## Dear Dr. John M. Huthnance

Many thanks for the points on "Some aspects of the deep abyssal overflow between the middle and southern basins of the Caspian Sea" by Javad Babagoli Matikolaei et al

We prepared the files as "our answers and further clarifications" and sent to anonymous Referee #1, privately, in 14 March 2017. Referee #1 had some points that we accept some points particularly concerning adding figures and some extra explorations to clarify .But we disagree with some points, especially, when it comes to the structure of the flow and the analytic models. Unfortunately, we have not received any answers from Referee #1 after it.

We kindly invite to observe our answers in following text. If you confirm our points, we will change some parts of paper as soon as possible.

Thank you very much for your cooperation.

Many thanks for the Interactive comment on "Some aspects of the deep abyssal overflow between the middle and southern basins of the Caspian Sea" by Javad Babagoli Matikolaei et al. by the anonymous Referee #1.

Our answer and further clarifications are as follows:

As mentioned in the paper, not much work on the deep flows and waters of the Caspian Sea basins, the largest water body in the world have been done, hence, the present work may be the first attempt to concentrate on such issues which are crucial for the future fate of this sea, as have been addressed in the papers conclusion section. Although there are shortcomings in the paper, particularly direct observations of deep currents in the Caspian Sea, however, I am surprised to hear that there is not much NEW in the paper by the referee #1!

There are two important points concerning the importance of this research. The first point is that there are a large number of evidences which show signs of life in the depths of the Caspian Sea, especially in the Southern basin of Caspian Sea. For example, you can observe this signs in below Scheme (**Biological Features and Resources Ocaspian Sea**, M.G. Karpinsky · D.N. Katunin · V. B. Goryunova · T.A. Shiganova, In Caspian Sea Environment, Spinger, Part P (2005): 191–210) .

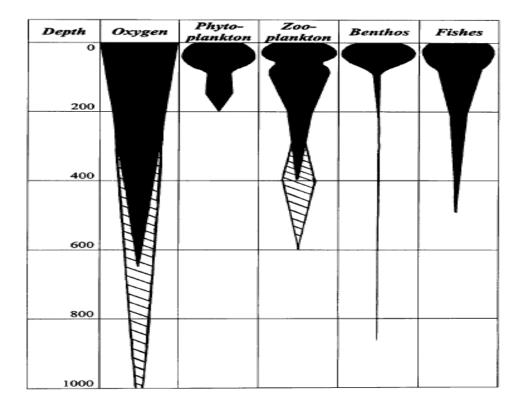


Fig1. Scheme of vertical distribution of oxygen phytoplankton, zooplankton benthos, and fishes.

Which means that the Caspian Sea is not as Black sea which is due to lack of such ventilation is nonproductive in deep parts. But you cannot also find any research to answer this question as why the deeper part of the Caspian Sea is rather alive. The current study shows that such abyssal flow can ventilate the deeper parts of the Caspian Sea.

Another point is oil pollution due to oil exploration, especially in the Strait of Apsheron, which is under way that can affect the deep parts of the southern basin of this Sea (see Fig 2).

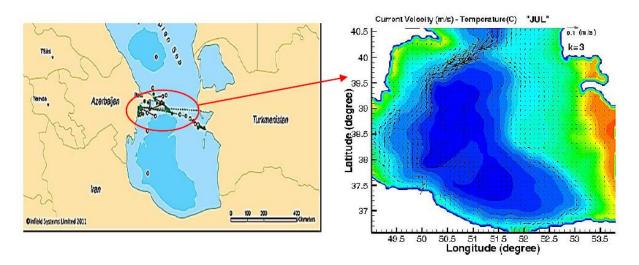


Fig2.

1. The density or better sigma (will be corrected in the paper) fields of the two middle and southern basins inferred by the direct observations indicate that the middle basin, especially in the deeper parts is denser than the southern one (as an example of density profiles, corrected for pressure effect, is shown in the paper). This is also expected as the higher latitude of the northern and middle basins are more prone to be exposed colder and stronger westerly winds and mid-latitude cyclones and cold fronts than the southern basins, the northern basin also freezes in the cold season (a figure of measured monthly mean air temperature over the basins will be added to the paper, fig. 4). For this purpose we used meteorological data and MODIS data in Caspian Sea for three positions of this points in Fig3.

