We would like to thank both reviewers for their highlighting the weaknesses of the first version of the manuscript. We tried to take their suggestions into account to make corrections and clarifications in the text. Below, we answer to each comment one by one.

# **Review 1**

RC: Textbooks tell that the smoothing algorithms provide more accurate solutions dynamically and statistically compared to Kalman filtering. Thus the key phrase found in the abstract "Results show that the smoother leads to a better estimation" is too obvious.

AC: Indeed this phrase seems too obvious as it, but in the details it is not. To mitigate the scope of this sentence, we reformulate it as :

Despite model's non-linearities and the various approximations used for its implementation, the smoother leads to a better estimation of the ocean state, both on statistical (i.e. mean error level) or dynamical point of view, as expected from linear theory.

Also, further considerations are added in the introduction (paragraph 2), to remind the reader that the relevance of using a smoother for high dimensional problems is still unclear :

The relevance of using a smoother for high dimensional oceanic or atmospheric problems is still an open question. Even if linear theory says that smoothing decreases residual filter errors, the usual approximations (on non linearity, rank reduction, localisation, etc) take these problems a long way from theory. In the study of of Zhu et al (2003), the smoother was producing apparently poor improvements over the filter, but the meteorological forecasts started from smoother estimates were better. On the contrary, with very different settings (ocean model, forward-backward smoother), Lermusiaux et al (2002) obtained better estimates with the smoother (in terms of errors), but poorer forecasts. Khare et al (2008) tried to identify regimes where the smoother is particularly efficient with an atmospheric model, but this is very case-dependent. The work reported in this paper is part of an effort to determine the relevance of smoothing for realistic oceanic problems.

**RC:** This paper should recognize another important obstacle: time length of lag used in the fixed-lag smoother. Considering the long time-scale of ocean current and waves, I cannot believe "We verified that extending the lag to more than 10 days does not improve the smoother results" (page 1203). Please show some evidences how the lag length affects the assimilated results and accuracy. It must be at least "Extending the lag to more than 10 days hardly (only weakly) improve the smoother results".

AC: Indeed this sentence must be mitigated, as illustrated by Figure 1 of this document. Following Reviewer's suggestion, it is replaced by :

Extending the lag to more than 10 days barely improves the smoother results.

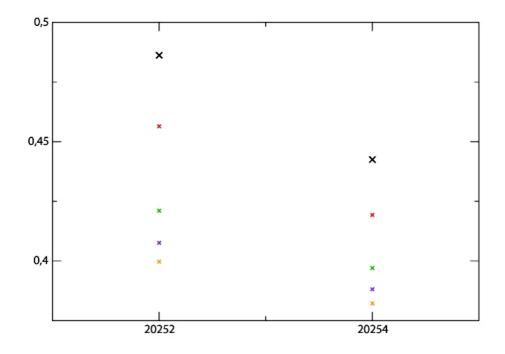


Figure 1 : RMS error on temperature (on the whole domain), after the filter analysis (black), and the smoother analyses with : 2 (red), 6 (green), 10 (purple) or 16 (orange) days retrospective assimilations.

**RC:** One more obstacle may be the localisation. I do not find evidence or verification about the size of influence zone of 15x10 degrees. If the influence size were larger, the lag window should be longer.

**AC:** Numerous tests have been carried out to identify the "best" influence zone for localisation. By "best", we mean a trade-off between CPU performance and analysis accuracy. Figure 2 reports our results, but we think it is not essential to show it in the manuscript. But our experiments are now mentioned in the text, section 3.5:

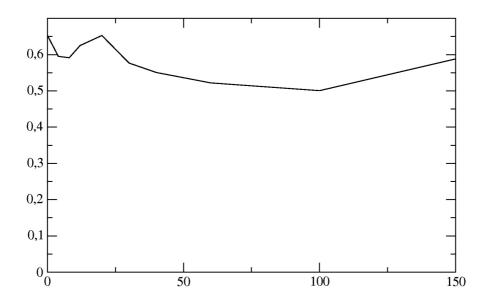
The size of the neighbourhood is defined as 15° zonal and 10° meridional in length (illustrated by the black boxe on figure 1), as a trade-off between analysis accuracy and computational efficiency, after a large number of sensitivity experiments.

