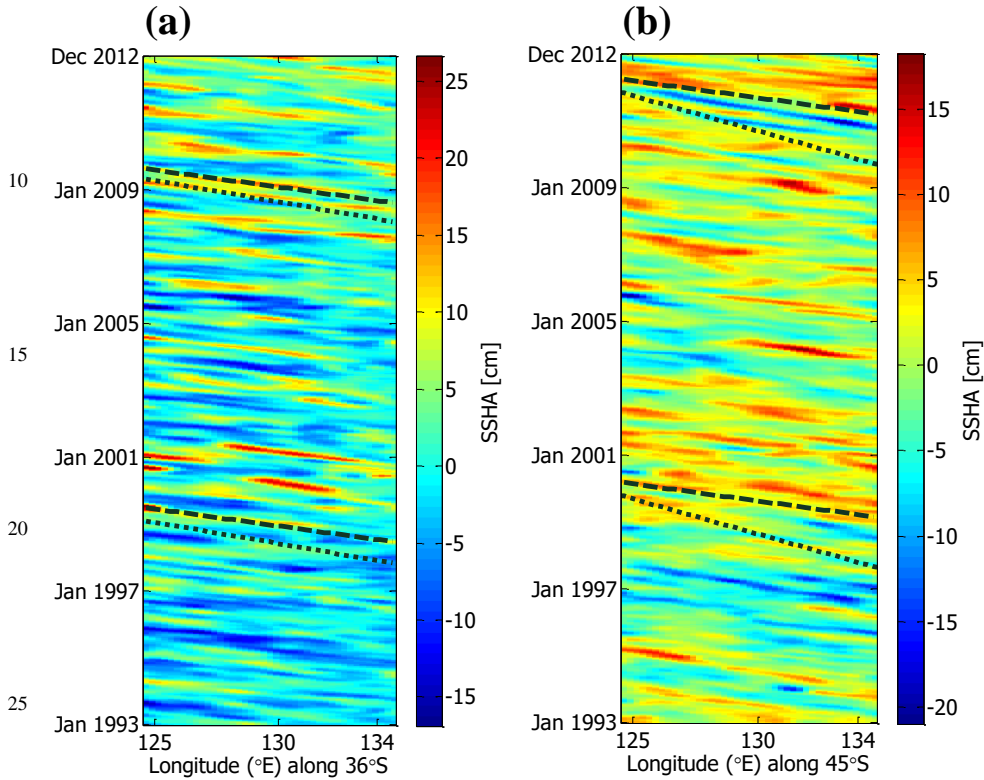


Figure 3. Time-longitude (Hovmöller) diagrams at $\phi_m = 36^\circ$ S (panel a) and at $\phi_m = 45^\circ$ S (panel b), where the abscissa is longitude and the ordinate is the date. The temporal average was subtracted from the record of each grid point. Dashed lines: trapped wave phase speed (Eq. (4)); Dotted lines: harmonic wave phase speed (Eq. (2)).

The various objective methods for obtaining the phase speed from Hovmöller diagram are now applied to the diagram in Fig. 3(b). The distribution of the sum-of-squares (or standard deviation) of the Radon transform as a function of the angle θ (for θ values near the peak) is shown in Fig. 4(a) (solid blue curve) where the maximum is at $\theta \approx 30^\circ$ (i.e., $C \approx 1.9 \text{ cm s}^{-1}$, in absolute value, hereafter). The distribution of the mean of variances as a function of the angle θ is shown in Fig. 4(b) where the mean of variances is minimal at $\theta \approx 33^\circ$ (i.e., $C \approx 2.1 \text{ cm s}^{-1}$). The θ -values corresponding to the phase speed of trapped waves (obtained from Eq. (4), $\theta \approx 37^\circ$ i.e., $C \approx 2.4 \text{ cm s}^{-1}$; solid red vertical line) and to the harmonic phase speed (obtained from Eq. (2), $\theta \approx 20^\circ$ i.e., $C \approx 1.2 \text{ cm s}^{-1}$; dashed green vertical line) are also shown in panels (a), (b) and (d) of Fig. 4.

