Reply to the comments of the Referee#1 on "Mesoscale cascades and the "conundrum" of energy transfer from large to dissipation scales in an adiabatic ocean"

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First of all, I would like to thank the Referee for emphasizing the fact that the results of the manuscript under the discussion *"are very much at odds with the general understanding of mesoscale turbulence"*. However, namely "*the general understanding"* leads to one of the most enigmatic conundrums of ocean general circulation which is *"how does the energy of the general circulation cascade from the large climate scales, where most of it is generated, to the small scales, where all of it is dissipated? In particular, how is the dynamical transition made from an anisotropic, 2D-like, geostrophic cascade at large scales-with its strong inhibition of down-scale energy flux-to 3D-like, down-scale cascades at small scales."* (Muller et al., 2002). Specifically, the Referee states that:

(1). *"it is widely recognized that strong conversion* $EPE \rightarrow EKE$ *occurs at the radius of deformation (see e.g. the text book by Geoff Vallis)"*. Indeed, in the chapter 6.8 titled ***"The energetics of linear baroclinic instability"*** Vallis studies the problem of the baroclinic instability and concluded in the end of the chapter that *"baroclinic instability converts potential energy into kinetic energy."* This conclusion was drawn on the basis of the linear analysis within which the energy exchange between different Fourier modes is absent at all, as well as the energy cascades. Meanwhile, those phenomena and the non-linear (NL) interactions are crucial for the mesoscale dynamics and observational effects, as Dubovikov (2003, D3) and Canuto and Dubovikov (2005, CD5) showed theoretically. An analogous conclusion was drawn by Chelton et al. (2011) from the analysis of observational data: "*essentially all of the observed mesoscales features are non-linear*", "*mesoscales do not move with the mean velocity but with their own drift velocity"* and the latter is "*the most germane of all the non-linear metric*s". In D3 and CD5 we derived the mesoscale drift velocity u_d theoretically. In Fig.1 borrowed from Canuto et al. (2017a), we present the comparison of the predicted u_d with observational data which were obtained later (Fu, 2009; Chelton and Schlax, 2013). In D3 and CD5 we parameterized the NL terms of the dynamical mesoscale equations on the basis of the general approach to modeling NL interactions in turbulent flows developed by the authors before (see the list of those articles in the manuscript under the discussion). The basis of the D3, CD5 mesoscale parameterization is the generation of the inverse energy cascade in mesoscale turbulence whose existence is now commonly recognized (Ferrari and Wunsch, 2009; Bruggemann and Eden, 2015; Jansen et al., 2015) and confirmed by sea surface height data (Scott and Wang, 2005; Scott and Arbic, 2007). As Kraichnan (1975) showed, that cascade generates the negative turbulent viscosity which drastically changes the mesoscale equations whose solution has no fitting parameters and can be tested against data of observations and OGCMs numerical computations. Some validations of D3, CD5 are demonstrated below in Fgs.1-3 borrowed from the submitted papers by Canuto et al. (2017a,b). Thus, we expect that the NL mesoscale dynamics radically modifies the transformation of EPE and EKE in comparison with the results of the linear analysis presented in the quoted above Vallis's text book. In particular, consider Eq.(5.7) of the manuscript under discussion which yields the EKE production $P_K(r_d)$ by EPE at scales of the deformation radius r_d:

$$P_K(r_d) = -2r_d^{-1}K^{3/2} < 0 \qquad \text{(a)}$$

where K is EKE. The mesoscale characteristics K and r_d demonstrate the fact that $P_K(r_d)$ is due to the cascades, i.e. due to the NL interaction. The negative sign in Eq.(a) means that at scales $\sim r_d$ EKE transforms into EPE. By contrast, at scales $\sim \ell$ given by Eq.(6.3) we have the conversion EPE$\rightarrow$EKE. The sign of the total EKE production $P_K(\text{total}) = P_K(r_d) + P_K(\ell)$ given in (5.10), is positive.

Even without any mesoscale model it is clear that the negative sign of $P_K(r_d)$ straightforwardly follows from the existence of the strong inverse energy cascade and the observational fact that the transfer of EKE to large scales is much less than the energy exchange between EKE and EPE. The latter follows from the oceanic analog of the observed atmospheric Lorenz (1960) energy cycle summarized by Holton (1992), Fig.10.13 adapted from Oort and Peixoto (1974). The same conclusion follows from the numerical simulations by Boning and Budich (1992, Figs. 8,9). The result (a) is odd with the discussed statement of the Referee cited in the beginning of (1).

