This manuscript attempts to derive meaningful indices for Kelvin wave activity along the equatorial and coastal waveguide in the Indian Ocean. It is a worthwhile goal, and the authors have certainly invested a great deal of effort in the pursuit, but I regret that I cannot recommend the manuscript for publication. There are too many unsupportable and erroneous mathematical manipulations that don't do what the authors intend them to, and I cannot come up with suggestions for straightforward corrections that would fix the problems. Below I will list problems as they arise in the manuscript, but this list is not necessarily comprehensive. By the point in the manuscript where the list stops, I could no longer believe in the product of the manipulations.

1) Line 58: The authors claim to be building on the methodology of Boulanger and Menkes, 1995, 1999, but this is not true. The problem with deducing equatorial wave amplitudes from only Sea Level Anomalies (SLAs) is that the signals of separate wave modes are not orthogonal (as the authors acknowledge). The individual wave amplitudes could only be deduced if the observed meridional profile of SLA were composed of only a finite number of modes. Boulanger and Menkes made the reasonable approximation that MOST of the SLA profile could be described by a large but finite number of modes (21) and then they derived an approximate solution for the singular matrix associated with projections onto all these modes. This manuscript only attempts a projection onto the lowest Hermite function, which if done properly, would not be able to distinguish between the amplitudes of the Kelvin wave and the first meridional mode long-Rossby wave; both of which can be expected to be present in the SLA signal. Presumably, these can be separated later by separating the eastward from the westward propagating signals, but the ambiguity should be acknowledged up front. It is misleading to call the projection attempted in equation (2) the "Kelvin wave y-projection."

2) The "projections" in (2) and (3) are not really projections, the mathematical forms and the choice of integration limits are puzzling, and the authors provide no justification for their choices. In Fourier analysis it is common to subtract the mean of a data set prior to projecting onto the sines and cosines, but this works because the basis functions all have zero mean. This is not true of the Hermite functions, and it is not true of the exponential profile of the coastal wave. Subtracting the means before integration introduces extra terms that have nothing to do with the desired projection, and limiting the integration limits to 5 degrees latitude ensures that the Kelvin wave structure will not even be orthogonal to structures that it should be orthogonal to.

I'll use a simple idealized situation to illustrate the problems with (2). On an unbounded equatorial β-plane, the orthonormal Hermite functions, $\psi_n(\hat{y}),\quad n = 0, 1, 2, \cdots$, provide a complete basis. The argument of the Hermite function is the latitude normalized by the deformation radius: $\hat{y} = y/L_e$, with $L_e = \sqrt{c/\beta} \approx 3°$ latitude for $c = 2.5$ m/s. Even in a bounded basin like the Indian Ocean, a truncated set of these functions does a decent job of describing meridional structures while remaining approximately orthonormal. An exception might be the region just south of Sri Lanka, but this is only a small part of the longitudinal span of the basin.

The meridional structure of an equatorial Kelvin wave's SLA is ψ_0: the Gaussian part of the authors' equation (1), with a normalizing factor of $\pi^{-1/4}$ (so that $\int_{-\infty}^{\infty} \psi_0^2 \, d\hat{y} = 1$). The structure of the mode-1 long-Rossby wave's SLA is $(2^{-1/2}\psi_2 + \psi_0)$. Suppose the measured equatorial SLA contains only a Kelvin wave of amplitude A_K, a 1st mode long-Rossby wave of amplitude A_1 and a background of other variability that is orthogonal to the Kelvin wave: $B(y) = \Sigma_{n=1}^{\infty} b_n \psi_n$. A true projection onto the Kelvin wave structure would be

$$
\begin{aligned}
K_y &= \int_{-\infty}^{\infty} h_{SLA} \, \psi_0 \, d\hat{y} && (1) \\
&= \int_{-\infty}^{\infty} [A_K \psi_0 + A_1(2^{-1/2}\psi_2 + \psi_0) + B] \, \psi_0 \, d\hat{y} && (2) \\
&= (A_K + A_1) \int_{-\infty}^{\infty} \psi_0^2 \, d\hat{y} + 2^{-1/2} A_1 \int_{-\infty}^{\infty} \psi_2 \psi_0 \, d\hat{y} + \Sigma_{n=1}^{\infty} b_n \int_{-\infty}^{\infty} \psi_n \psi_0 \, d\hat{y} && (3) \\
&= A_k + A_1. && (4)
\end{aligned}
$$