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The frequency-wavenumber diagram obtained by applying 2D FFT to the time-longitude diagram at this latitude is shown in Fig. 4(c) in the range of low frequency and low wavenumber (in the rest of the frequency-wavenumber plane the amplitudes vanish). The maximum amplitude (outside $k=0$ since only $k \neq 0$ values yield finite westward phase speeds by ω/k) of the frequency-wavenumber diagram shown in Fig. 4(c) occurs at $k=0.1571$ which is a sufficiently small value that justifies the long-wave approximation made earlier ($k=0.1571$ corresponds to wavelength of about 160 degrees of longitude). The frequency with maximal spectral amplitude at this wavenumber is -0.09045 so the resulting phase speed of maximal spectral amplitude is $-0.09045/0.1571 = -0.5757$ (in degrees of longitude per 4 weeks, i.e., $C \approx 1.9 \text{ cm s}^{-1}$) and this phase speed equals the phase speed obtained independently by the Radon transform. Figure 4(c) also compares the phase speeds of the two theories with the observed speed and it demonstrates that the phase speed of trapped waves (dashed red line) is slightly (but not significantly) closer to the observed speed (defined by both the maximum amplitudes and the directionality of the band of high amplitudes in frequency-wavenumber plane) than that of the harmonic waves (dotted light-green line).

The distribution of the sum-of-squares of the spectral coefficients along ω/k lines versus the inclination angle, $\arctan(C)$, is shown in Fig. 4(d) (blue curve) where the curve attains its maximum at $\arctan(C) \approx 151^\circ$ i.e. $\theta \approx 29^\circ$ in terms of the Radon transform method ($C \approx 1.8 \text{ cm s}^{-1}$).

For this time-longitude diagram the phase speed obtained by the variance method differs by about 11 % to 14 % from that obtained by the Radon and 2D FFT methods that yield nearly identical phase speeds, so according to our criteria mentioned in the end of Sect. 2.2 the latter estimate for the phase speed is accepted. However, this observed phase speed does not clearly validate any of the two theoretical phase speeds since the corresponding vertical lines in panels (a) and (d) of Fig. 4 are located at nearly the same distance on both sides of the observed peak. In contrast, the estimate of the observed phase speed obtained by the variance method (Fig. 4(b)) is much closer to that of the trapped wave phase speed than the harmonic one. Thus, the determination of the relevant theory that yields the correct phase speed that matches the propagation rate determined from observations cannot rely solely on the match at any particular latitude and therefore match over an entire range of latitudes was also examined.

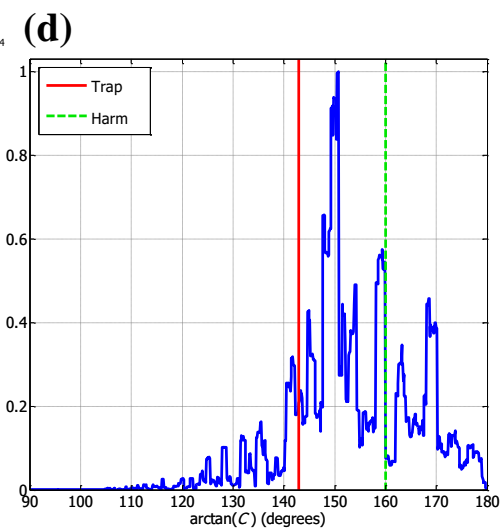
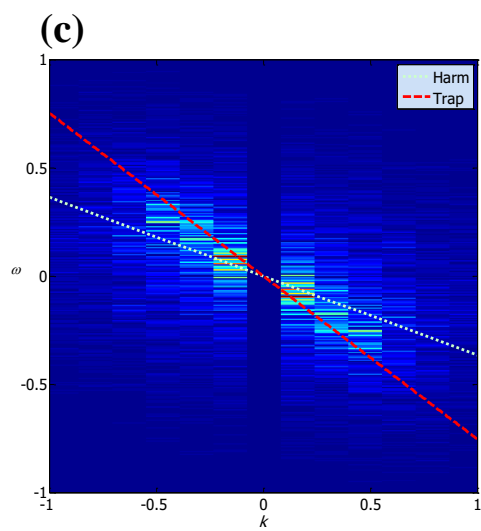
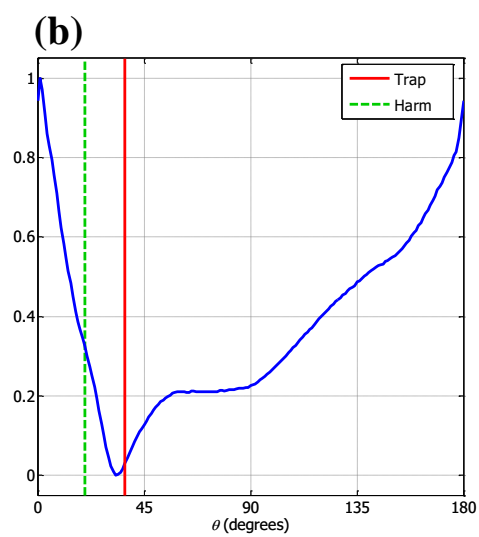
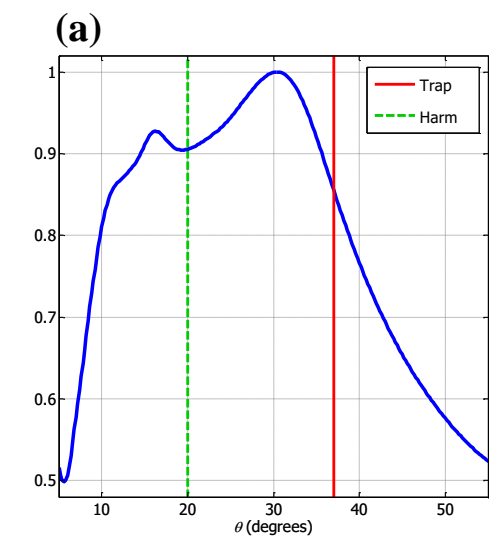