(2). The Referee states that "*it is widely recognized that **total eddy energy** is transferred to larger scales*". This is not correct. Exactly the opposite is true: the **total eddy energy** is fed mostly by the large scale available potential energy which is due to the baroclinic instability. Specifically, the production of EPE which ultimately converts into EKE and finally is dissipated, is mostly contributed by the transfer of available potential energy from large scales, the conclusion which follows from, say, the Gent-McWilliams model as well as from D3 and CD5 ones. Thus, the large scale energy is transferred to the ***total eddy energy.***

(3). The Referee *"strongly disagrees"* with our input that *"intense release EPE→EKE begins at scales where the spectral Rossby number* $Ro(k)$ *which at large scales is small, increases to unity"*. Nevertheless, this conclusion follows straightforwardly from mesoscale equations (5e)-(6b) of D3 or Eqs.(4i) –(5b) of CD5 which account for NL terms. In fact, EKE is produced by the a-geostrophic component of the velocity $\mathbf{u}_a$. In Fourier space we have:

$$P_K(\mathbf{k}) = -\operatorname{Im}\left[p_*(\mathbf{k})\mathbf{k}\cdot\mathbf{u}_a^*(\mathbf{k})\right] \quad \text{(b)}$$

where $p = \rho_0 p_*$ is the pressure, $\rho_0 = 10^3 kg/m^3$ is the reference density. In the case of a small $Ro(k)$ from the referred above equations of D3 or CD5 to the main order of EKE using the manuscript notations we deduce:

$$\mathbf{u}_a(\mathbf{k}) = -Ro(k)\mathbf{e}_z \times \mathbf{u}_g(\mathbf{k}), \qquad f\mathbf{e}_z \times \mathbf{u}_g(\mathbf{k}) = -i\mathbf{k}p_* \quad \text{(c)}$$

where $\mathbf{e}_z$ is the unit vertical vector, f is the Coriolis parameter. From Eqs.(b), (c) we get

$$P_K(\mathbf{k}) = -k^2 f^{-1} Ro(k)\left|p_*(\mathbf{k})\right|^2 < 0 \quad \text{(d)}$$

i.e. at small $Ro(k)$ EKE transforms into EPE but not vice verca. It is worth recalling that this result is obtained with account for the negative turbulent viscosity in the referred mesoscale equations which, in turn, is due to the inverse energy cascade created by NL interactions which is absent in the linear approximation. In the opposite case of a large $Ro(k)$ the effect of rotation is weak and the velocity equation yields the usual EPE→EKE conversion.

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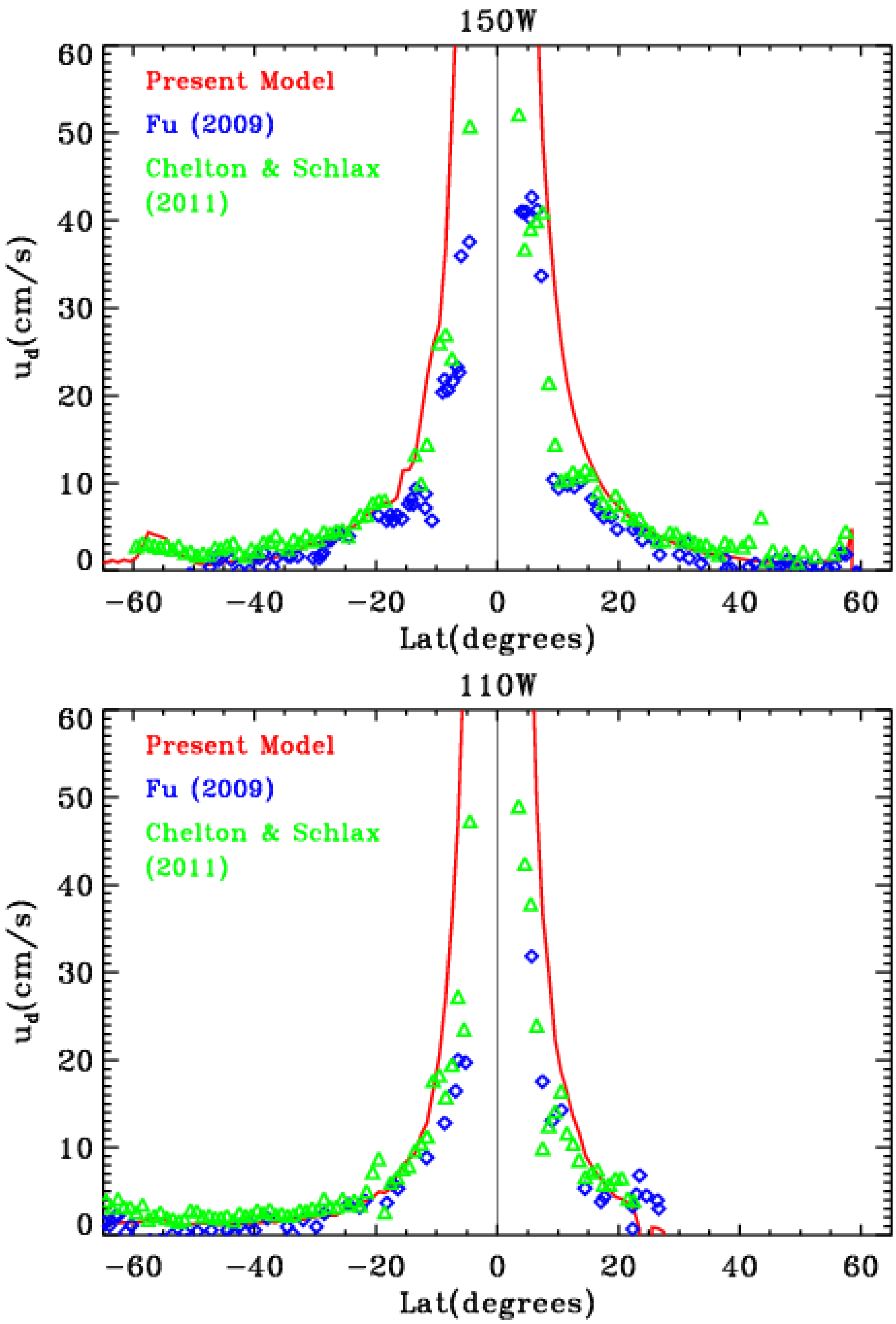