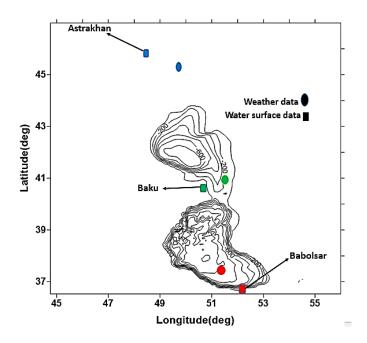
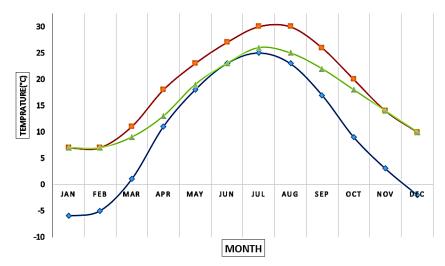
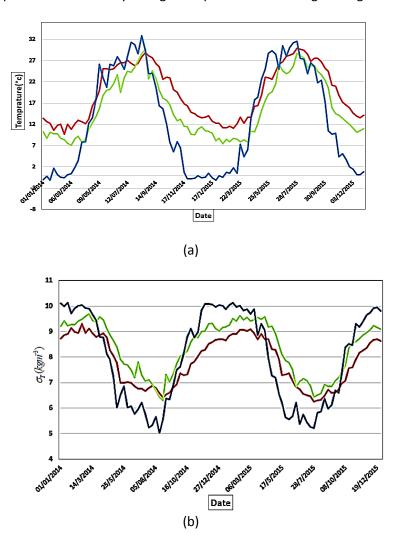


Fig3.



**Fig 4.** Comparison the air temperature in North (blue), Middle (green) and South (red) Caspian Sea.

As shown in the northern basin, the temperature can reach -8°c in winter. In the next step, we plot time series of surface temperature and density changes for points shown in Fig 3 using MODIS data.



**Fig 5.** Comparison the surface temperature and sigma-T in North (blue), Middle (green) and South (red) the Caspian Sea.

Another important evidence about the overflow is the numerical model output. For example, you can see the overflow in the cross-section along the Latitude in fig. 6.

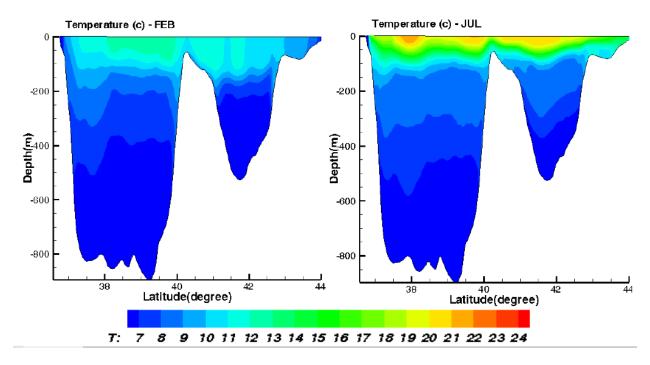


Fig 6. Cross-Section of the mean temperature obtained from model simulation during Febuary and July.

Hence, overflow from the middle to southern basin is expected, although direct measurement of this flow has not been carried out. This is why we used numerical simulations which have been verified against some existed measurements only in the southern basin that will be added to the paper (fig. 7). The overflow certainly has seasonal changes which we have also indicated and have been taken into account in the paper, but its detail possible intermittent behavior should be the subject of another work. The introduction will also be amended accordingly, as pointed out be the referee #1, although as was mentioned before not much work have been done on this overflow.

2. About validation of numerical model, yes we validate the results of numerical model by CTD and ADCP data. For this reason, we run this model in certain years to compare it results with observation data. For example, you can see ccomparison between the numerical model results and observation in Fig 7, near Bandar-e Anzali (long.: 37.5°, lat.: 50.5°), indicating rather close agreements.

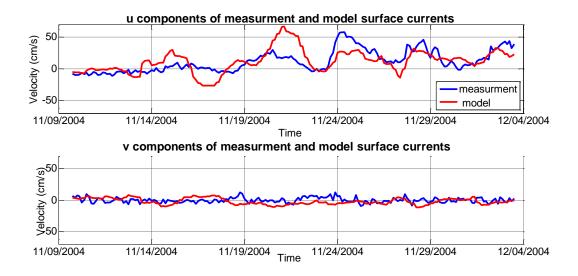


Fig7. Comparison between numerical model results of current components and observation.