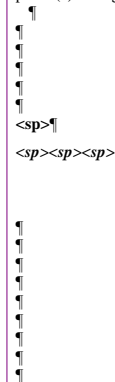
Figure 4. Objective analysis of the phase speeds associated with the time-longitude diagram of Fig. 3(b). (a) The distribution of the sum-of-squares of the Radon transform versus θ (near the peak) normalized such that the maximum value equals 1 (solid blue curve) and the two angles corresponding to the phase speeds of the harmonic wave theory (Eq. (2); dashed green vertical line) and to the trapped wave theory (Eq. (4); solid red vertical line). Note that the same two vertical lines appear also in panels (b) and (d). (b) The distribution of the mean of variances versus θ , normalized such that the maximum value equals 1 and the minimum is 0 (solid blue curve). (c) The 2D FFT frequency-wavenumber diagram in the low frequency-low wavenumber regime (where the wavenumber k is measured in units of $(\frac{1}{4}$ degrees of longitude) $^{-1}$, the frequency ω is measured in units of week $^{-1}$ and the amplitude is calculated in arbitrary units). Dashed red line: phase speed of the trapped wave theory (Eq. (4)). Dotted light-green line: phase speed of harmonic wave theory (Eq. (2)). The line corresponding to trapped wave connects the two maximal values of the 2D FFT at the smallest $\pm k \neq 0$ which also passes through the origin but does not follow the location of maximal amplitude at larger wavenumbers better than the line corresponding to harmonic waves. (d) The distribution of the sum-of-squares of the 2D FFT amplitudes along different lines (sweeping) versus $\arctan(C)$, normalized such that the maximum value equals 1 (blue curve). Only values of $\arctan(C)$ in the range of 90° - 180° are shown since only these values correspond to westward propagating phase speed.

The implications from similar comparisons carried out every 0.5° between 33° S and 45.5° S can be summarized as follows: At about third of the diagrams analyzed the signal was too blurred or the three methods yielded three different phase speed estimates. The application of a single method over the entire range of latitudes yields estimates that occasionally vary by over 50 % between adjacent 0.5° latitudes so the latitudinal continuity of the phase speed rules out the use of a single method. In only one or two latitudes (out of 22) all three methods have yielded the same (up to 10 %) estimate. Our conclusion from these comparisons bolsters our criteria that only when at least two of the three methods yield phase speed estimates that are closer to one another than 10 %, the resulting phase speed estimate can be considered reliable.

