“A harmonic projection and least-squares method for quantifying Kelvin wave activity”
Response to Reviewer #1

This paper proposes a statistical method for extracting the Kelvin wave signal from altimetric data in the Indian ocean both along the equator and along the coast. While I find the topic worth addressing and relevant, I have reservations regarding the method.

General comments: Many thanks to the reviewer for this thoughtful review. Based on feedback received from both reviewers, the manuscript has been reworked so that the method is clearer and easier for others to replicate; the revised method also enables more direct comparisons between our results and the results of other methods that have extracted the Kelvin wave signal from SSH data. Of particular note: since earlier work by Delcroix et al. (1994), Boulanger and Menkes (1995; 1999), and others have simultaneously extracted the signal from Kelvin waves and the lowest meridional mode Rossby waves, we have revised our method to also recover signals representing both Kelvin and Rossby waves. Hence, the title of the article has been changed to “A harmonic projection and least-squares method for quantifying equatorial wave activity”. Moreover, the output of our method is now an SLA field associated with each mode (rather than the Kelvin wave coefficient, whose physical significance was not obvious). In the revised manuscript we also exclude the coastal wave part of the decomposition and focus on the more straightforward equatorial region to refine our method; it is hoped that coastal waves can be tracked using a similar method in a future study.

Note about the manuscript: All of the text that has been revised in the article (which includes most of the material) is in italics.

1) The paper mentions the study by Boulanger and Menkes (1999) which is a method that provides estimate of the equatorial Kelvin based on the projection of the theoretical meridional wave structures. We wonder why the authors do not test this method to compare with theirs, since the method by BM99 was validated from independent observations
(by reconstructing the wave-induced zonal currents and comparing to TAO data). This would provide more confidence in their results if both methods compare to some extent. The method should be at least applied to the equatorial Pacific and compared with estimates from Boulanger and Menkes (1995, 1999).

The reviewer makes a very good point here; in the original manuscript we applied our method only to the Indian Ocean so a direct comparison could not be made between our results and those from Boulanger and Menkes (1999). This has been rectified; in the revised manuscript the method is applied to SLA data in both the equatorial Indian and Pacific Oceans. To facilitate the comparison with Boulanger and Menkes (1999) in particular, Figure 7 shows the Kelvin and mode 1 Rossby waves in the same format and using the same units as in Figure 6 of Boulanger and Menkes (1999). The results are broadly similar, though our method better highlights the higher-frequency (intraseasonal) propagating waves more as described in Section 4 (lines 290-304).

2) The equatorial Kelvin wave while impinging along the coast of Indonesia could be trapped or reflect as Rossby wave depending on its vertical structure and frequency (Clarke and Shi, 1991). Since the method does not discriminate explicitly the frequencies in the raw SLA, it is not clear to which extend it is able to actually grasp the share of the variability that is trapped along the coast from the one that radiates off-shore as Rossby wave. In fact, the cross-shore scale of the Kelvin wave that is extracted indicates that there is probably a mixture between Kelvin and Rossby waves, and might explain the large residual at some places (Figures 7 and 8). Could the authors comment on that? How are altered the results of the projection when the raw SLA data are filtered in the frequency domain so as to retain intraseasonal frequencies?

As the reviewer noted, when the Kelvin wave reaches the eastern boundary and starts to propagate poleward along the coast, it can shed Rossby waves. In the earlier manuscript, it was assumed that the loss in Kelvin wave energy that resulted from Rossby wave shedding would be adequately represented using the tapered basis functions, but we agree that the distinction between the Kelvin and Rossby wave signals was likely muddled along coastal Sumatra.

Hence, the reviewer's point is a major reason that we decided to focus on the equatorial oceans in the revised manuscript, in order to validate the method in a context that has been extensively studied using theory and observations. In the revised manuscript, the (low-mode) Rossby wave signals are also explicitly extracted, and both Kelvin and Rossby wave signals are validated using correlations with SLA data and residuals (Section 3).

3) While in the Pacific the equatorial Kelvin wave structure can be assumed to reduce to the one of the first baroclinic mode, this is certainly not the case for the Indian ocean where the wave structure is more complex and result from the superposition of a number of baroclinic modes. This is a difficulty that is not discussed in the paper and that could be of importance. In fact it is not clear what is the purpose of doing the y-projection considering that this projection does not provide the Kelvin wave coefficient. We would expect that if the method is applied on the raw SLA data at each latitudes (without performing the y-projection), the results of the projection onto the basis functions (i.e. K(y)) should have a meridional structure of a Kelvin wave (i.e. a Gaussian curve) from which a phase speed could be inferred. Could the authors verify that? This would be a stringent test that the method does allow extracting the Kelvin wave signal from the raw SLA. It is also expected that the zonal change in the meridional structure reflects the sloping thermocline (i.e. larger value of c to the west).

In the earlier manuscript, the projection of the meridional (or cross-shore) structure of the wave was carried out first to obtain the Kelvin wave "y-projection", followed by the projection of the harmonic basis functions to obtain the Kelvin wave coefficient. The separation of these two steps was a consequence of the evolution of our thinking and implementation of the code to compute the Kelvin wave coefficient. However, the method has been revised, so that the projections are carried out in x, y, and t simultaneously. This is accomplished by projecting three-dimensional basis functions of the form given in equation (1) of the revised manuscript.

The new three-dimensional basis functions contain both the meridional structures and the harmonic functions that were used previously. Projecting these three-dimensional structures allows the waves to be differentiated based on both their meridional structures and approximate phase speeds, in a more efficient way than projecting the meridional structure separately.