About temperature differences in the Strait and the South Caspian Sea, it should be noted that model results for Sigma coordinate have been presented so, depth is not equal in each k along the flow. The actual depths of some points along the flow in the Caspian Sea are shown in Fig 8. For this reason, temperature is not the same in the Strait and parts of South Caspian, see also Fig 6.

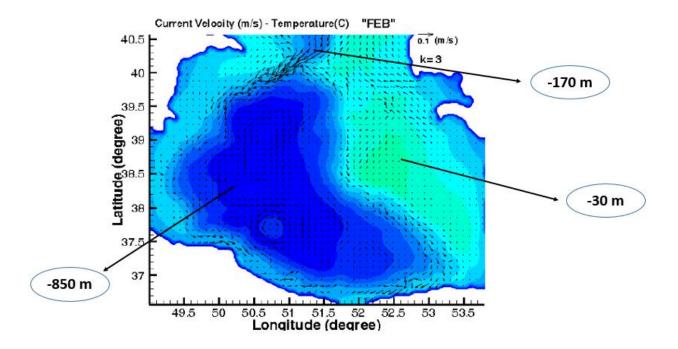


Fig 8.

Yes, as measurements are scarce (mentioned in the paper) we used some of the numerical results to provide parameters for the analytical mode. Yes mixing parameterizations, within the model limitations, are included with proper turbulence scheme as Miller-Yamada in numerical simulations. The flow descends as a density driven (as g' the reduced gravity of the over flow is negative, hence down ward flow) current then is trapped by the side boundary of the basin, due to rotational effect (Fig 6). Due to bottom friction it should spiral down the basin as indicated to some extent by the numerical results.

3. The choice of the coordinates for the first analytical model may not be as popular, but the model is an initial condition Lagrangian one without entrainment (or interaction of fluid particle with the surrounding ones, a shortcoming for this stage), hence yes, it only shows the path of the fluid particles. The bottom slope of model was also estimated from the topography of Absheron sill. About the path of two particles, a particle cannot have an effect on another particle because we ignore the effects of stirring diffusion. If we consider this effect, we cannot present such an analytical model for this flow which is time non-steady and three dimensional, but there is some paper deal with Stirring diffusion only for a steady case (see Cenedese, C. J. and Whitehead, A., 2002. "A dense current flowing down a sloping bottom in a rotating fluid", J. Phys. Oceanogr, Vol. 34, 188-203). We had to choose either steady with stirring diffusion or non-steady without diffusion to solve momentum equations, as the path of the flow was important.

Concerning particle shift as much as 30 km, we should note that a particle cannot move about 30 km in South Caspian due to topography, while sinking down. In Fig 9 (paper) we just show that the path of flow in the absence of topography. As  $\theta$  is constant, in reality the flow move about 10 km (maximum) as the strait is short, then the flow is trapped by topography and does not find the opportunity to do oscillating motion (Fig 7 in paper, or the below figure).

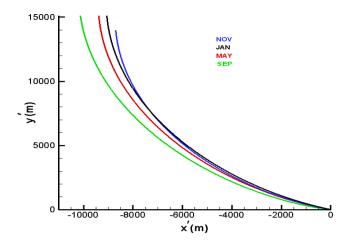


Fig 9.

As seen in Fig 10, flow moves about 8.5 km, then with your estimation the flow descent only about 170 meters (8500×0.02=170 m). If you note the numerical model results (Fig 10), flow descent about 140 or 150 meters. It is logical because of our assumption (with no diffusion).

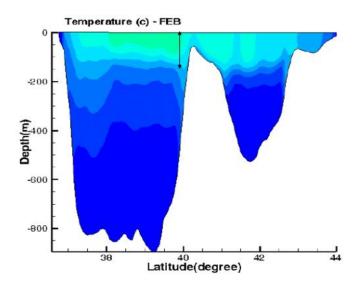


Fig 10.

Concerning accuracy of this model, you can see Appendix A.