Phase speed estimates north of 35° S and between 37° S and 39° S have not satisfied the agreement criteria between methods outlined in the end of Sect. 2.2. The lack of reliable phase speed estimates at these latitudes even though the amplitudes of the SSHA there are higher than in adjacent latitudes in which the phase speed estimates were deemed reliable (and especially north of 35° S) requires an explanation. In linear theories amplitudes can only be determined up to a multiplicative factor while phase speeds are determined completely. Accordingly, the harmonic theory where the solution is determined by ϕ_m alone, does not provide any information on the variation of the SSHA amplitude with ϕ_m , while in the trapped wave theory the variation of the amplitude with ϕ_m is determined up to an overall multiplicative constant. Regardless of whether the meridional structure of SSHA is determined or not it should be stressed that higher/lower amplitudes do not necessary imply that the corresponding phase speed estimates are more/less reliable and it is possible for the amplitude to be high while the phase speed estimates are not reliable (using the methods and criteria we apply) or for the phase speed to be significant where the amplitudes are small (e.g. south of 40° S).

Figure 5 shows the observed and the two theoretical speeds as a function of ϕ_m between 35° S and 45.5° S where reliable estimates are obtained. The theoretical trapped speed is calculated using Eq. (4) and the theoretical harmonic speed is calculated using Eq. (2). It is clear that the trapped speeds (solid red line) are closer to the observed speeds (blue dots,

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squares and triangles) than the harmonic speed (dashed green line). A quantitative confirmation of this qualitative conclusion can be obtained by calculating the sum of squares of the distances between the observed and theoretical speeds. This calculation shows that trapped speeds with sum of squares that equals 3.5 are much closer than harmonic speeds where the sum of squares is 15.3 i.e. more than four times that of trapped waves. Since the value of 10 % agreement (shown by blue circular dots) is somewhat arbitrary, we also include in Fig. 5 estimates of 11 and 12 % agreement (light-blue triangles) and estimates that agree by 25 % (light-blue squares).

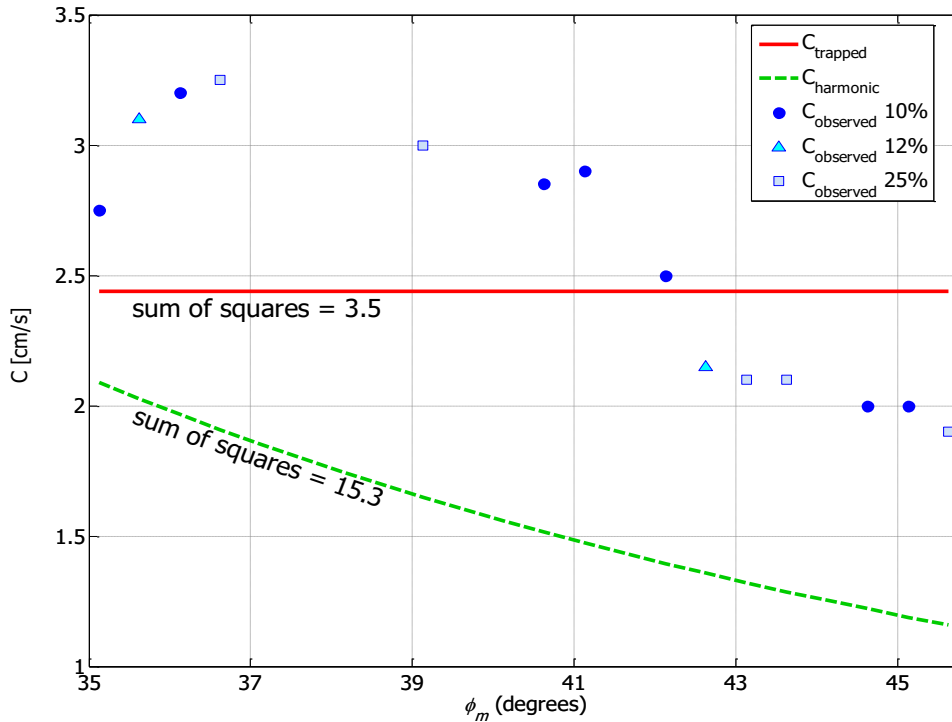


Figure 5. The observed phase speeds and the two theoretical phase speeds (trapped and harmonic) as a function of ϕ_m in intervals of 0.5° latitude. Blue dots denote latitudes where the estimates of at least two methods agreed by 10 % or less, triangles denote latitudes where such estimates agreed by 11 to 12 % and squares denote latitudes where the agreement is 25 %. No reliable estimates were obtained north of 35° S and in some more latitudes. The sum of squares of the distances in $(\text{cm s}^{-1})^2$ between trapped wave phase speeds and observed speeds (3.5) is much smaller than that of harmonic phase speeds (15.3).