**RC:** Contents of section 6 "Smoother based on a static filter" are interesting but very independent from the previous sections. It should be another article with more number of figures and equations to demonstrate the effectiveness of "half-fixed smoother".

**AC:** We definitely agree on the need of further investigations on this scheme. Those were cut due to shameful reasons of missing workforces, but the use of such scheme should be soon considered in the framework of a "Reanalysis" project at the French oceanographic service, Mercator-Ocean. As such, we think it is better to keep this section as is. The introduction of Section 6 has been modified to clarify the fact that this half-fixed approach is really a recipe to make the smoother work in an operational framework:

It is well known that the main obstacle to accurate Kalman filtering in oceanography is models dimension. At the French ocean forecast service Mercator-Ocean (<u>www.mercator-ocean.fr</u>), the model is presently run at a  $1/12^{\circ}$  resolution and 50 vertical levels, featuring more than a billion state

variables and a prohibitive cost for ensemble methods. Consequently, data assimilation is performed with a rather simple optimal interpolation (OI) scheme, as in several other centres. In this section, we present and experiment a formulation of the fixed-lag smoother that complies with these operational constraints. The filter described and implemented previously is modified to work as an OI scheme. The same parameterisations are used, except...



*Figure 2 : RMS error on temperature, after a filter analysis, given a lenght of the localisation. The table below gives the correspondance between the lenght numbers and the localisation in degrees.* 

Localisation number	Localisation in degrees (E-W x N-S)
4	1 x 0.75
8	2 x 1.5
12	3 x 2
20	5 x 4
30	7.5 x 5
40	10 x 7.5
60	15 x 10
100	25 x 20
150	37.5 x 30

**RC:** How did you estimate x0 and P0 (or S0)? Their structure and accuracy are critical for the half-fixed smoother. This is actually a part of the "major obstacles" as the error sources linked to the initial condition. The related question arises to "One of the main obstacles : the error covariances of the filter" in the abstract. This problem must be true for both constant and evolutive parts. How can you neglect the effect of the initial or fixed error by investigating the operational system?

**AC:** Definition of initial statistics is indeed a sensitive point, especially for a half-fixed smoother that use P0 for each analysis. We chose a simple but widely used method based on interannual variability. Some other methods can be used, as it has been discussed for instance in *Fukumori*, 2001.

Though our configuration of P0 is consistent with the experiment settings (error based on the interannual variability), another method could allow the filter and smoother to perform more accurate analyses.

## Review 2

## RC: 1. The scheme.

The scheme of the smoother detailed in Sec. seems to be equivalent to the EnKS (Evensen and van Leeuwen 2000, Evensen 2003, Evensen 2009). It can be summarised as applying the ensemble transforms obtained with the EnKF back in time. This is a generic, scheme independent, formulation. If the above is correct, then I suggest that the authors drop most or the whole Section 2 and concentrate on the oceanographic aspects of the study. Otherwise, it is necessary to detail the differences with the existing approaches.

AC: Indeed the scheme is close to the EnKS, as it was already (perhaps too quickly and not explicitly) acknowledged in the previous manuscript. This is now clarified in Section 2. Various descriptive aspects cannot be removed because they are used in the following text, including the model error and covariance definition, the discussion of the oceanographic aspects of the study, and in particular in the Half-fixed smoother section. The more "classical" parts (standard Kalman filter and smoother) are more superfluous but still useful to fix the notations and make the paper self-sufficient with a rather limited amount of material. Moreover, as it is still not clear to everyone that different smoother schemes exist, we chose to explicitly introduce the equations to proscribe any ambiguity.

Finally, we chose to focus mostly on the data assimilation presentation instead of oceanographic aspects, because the results exposed in this paper are also focussed on data assimilation problems (though some dynamical structures illustrate the results, such as North Brazil rings).

#### RC: 2. Experiment settings.

In contrary to the suggestions that could be drawn from the title, the study concentrates on results of a specific experiment with very lenient settings, rather than trying to replicate the conditions encountered in practice. The FALSE and REF runs are conducted with identical forcing; both the model and observations are perfect; and the observation network is dense and on a regular grid. The RMS difference between the free run and assimilated runs seems to be stable in time, which points either to a non-chaotic system or to nonlinear saturation. Assuming that it is a result of nonlinear saturation, the system perhaps needs more time for stabilisation than it is given in the experiment. Analysing results obtained with the smoother on day 2 after the beginning of assimilation is likely to be premature, as the system is probably still in the transient and/or nonlinear regime.