4. In this analytical model, we use conservation of potential vorticity (PV) and assume that vorticity of flow is  $\xi_1$  in the strait of Apsheron (Fig 11) and  $\xi_2$  (= $\xi_1$ +c) when the flow is trapped. We consider c=0, so potential vorticity is conserved. We have to consider c=0 because it is impossible to estimate c. In last article (Bidokhti and Ezam)  $\xi_1$  is divided into two vorticities, relative and planet vorticity, because they consider geostrophic flow. In this paper, we write momentum equation based on Quasi-geostrophic flow. In this situation, the angle between the pressure gradient and Coriolis is not 90 degree, due to bottom friction which was considered in our model (please see Fig 12).

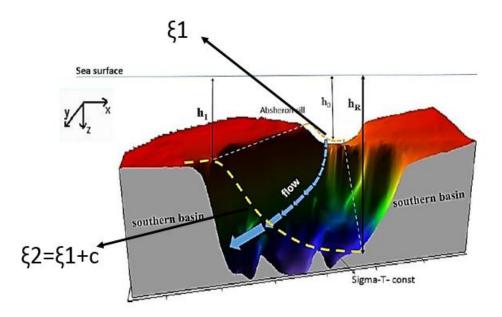
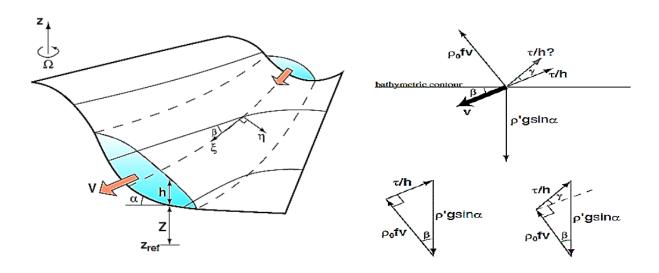


Fig 11.



**Fig 12** A model of steady trapped flow affected by bottom topography (Girton, J. B., Sanford, T. B., 2001. "Descent and Modification of the Overflow Plume in the Denmark Strait", Geophys. Res. Lett., 28, 1619–1622).

In this model, part of vorticity (relative) is converted due to friction. For this reason when we increase r in the model Rossby deformation radius decrease because part of relative vorticity is more converted or the angle between the pressure gradient and the Coriolis is further reduced. It is not against the Conservation of potential vorticity because  $\xi_1 \approx \xi_2$ . Vorticity is distributed between three factors unlike in the case with the case with perfect PV conservation as has been used in the past.

Here we have used the model based on two layers and considered a isopynal line after the strait in the South Caspian then, we write momentum equation based on this isopynal line. In this model two layers move together and we can define boundary conditions (BC) better when consider the surface BC. This method is common in other paper, e.g. please Bidokhti, A. A. and Ezam, M., 2009. The structure of the Persian Gulf outflow subjected to density variations, Ocean Sci., 5, 1–12, doi: 10.5194/os-5-1-2009).

In this paper, coefficient of friction is a linear function of velocity. Some paper assume linear for example see this equation:

$$-r\underline{u} = \frac{rg'}{f}\underline{k} \times \nabla(H - h) - \frac{r^2}{f}(\underline{k} \times \underline{u}).$$

In Mansley, H.C., and Marshall, D., 2004. "Dense Water Overflows and Cascades", University of Reading, pp.1-65.

5- In this article the following equation is used to calculate volume

$$Q_V = \int v dA$$

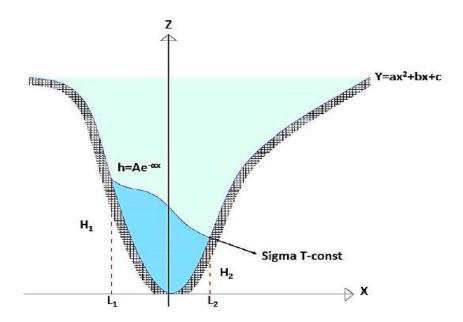


Fig 13.