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6 Discussion and Summary

The phase speed of harmonic waves decreases monotonically with the latitude of observation, ϕ_m , as is evident from Eqs (1)-(2). In contrast, the phase speed of trapped waves depends on ϕ_w only (i.e. the latitude of the zonal boundary) and is independent of ϕ_m . Our analyses of the propagation speeds of SSHA signals show that the rate at which the observed speed decreases with ϕ_m (the trend of the data in Figure 5 is $0.12 \text{ cm s}^{-1} \text{ deg}^{-1}$) exceeds the rates of decrease of both harmonic (where the trend is $0.09 \text{ cm s}^{-1} \text{ deg}^{-1}$) and trapped (no trend) phase speeds. In contrast, the values of the observed phase speeds are much closer to the trapped phase speeds than to the harmonic speeds.

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Colin de Verdière and Tailleux (2005) argue that the addition of mean flows affects the propagation speed of Rossby waves via its curvature: increase (decrease) of the westward phase speed for eastward (westward) surface mean flow. However, in the domain of the Indian Ocean studied here it is not clear whether or not a mean flow exists (in contrast to west of Australia where a subtropical gyre has been observed, see e.g. Stramma and Lutjeharms, 1997) and what it is its direction (some of the flows vary seasonally, see Wyrtki, 1973) so even if the numerical values of parameters such as Richardson number or buoyancy could be somehow estimated it is still unclear whether the mean flow increases or decreases the phase speed.

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We should note that the simple choice, here and in many other prior studies, to interpret that observed SSHA propagation as that of the first baroclinic mode is not the only possible choice. Other choices of a single mode to fit the observation require detailed analyses of the hydrography while a (linear) combination of several modes (including the fast barotropic mode) with weights that are tuned so as to fit the observed speed can yield a better fit (see Hochet et al., 2015).

Chelton et al. (2007) and Chelton et al. (2011) argue that most of the observed SSHA features in the global ocean are nonlinear mesoscale eddies whose propagation speed is close to the phase speed of long harmonic Rossby waves (but linear eddies move much faster). The nonlinearity in those studies is determined by a combination of second order spatial derivatives of the SSHA that are used in the calculation of the Okubo-Weiss parameter. Since no derivatives can be computed by the methods in the present study it is impossible to use these methods to directly determine whether the SSHA features examined in the present study are linear or not. However, all observed propagation speeds calculated here move faster than the phase of long harmonic Rossby waves (see Fig. 5) which implies that only linear (in the sense defined in Chelton et al., 2007) eddies that propagate faster than the phase speed of long harmonic Rossby waves exist in the Indian Ocean south of Australia.

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As was concluded in De-Leon and Paldor (2016) an estimation of the observed phase speed using one method only, is not reliable in most of the observed signals. On the other hand, even when estimates of the observed speed of at least two methods agree with each other, a comparison of the observed speed and the theoretical speeds varies in accordance with the method used for obtaining the observed speed. For example, in Fig. 4 the observed speed obtained by the variance method

(panel b) is much closer to the trapped speed than to the harmonic speed while the observed speed obtained by the Radon and 2D FFT methods does not fit either the trapped or the harmonic speed preferentially. These differences between different methods point to the low accuracy/reliability of existing SSHA data.

As mentioned in the introduction, many studies compared observations of Rossby waves in the ocean with the harmonic Rossby waves. However from a theoretical point of view, the harmonic theory [in mid-latitudes](#) is valid only in domains narrower than a few hundred kilometres so it is not clear why one should expect the harmonic speed to match the observed speed at unbounded domains. The case of Australia is unique since the trapped wave theory applies there while no other place exists that has a sufficiently wide, nearly straight, zonal coast line and meridional extent of the ocean that spans over 10° poleward of the equatorward boundary. In cases of narrower straight zonal coast line such as Puerto Rico the trapped wave theory is inapplicable. In unbounded domains of the world ocean the trapped wave theory does not apply straightforwardly and additional theoretical considerations have to be developed.

Acknowledgements

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