Fig.1. Borrowed from Canuto et al. (2017a). Comparison of $|\mathbf{u}_d|$ derived in D3 and CD5 with the data of Fu (2009) and Chelton and Schlax (2011) at 150^0 W and 110^0 W. The data are reproduced satisfactorily. In all the figures, the model results were obtained from an average of the last 3 years of a simulation with the GISS ER stand-alone OGCM which was run for 300 years.

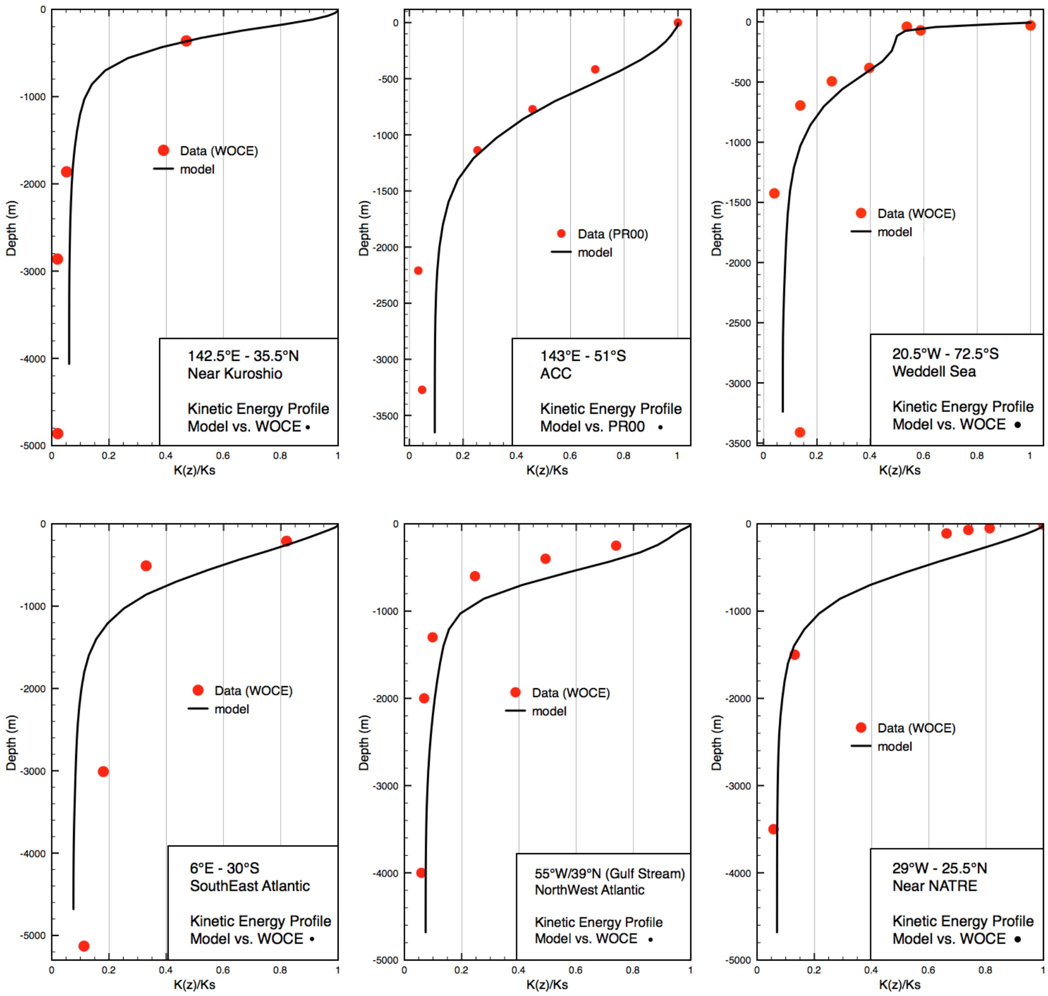

Fig.2. Borrowed from Canuto et al. (2017b). Comparison of the z-profile of the EKE derived in