To calculate velocity and area which water enter the south Caspian Sea, we use the shape of the reference isopycnal when the Strait topography crosses the isopycnal at two points. We use geostrophic flow  $(v=g'\frac{\partial h}{\partial x})$  to calculate velocity, v which means that:

$$\frac{g'}{f} \frac{H_1 - H_2}{L_2 - L_1}$$

To calculate dA, we integrate the isopynal while using the parabolic equation of the strait topograpgy, then calculated area is:

$$Q_{V} = \frac{g'}{f} \frac{H_{1} - H_{2}}{L_{2} - L_{1}} \left[ \left( -\frac{A}{\alpha} (e^{-\alpha L_{2}} - e^{-\alpha L_{1}}) - \frac{a}{3} (L_{2}^{3} - L_{1}^{3}) - \frac{b}{2} (L_{2}^{2} - L_{1}^{2}) \right]$$
 (1)

At first, we use  $L_1$  and  $L_2$  because the shape of topography is approximately symmetric, hence  $L1 \approx L2$ . Please see the following Fig

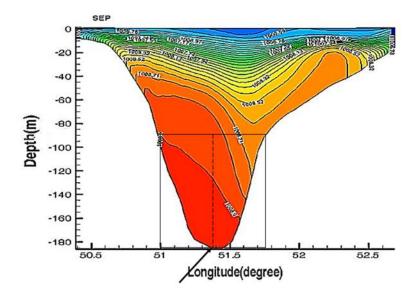


Fig 14. To simplify, we can also assume L1=L2=L which L=(|L2|+|L1|)/2 , So we earn the following equation:

$$Q_{V} = \frac{g'}{f} \frac{H_{1} - H_{2}}{L_{2} - L_{1}} \left[ \frac{2A}{\alpha} \sinh(\alpha L) - \frac{2}{3} a L^{3} \right]$$
 (2)

Now, we show that this assumption does not produce large differences. For example in January L2|+|L1|=32000~m we assume that under the worst conditions  $|L_2|=17000~m$ ,  $|L_1|=15000~m$  and L=16000.

We calculated  $Q_V$  by (1) and (2) formula above, and compare the results with each other in following table, which are not much different.

Formula	a	b	α	A	Qv(SV)
1	4.43×10 <sup>-7</sup>	-9.877×10 <sup>-4</sup>	1.669×10 <sup>-5</sup>	109.18	0.109
2	4.49×10 <sup>-7</sup>	-1.875×10 <sup>-3</sup>	1.669×10 <sup>-5</sup>	111.018	0.115

Concerning other formula to calculate  $Q_V$ , you suggested that we use this formula (Borenäs and Lundberg, 1988). We found the following relations.

$$Q = \frac{h^2}{2+r} \sqrt{\frac{3f^2}{2r}}$$
$$r = \alpha f^2 / g'$$

in Wilkenskjeld, S. and Quadfasel, D.,2005. "Response of the Greenland-Scotland overflow to changing deep water supply from the Arctic Mediterranean", GEOPHYSICAL RESEARCH LETTERS, VOL. 32, L21607.

The equation is presented to calculate  $Q_V$  in the **Faroe Bank Channel** based on field observations in **Faroe Bank**, which seems useful to estimate  $Q_V$  for that area. They used this formula to calculate the maximum volume in the current case. The values for the two method are shown in the following table, indicating that their formula gives much larger values giving the flushing time of about 10 years which will be added to the present paper.

	Borena"s and Lundberg-Qv(SV)	Current paper- Q <sub>V</sub> (SV)	
NOV	0.028	0.016	
JAN	0.42	0.115	

In the paper table 3 L should be changed to 2L.

	$H_1(m)$	$H_2(m)$	<i>2L</i> (m)	$Q_V(Sv)$
NOV	55	10	19000	0.016
JAN	145	85	32000	0.115
MAY	145	55	34000	0.146
SEP	135	45	27500	0.116

Concerning accuracy of this model, you can see Appendix B.

6. The editing error in the text will be corrected, while noting the mentioned ones by the Referee #1

## Appendix A

Derivation of the first analytical model:

$$\frac{du'}{dt} = -g' \tan \theta + fv' - ru'$$

$$\frac{dv'}{dt} = -fu' - rv'$$

$$u'(t) = c_1 e^{-rt} \sin(ft) + c_2 e^{-rt} \cos(ft) - \frac{g'r \tan \theta}{f^2 + r^2}$$
$$v'(t) = c_1 e^{-rt} \cos(ft) - c_2 e^{-rt} \sin(ft) + \frac{g'f \tan \theta}{f^2 + r^2}$$