Additionally, the correlations of the mode 1 SLA field generated by the method with the original SLA data (Figures 2b,d) allow our assumption that the phase speed c = 2.5 m/s in the equatorial Indian Ocean to be tested. We expect that the correlation values will be highest where the amplitude of the true mode 1 Rossby wave is largest, and indeed the correlation has its maximum values along the axis of maximum amplitude for a mode 1 Rossby wave with a Kelvin wave phase speed of c = 2.5 m/s. We are also aware that the second baroclinic mode, while not explicitly resolved in this analysis, makes significant contributions to the SLA field in the equatorial Indian Ocean. As the text in Section 3 of the revised manuscript now reads:

"The 2.5 m s^{-1} phase speed is consistent with the Argo-based 1st baroclinic mode phase speed estimate by Nagura and McPhaden (2012); according to the estimate by Drushka et al. (2010), also based on Argo data, this value would lie between the 1st and 2nd baroclinic mode Kelvin wave phase speeds (2.8 and 1.8 m s^{-1} respectively). As Drushka et al. (2010) and others have found, the first and second baroclinic modes make substantial contributions to intraseasonal wave activity in the equatorial Indian Ocean, so the 2.5 m s^{-1} phase speed value may represent waves that contain both baroclinic modes." (lines 238-244)

Other comments:

p. 4, l. 105: please provide a reference for this formula (1)

As the focus of the manuscript is now exclusively on equatorial wave modes, this formula (for a Kelvin wave in the equatorial-coastal transition) is no longer included.

p. 4, l. 115: this is surprising. Could you illustrate that? I think this is due to the fact that r=5_S. Please explain what is the purpose of the y-projection.

The fact that the Kelvin wave projections did not depend much on the value of c (at least varying between 2.0 and 3.0 m/s) probably reflects that the Kelvin wave meridional profile is a Gaussian centered along the equator regardless, with slightly different radii. Specifically, the Kelvin wave profile for c = 2.0 m/s projects highly onto the Kelvin wave profile for c = 3.0 m/s. This is somewhat less the case for Rossby waves, where the latitude of peak amplitudes actually changes position depending on the value c.

As explained in the response to major comment 3 above, the "y-projection" is no longer a part of the method in the revised manuscript. Instead, the projections of the meridional structure and the zonally-propagating basis functions are carried out at the same time, by projecting three-dimensional functions of the form given in equation (1) onto the SLA data.

p. 5, l. 137-142: It is not clear if with such a tapering, the basis functions are still a basis? Also this tapering is in fact modelling the dissipation of the waves, it may need to

be better justified physically and the sensitivity of the results to the tapering parameters would need to be evaluated.

The “basis” functions that are used are not a basis in the traditional sense: a set of functions, orthogonal to one another, that can be summed up (with coefficients) to explain all of the variability in a numerical field. However, even though our “basis” functions are not orthogonal to one another, their application in our method is to achieve a similar goal: to represent the variability present in the field using a set of functions that represent intrinsic modes of variability. To avoid confusion with the traditional mathematical meaning of “basis”, these functions have been renamed wave functions.

As for the sensitivity of the results to the tapering parameters, this was tested to some extent by varying the value of w; the parameter w controls the relative weighting in the cost function of the forcing/dissipation associated with the tapers, vs. the cumulative projection values across the basin. The optimization of the parameter w is described in Section 2.3.

p. 7, l. 195-200: What is called “The Kelvin wave y-projection” has nothing to do with the wave coefficient associated to the Kelvin wave. The wording is confusing. This quantity does contain variability associated to the Rossby wave, so there is no reason why Figure 3b should exhibit more Kelvin-wave-like properties than the raw SLA. In fact performing the y-projection might be a problem since you mix waves having different propagating characteristics (i.e. phase speed and dissipation rate) so that where the amplitude of Ky accounts for the contribution of the Rossby wave and Kelvin wave in a proportion that does not correspond to the actual ratio expected from the local wind forcing.

It is certainly true that the “Kelvin wave y-projection” as used before contained Rossby as well as Kelvin wave signals; the Rossby wave signals then had to be removed by extracting the westward-propagating signals. As explained in the response to major comment 3, the “y-projection” is no longer carried out prior to the projection of the zonally-propagating functions; instead, three-dimensional wave functions are projected onto the SLA data all at once.

p. 8, l. 210: it would be more convincing if a hovmuler of SLA along the coast of Sumatra was provided over an extended period of time.

As the revised analysis no longer includes the coastal part of the waveguide, this comment does not apply.

p. 8, section 3.2: While mathematically correct, I find the test not really relevant. You use the basis functions of propagating modes to construct a surrogate sea level field that you perturb with noise, and then project again on the same basis functions. It is not so surprising that you come up with something you want. However this is not testing if the method is actually able to extract the Kelvin wave signal from altimetric data. It would be more relevant to compare with BM99’s method and apply it on data over the equatorial Pacific.

The Monte Carlo simulations were an attempt to quantify errors that resulted from the least-squares deconvolution of the non-orthogonal basis functions, and did not include errors that might result from other aspects of the projections. We acknowledge the limits of this approach, and have removed that section in the revised manuscript. Instead, correlations were computed between the SLA associated with each mode and the original SLA data (Figure 2), as well as between the mode SLA and the SLA data residual with SLA from all of the projected modes removed (Figure 3). This was done as a means of testing whether the amplitude and phase of the mode contributions to the SLA field are being accurately represented (lines 207-224).