This demonstrates the non-orthogonality of the pressure structures of equatorial waves, but at least we're only left with two modes to be sorted out later. If the integration limits are reduced to $\pm 10°$, the results are essentially the same, with only a small error. At the authors' chosen integration limit of 5° latitude, however, the Kelvin wave structure is still almost 25% of it's maximum value, and over this interval it is not even approximately orthogonal to any of the other even Hermite functions. In this case, a proper attempt at a projection would yield:

$$
\begin{aligned}
K_y &= (A_K + A_1) \int_{-5°/Le}^{5°/Le} \psi_0^2 \, d\hat{y} + 2^{-1/2} A_1 \int_{-5°/Le}^{5°/Le} \psi_2 \psi_0 \, d\hat{y} + \Sigma_{n=1}^{\infty} b_n \int_{-5°/Le}^{5°/Le} \psi_n \psi_0 \, d\hat{y} && (5) \\
&= 0.98(A_k + A_1) - 0.06 A_1 + 0.08 b_2 + 0.06 b_4 + \Sigma_{n=6}^{\infty} b_n \int_{-5°/Le}^{5°/Le} \psi_n \psi_0 \, d\hat{y}. && (6)
\end{aligned}
$$

The additional terms may be individually small, but a realistic background would contain a large number of them, and they can add up to a significant number that has nothing to do with the Kelvin wave amplitude, all because the integration was not carried to a latitude where the Kelvin wave is truly insignificant.

An even worse situation arises when the means are subtracted prior to the integration, as in the manuscript's equation (2). I will continue to integrate in the nondimensional coordinate $\hat{y}$ for consistency with the above equations, but note that with the exception of a normalizing constant, the "projection" below is identical to (2) in the manuscript. Integrating over $\hat{y}$, the authors' definition of mean becomes

$$
\overline{a} \equiv \frac{1}{2r/Le} \int_{-r/Le}^{r/Le} a \, d\hat{y}, \qquad (7)
$$

and the "projection" in their equation (2) is

$$K_y = \frac{1}{2}\int_{-r/L_e}^{r/L_e}(h_{SLA} - \overline{h_{SLA}})(\psi_0 - \overline{\psi_0})\ d\hat{y} \tag{8}$$

$$\begin{aligned} &= \frac{1}{2}\int_{-r/L_e}^{r/L_e} h_{SLA}\ \psi_0\ d\hat{y}\ -\ \overline{h_{SLA}}\left(\frac{r}{L_e}\right)\frac{1}{2r/L_E}\int_{-r/L_e}^{r/L_e}\psi_0\ d\hat{y} \\ &\quad -\ \overline{\psi_0}\left(\frac{r}{L_e}\right)\frac{1}{2r/L_E}\int_{-r/L_e}^{r/L_e} h_{SLA}\ d\hat{y}\ +\ \overline{h_{SLA}}\ \overline{\psi_0}\frac{1}{2}\int_{-r/L_e}^{r/L_e}\ d\hat{y} \end{aligned} \tag{9}$$

$$= \frac{1}{2}\int_{r/L_e}^{r/L_e} h_{SLA}\ \psi_0\ d\hat{y}\ -\ \frac{r}{L_e}\overline{\psi_0}\ \overline{h_{SLA}}. \tag{10}$$

Using the manuscript's values of r and c, the equation is

$$K_y = \frac{1}{2}\int_{-5^\circ/L_e}^{5^\circ/L_e} h_{SLA}\ \psi_0\ d\hat{y} - 0.85\ \overline{h_{SLA}}. \tag{11}$$

The first term on the right-hand-side is half of the true projection of the SLA onto the Kelvin wave structure (which includes the mode-1 Rossby wave amplitude), except that it will contain the extraneous terms noted above because the value $r = 5^\circ$ is too small. The second term is a fraction of the mean SLA and has nothing to do with the Kelvin wave amplitude. Furthermore, the factor in front of $\overline{h_{SLA}}$ asymptotes to 0.94 as $r/L_e \to \infty$. This term cannot be removed by choosing larger integration limits. It is the consequence of erroneously subtracting the means of h_{SLA} and ψ_0 in (2).