$$x'(t) = \frac{fc_2 - rc_1}{f^2 + r^2} e^{-rt} \sin(ft) - \frac{fc_1 + rc_2}{f^2 + r^2} e^{-rt} \cos(ft) - \frac{g'r \tan \theta}{f^2 + r^2} t + \frac{fc_1 + rc_2}{f^2 + r^2}$$

$$y'(t) = \frac{fc_1 + rc_2}{f^2 + r^2} e^{-rt} \sin(ft) + \frac{fc_2 - rc_1}{f^2 + r^2} e^{-rt} \cos(ft) + \frac{g'f \tan \theta}{f^2 + r^2} t + \frac{rc_1 - fc_2}{f^2 + r^2}$$

$$\begin{split} \frac{dx'}{dt} &= \frac{fc_2 - rc_1}{f^2 + r^2} \Big[ - r \, e^{-rt} \sin(ft) + f \, e^{-rt} \cos(ft) \Big] - \frac{fc_1 + rc_2}{f^2 + r^2} \Big[ - r \, e^{-rt} \cos(ft) - f \, e^{-rt} \sin(ft) \Big] + \frac{-g'r \tan \theta}{f^2 + r^2} \\ u' &= \frac{fc_2 - rc_1}{f^2 + r^2} \, e^{-rt} (f \cos(ft) - r \sin(ft)) + \frac{fc_1 + rc_2}{f^2 + r^2} \, e^{-rt} (r \cos(ft) + f \sin(ft) + \frac{-g'r \tan \theta}{f^2 + r^2}) \\ u' &= \frac{fc_2 - rc_1}{f^2 + r^2} \, e^{-rt} \times f \cos(ft) - \frac{fc_2 - rc_1}{f^2 + r^2} \, e^{-rt} r \sin(ft) + \frac{fc_1 + rc_2}{f^2 + r^2} \, e^{-rt} \times r \cos(ft) + \frac{fc_1 + rc_2}{f^2 + r^2} \, e^{-rt} f \sin(ft) - \frac{g'r \tan \theta}{f^2 + r^2} \\ u' &= \cos(ft) \, e^{-rt} \Big[ f (\frac{fc_2 - rc_1}{f^2 + r^2} + r(\frac{fc_1 + rc_2}{f^2 + r^2}) \Big] + \sin(ft) \times e^{-rt} \Big[ f \frac{(fc_1 + rc_2)}{f^2 + r^2} - r \frac{(fc_2 - rc_1)}{f^2 + r^2} \Big] - \frac{g'r \tan \theta}{f^2 + r^2} \\ u' &= \cos(ft) \times e^{-rt} \Big[ \frac{f^2c_2 - rfc_1 + rfc_1 + r^2c_2}{f^2 + r^2} \Big] + \sin(ft) \times e^{-rt} \Big[ \frac{f^2c_1 + frc_2 - rfc_2 + r^2c_1}{f^2 + r^2} \Big] - \frac{g'r \tan \theta}{f^2 + r^2} \\ u' &= c_2 \, e^{-rt} \cos(ft) + c_1 \, e^{-rt} \sin(ft) - \frac{g'r \tan \theta}{f^2 + r^2} * *ok \end{split}$$

$$\frac{dy'}{dt} = \frac{fc_1 + rc_2}{f^2 + r^2} \left[ -re^{-rt} \sin(ft) + fe^{-rt} \cos(ft) \right] + \frac{fc_2 - rc_1}{f^2 + r^2} \left[ -re^{-rt} \cos(ft) - fe^{-rt} \sin(ft) \right] + \frac{g'f \tan \theta}{f^2 + r^2}$$

$$v' = e^{-rt} \sin(ft) \left[ \frac{-rfc_1 - r^2c_2}{f^2 + r^2} \right] + e^{-rt} \cos(ft) \times \left[ \frac{f^2c_1 + rfc_2}{f^2 + r^2} \right] + \frac{r^2c_1 - rfc_2}{f^2 + r^2} \cos(ft) \times e^{-rt} + \frac{-f^2c_2 + rfc_1}{f^2 + r^2} e^{-rt} \sin(ft) + \frac{g'f \tan \theta}{f^2 + r^2}$$