**AC:** It is true that experiments are carried out with "lenient settings", but still, it is hard to make the filter and the smoother very efficient with such settings. The Tropical Atlantic ocean is a thorny problem for data assimilation, because of the quasi-absence of geostrophy (connexion between SSH and U-V), and because of large scale dynamics (waves) hardly corrected as a whole with localized filters. Here, even if the observation network is somewhat idealized, it remains realistic in terms of space/time density and under-estimated in terms of observation nature (SST is not assimilated for instance). And it is not that dense, because only a sub-network is assimilated every two days. The global observation network is assimilated only after 18 days (for T/S) and 10 days (for the SSH).

Also, the Tropical ocean dynamics are strongly controlled by forcings. It is then true that the FALSE system converges to the REF one after some time, but this takes more than 6 months. The assimilation starts after a 1 month spin-up, which seems to be a nice trade-off between a transient and a stabilized regime, appropriate to look at the effects of data assimilation.

In any case, Tropical Instability Waves can be generated at any time, depending on the ocean state and forcings, and this is not associated with a transient or a non-linear saturation regime. In the title, "Atlantic Ocean" has been replaced by "Tropical Atlantic Ocean" to acknowledge that this study is very specific to this region, as it is already clear in the text, we believe.

## RC: 3. Observations.

The study uses perfect observations, without any justification. This is completely unrealistic and unnecessary for the goals of the study.

**AC:** Justification is now given in Section 3: the preliminary experiments have been carried out this way, and since the appropriate value of R to account for representativeness is much larger than the actual observation error, perturbing the obs was considered somewhat superfluous. Re-running the experiments would be extremely expensive and would not change the results significantly.

**RC:** 4. Dynamical consistency of the analysis.

The claim that the smoother is able to produce analysis "more consistent with the dynamics" seems to be one of the main results of the study. This indeed can only be possible in a nonlinear system, as in a linear system a smoothed and filtered solutions that assimilate the same observations do coincide. This is an interesting observation, but it is based on a single experiment, and this is absolutely not sufficient to justify the general conclusion, particularly when there are no theoretical arguments presented to support it.

AC: There is indeed no theoretical argument to support the fact that the smoother provides more dynamically consistent states than the filter, because none exists. It is a quite loose and empirical result. We note however that in the earlier smoother study of Zhu et al (2003), a similar result was observed (on meteo forecasts), despite the absence of clear improvement of the raw state estimations by the smoother. Also, Lermusiaux et al (2002) did not find a similar behaviour in a quite different setting (different model and obs, forward-backward smoother...). This is just to say that the advantage of a smoother to provide better forecast or more dynamically consistent estimates is still an open question, and here we simply give our opinion based on our (well-documented) experiment. We may precise again that these results are specific to our configuration (but still suggest some optimistic leads for further works on smoothing problems).

**RC:** 5. The smoother based on a static filter (section 6).

This is an interesting scheme that deserves a more thorough investigation. In particular, it would be valuable to compare it performance in experiments with small models, both linear and nonlinear, as well as to get some theoretical insight on expectations of performance of such system. I do not think that this material in its present form is ready for publication.

**AC:** We agree with Reviewer 2. As suggested in the answer to a similar comment by Reviewer 1 (please refer to it), our group does not have the workforce at present to further investigate these aspects. We chose to keep this section as it is because this scheme might be tested by the French oceanographical service soon, in a reanalysis experiment.

# **RC:** Conclusion

In my view the paper can not be published in its present form. The importance of the theoretical part is not clear, the experimental settings are doubtful, and the conclusion about better dynamic consistency of the smoother compared to the filter is not substantiated.

# AC:

In the revised version of the manuscript, it is was re-emphasized that the objective of the study was not to make a theoretical demonstration of the strengths and weaknesses of a smoother w.r.t. a filter in the absolute sense, but rather to document a number of issues arising from the practical implementation of a smoother in the particular case of tropical dynamics. Capturing the initial development phase if TIW indeed remains a highly relevant and challenging assimilation problem that has not received a lot of attention so far. The form of the manuscript has been updated to make this point clearer.