d3 and CD5 in units of its surface value vs. WOCE data in different locations. The model results reproduce the data satisfactorily.

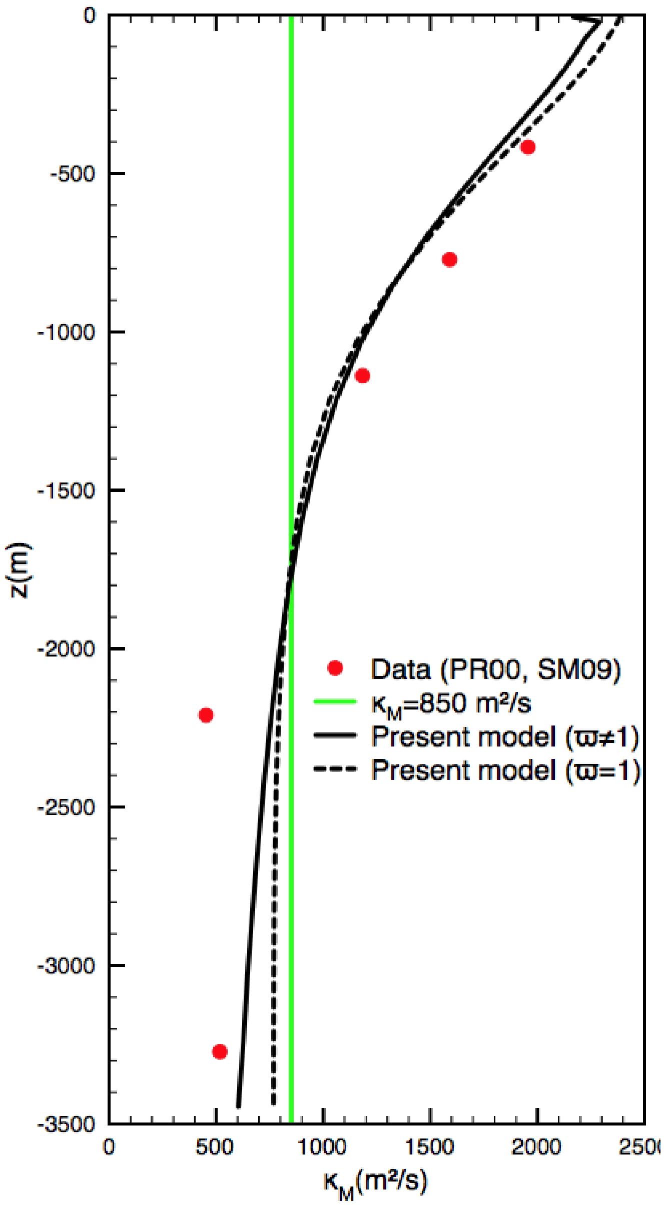

Fig.3 Borrowed from Canuto et al. (2017b). Comparison of mesoscale diffusivity (in $m^2 s^{-1}$) computed within D3 and CD5 model vs. the measured data of Philips and Rintoul (2000, PR00) in the ACC (143E, 51S). The $\varpi = 1$ case is with the contribution of corrections of the higher order in the small parameter equal to the ratio (mean K/EKE).