$$v' = e^{-rt} \times \sin(ft) \times - \left[ \frac{f^2c_2 - rfc_1 + r^2c_2 + rfc_1}{f^2 + r^2} \right] + \cos(ft) \times e^{-rt} \left[ \frac{r^2c_1 - rfc_2 + f^2c_1 + rfc_2}{f^2 + r^2} \right] + \frac{g'f \tan \theta}{f^2 + r^2}$$

$$v' = -c_2 e^{-rt} \sin(ft) + c_1 e^{-rt} \cos(ft) + \frac{g' f \tan \theta}{f^2 + r^2} ***ok$$

$$\frac{du'}{dt} = -g'\tan\theta + fv - ru$$

$$-rc_1 e^{-rt}\sin(ft) + fc_1 e^{-rt}\cos(ft) - rc_2 e^{-rt}\cos(ft) - fc_2 e^{-rt}\sin(ft)$$

$$= -g'\tan\theta + fc_1 e^{-rt}\cos(ft) - fc_2 e^{-rt}\sin(ft) + \frac{g'f^2\tan\theta}{f^2 + r^2} - rc_1 e^{-rt}\sin(ft) - rc_2 e^{-rt}\cos(ft)$$

$$+ \frac{g'r^2\tan\theta}{f^2 + r^2} = 0$$

$$- g'f^2\tan\theta - g'r^2\tan\theta + g'f^2\tan\theta + g'f^2\tan\theta - g'r^2\tan\theta = 0$$

$$0 = 0 ***ok$$

$$\frac{dv'}{dt} = -rc_1 e^{-rt} \cos(ft) - fc_1 e^{-rt} \sin(ft) + rc_2 e^{-rt} \sin(ft) - fc_2 e^{-rt} \cos(ft)$$

$$\frac{dv'}{dt} = -fu' - rv'$$

$$-rc_1 e^{-rt} \cos(ft) - fc_1 e^{-rt} \sin(ft) + rc_2 e^{-rt} \sin(ft) - fc_2 e^{-rt} \cos(ft) = -fc_1 e^{-rt} \sin(ft)$$

$$-fc_2 e^{rt} \cos(ft) + \frac{g'rf \tan \theta}{f^2 + r^2} - rc_1 e^{-rt} \cos(ft) + rc_2 e^{-rt} \sin(ft)$$

$$0 = 0***ok$$

$$u'(t = 0) = c_2 - \frac{g'r \tan \theta}{f^2 + r^2} = u_p$$

$$c_2 = u_p + \frac{g'r \tan \theta}{f^2 + r^2}$$

$$v(t = 0) = v_p$$

$$c_1 + \frac{g'f \tan \theta}{f^2 + r^2} = v_p$$

$$c_1 = v_p - \frac{g'f \tan \theta}{f^2 + r^2}$$

## **Appendix B**

Derivation of the analytical model for calculating Qv:

$$if(x = 0) = 0 \rightarrow y = 0$$

$$y = 0 \rightarrow c = 0$$

$$if(x = -L_1)Then$$

$$y_1 = aL_1^2 - bL_1 \rightarrow$$

$$H_1 = aL_1^2 - bL_1 ***(1)$$

$$if(x = L_2)Then$$

$$y_2 = aL_2^2 + bL_2$$

$$H_2 = aL_2^2 + bL_2 ****(2)$$

$$(1) \rightarrow bL_1 = aL_1^2 - H_1 \rightarrow$$

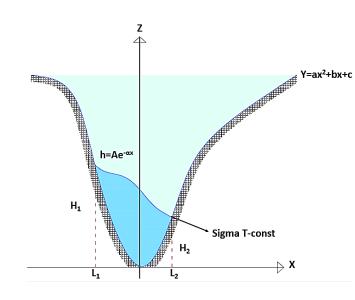
$$b = \frac{aL_1^2 - H_1}{L_1} ***(3)$$

$$(3),(2) \rightarrow H_2 = aL_2^2 + L_2(\frac{aL_1^2 - H_1}{L_1})$$

$$H_2 = aL_2^2 + aL_1L_2 - \frac{L_2}{L_1}H_1$$

$$H_2 + \frac{L_2}{L}H_1 = a(L_2^2 + L_1L_2) \rightarrow$$

$$a = \frac{H_2L_1 + L_2H_1}{L_1(L_1L_2 + L_2^2)} **(4)$$



$$(4), (3) \to b = aL_1 - \frac{H_1}{L_1} = \frac{H_2L_1^2 + L_2L_1H_1}{L_1(L_1L_2 + L_2^2)} - \frac{H_1}{L_1}$$