Similar issues arise with the "projection" in (3) onto the coastal Kelvin wave structure, but I won't go into detail. Even without the problem of subtracting the means, the exponential decay of the coastal wave would project onto just about anything. K_y would likely include contributions that have nothing to do with the Kelvin wave or even the Rossby waves represented by their limited set of $x - t$ "basis" functions. To repeat, the "projections" in (2) and (3) are not really projections onto Kelvin wave structures, and they are not reliable measures of either the Kelvin wave or low-mode Rossby wave amplitudes.

3) Lines 124-127: The authors note that the amplitude of a coastal Kelvin wave increases as the wave propagates poleward in the absence of dissipation, and this is correct. They claim, however, that the integral of the Kelvin wave's SLA structure is thus a better measure of the wave, and this is not true. What remains constant with changing latitude is the energy flux, and this is proportional to the integral of the **squared** SLA structure (which obviously includes the squared amplitude).

4) Equation (1) is not quite correct for Kelvin waves propagating meridionally. This may not matter given my next comment, but the authors should at least note the approximation and cite a reference when they present the equation.

5) For a harmonic solution of given frequency, ω, coastal Kelvin waves as described by (1) in the manuscript do not technically exist equatorward of the turning latitudes

$$Y_t = \pm\sqrt{(\omega/\beta)^2 + (c/2\omega)^2}. \tag{12}$$

Within these latitudes, Rossby waves can propagate freely, any meridional motions on an eastern boundary will shed long Rossby waves, and the transfer of energy flux from an equatorial Kelvin wave incident upon an eastern boundary to the outgoing coastal Kelvin waves poleward of the turning latitudes is more complicated than the simple scenario described by (1). For $c = 2.5$ m/s and a period of 45 days, the turning latitudes are at $\pm 7°$, which is about the latitude of the southwestern tip of Java. For periods this long or longer, it is not worth trying to describe a Kelvin wave propagating down the coast of Sumatra from the equator.

The actual Kelvin waves are pulse-like and so contain a spectrum of frequencies, some of which maybe high enough to sustain a true Kelvin wave along the coast of Sumatra, but even if you could describe the alongshore energy flux of the entire pulse as a pseudo-Kelvin wave, its energy flux would be decreasing steadily poleward as energy is shed westward into Rossby modes. This loss of along-shore energy flux would be much different than that produced by dissipation. The tapered pseudo-basis functions would not apply equally well to both situations.

6) The $x - t$ "basis" functions presume that all signals are moving along the coast in one direction or the opposite, and the suite of functions is truncated to waves moving in the Kelvin wave direction for a range of appropriate phase speeds and waves moving in the opposite direction for a range of Rossby wave speeds. If a Kelvin wave could be described along the coast of Sumatra, the Rossby waves would not propagate in the opposite direction from the Kelvin wave. The basis function would be trying to isolate signals propagating northwestward at the Rossby wave's westward phase speed. The Rossby wave's amplitude is embedded in the K_y vector, but the amplitude of the waves shedding westward at one latitude does not match the amplitude of the wave shedding westward at a different latitude in a way described by the basis function.

7) There is often an energetic eddy field south of Java and the Indonesian Islands that is close enough to the coast to be captured by the K_y integration in (3). This is a place where the Kelvin and Rossby waves do propagate in opposite directions, but the manuscript limits the westward propagating basis functions to phase speeds $-1.2\,\text{m/s} < c_{m,n} < -0.4$ m/s. The eddies propagate at -0.15 to -0.2 m/s (Feng and Wijfells, JPO, 2002), so even though their SLA signal will be included in the K_y vector, the basis functions will not be able to filter them out through their propagation characteristics.

8) The "basis" functions are not truly basis functions - they do not constitute a complete set. I would have to see more details of the mathematic involved in the "projections" onto the tapered functions to asses the value of this step.

Unfortunately, at this point I have lost confidence in the authors' understanding of precisely what each of their calculations does. It feels like a series of flawed mathematical manipulations that produces an answer of dubious value. The fact that their figures qualitatively resemble the raw SLA plots is probably due to the robustness of the Kelvin wave in this part of the world, but I can't believe that the final K product has more value than an amplitude derived by simply tracking alongshore time-longitude plots. The mathematical complexity of the procedure implies a rigor that is not really justified.

The authors have obviously put considerable effort into this procedure and it is possible that with the above comments in mind and a more careful approach, they could rework the procedure into something more meaningful. I can't suggest an obvious way forward, however, so I regret that I cannot recommend the present manuscript for publication.