$$b = \frac{H_2L_1^2 + L_2H_1L_1}{L_1(L_2^2 + L_1L_2)} - \frac{H_1}{L_1}$$

$$b = \frac{H_2L_1 + L_2H_1}{L_2^2 + L_1L_2} - \frac{H_1}{L_1} **(5)$$

$$Q_V = \int UdA$$

$$dA = \int hdx - \int ydx$$

$$h = Ae^{-\alpha x} \to H_1 = Ae^{\alpha L_1} \to A = \frac{H_1}{e^{\alpha L_1}} **(6)$$

$$if(x = L_{2}) \to H_{2} = Ae^{-\alpha L_{2}} \to H_{2} = \frac{H_{1}}{e^{\alpha L_{1}}}e^{-\alpha L_{2}}$$

$$\frac{H_{2}}{H_{1}} = e^{-\alpha(L_{1} + L_{2})} \to Ln\frac{H_{2}}{H_{1}} = -\alpha(L_{1} + L_{2})$$

$$\alpha = -\frac{1}{L_{1} + L_{2}}Ln\frac{H_{2}}{H_{1}} **(7)$$

$$\int hdx = \int_{L_{1}}^{L_{2}} Ae^{-\alpha}dx = \frac{A}{-\alpha}(e^{-\alpha L_{2}} - e^{\alpha L_{1}})$$

$$\int_{L_1}^{L_2} y dx = \int_{L_1}^{L_2} (ax^2 + bx) dx = \frac{a}{3} (L_2^3 + L_1^3) + \frac{b}{2} (L_2^2 - L_1^2)$$

$$Q_{V} = \frac{g'}{f} \frac{H_{2} - H}{L_{2} - L_{1}} \left[ \left[ \frac{-A}{\alpha} (e^{-\alpha L_{2}} - e^{\alpha L_{1}}) - \left[ \frac{a}{3} (L_{2}^{3} + L_{1}^{3}) \right] + \frac{b}{2} (L_{2}^{2} - L_{1}^{2}) \right]$$

$$\alpha = \frac{H_2 L_1 + H_1 L_2}{L_1 (L_1 L_2 + L_2^2)}$$

$$b = \frac{H_2 L_1 + L_2 H_1}{L_2^2 + L_1 L_2} - \frac{H_1}{L_1}$$

$$A = \frac{H_1}{e^{\alpha L_1}}$$

$$\alpha = \frac{-1}{L_1 + L_2} Ln \frac{H_2}{H_1}$$

$$if \rightarrow |L_1| \cong |L_2| \cong L$$

$$Q_{V} = \frac{g'}{f} \frac{H_{2} - H_{1}}{L_{2} - L_{1}} \left[ \left[ \frac{-A}{\alpha} \left( e^{-\alpha L} - e^{\alpha L} \right) - \left[ \frac{2a}{3} \left( L^{3} \right) \right] \right]$$

$$Q_{V} = \frac{g'}{f} \frac{H_{2} - H}{L_{2} - L_{1}} \left[ \left[ \frac{2A}{\alpha} \left( \frac{e^{\alpha L} - e^{-\alpha L}}{2} \right) - \left[ \frac{2a}{3} \left( L^{3} \right) \right] \right]$$

$$Q_{V} = \frac{g'}{f} \frac{H_{2} - H_{1}}{L_{2} - L_{1}} \left[ \frac{2A}{\alpha} Sinh(\alpha L) - \frac{2}{3} \alpha L^{3} \right]$$

$$Or$$

$$Q_{V} = \frac{g'}{f} \frac{H_{2} - H_{1}}{2L} \left[ \frac{2A}{\alpha} Sinh(\alpha L) - \frac{2}{3} \alpha L^{3} \right]$$

Where

$$\alpha = \frac{H_2 + H_1}{2L_2^2}$$

$$b = \frac{H_2 - H_1}{2L}$$

$$A = \frac{H_1}{aL}$$

$$\alpha = \frac{-1}{L_1 + L_2} Ln \frac{H_2}{H_1}$$