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On the use of the Strouhal/Stokes number to explain the dynamics and water column structure on shelf seas

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Abstract

In recent years coastal oceanographers have suggested the use of the “Strouhal” number or it’s inverse the “Stokes” number, which have been defined as the ratios of the frictional depth (δ) to the water column depth (h) or vice versa, to describe the effect of bottom boundary layer turbulence on the vertical structure of both density and currents. Although they have mention that the effects of rotation should be important, they have tended to omit it. This omission may be important when talking about tidal currents as the frictional depth from a fully cyclonic to a fully anticyclonic tidal ellipse can vary up to an order of magnitude in the mid latitudes; so that the stokes number might appear smaller (larger) than it is resulting in frictional effects being underestimated (overestimated). Here a way to calculate a Stokes number, in which the effect of the Earth’s rotation is taken into account, is suggested. Then the standard Stokes and the rotational Stokes numbers are used as predictors for the position of the tidal mixing fronts in the Irish Sea. Results show that the rotational number improves prediction of the front in shallow cyclonic areas of the eastern Irish Sea. This suggest that the effect of rotation on the water column structure will be more important in shallow shelf seas and estuaries with strong rotational currents.

1 Introduction

The Strouhal number was originally defined by Strouhal (1878) while experimenting with wires experiencing vortex shedding and singing in the wind. This number is now used mainly to explain vortex shedding. The Strouhal number is defined as:

$$\mathrm{Str} = \frac{\omega L}{U} \tag{1}$$

Where ω is the frequency of the vortex shedding, L is a characteristic length and U is the velocity of the flow.

OSD

9, 3723–3738, 2012

Dynamics and water column structure on shelf seas

A. J. Souza

A few years earlier, Stokes (1851) studying a flow over an oscillating plate (analogous to oscillatory flow over the bottom) found that the depth of frictional influence is given by the parameter δ. The ratio of δ to the total water column depth (h) is defined as the Stokes number.

$$\mathrm{Stk} = \frac{\delta}{h} \quad (2)$$

This is equivalent to the ratio of friction to local accelerations in the momentum balance. If we define the oscillatory boundary layer thickness following Lamb (1975)

$$\delta = \frac{c_1 U_*}{\varpi} \quad (3)$$

where ϖ is now the oscillatory frequency, e.g. the M_2 and U_* is the frictional velocity ($U_* = \mathrm{Cd}^{1/2} U$), where U is the M_2 velocity amplitude, Cd is the quadratic drag coefficient, and c_1 is a proportionality constant, so that it becomes

$$\mathrm{Stk} = \frac{c_1 U_*}{\varpi h} = \frac{\delta}{h} \quad (4)$$

Similarly the Strouhal number has commonly been defined as:

$$\mathrm{Str} = \frac{\varpi h}{c_1 U_*} = \frac{h}{\delta} = \frac{1}{\mathrm{Stk}} \quad (5)$$

as in the case of Baumert and Radach (1992) and Souza et al. (2012), or simply

$$\mathrm{Str} = \frac{\varpi h}{U} \quad (6)$$

e.g. Prandle (1982) or Burchard and Hetland (2010). Although this analysis is dimensionally correct, the use of the Strouhal number is dynamically incorrect as it is the ratio of local to advective accelerations.

2 Use of the Strouhal/Stokes number on the description of shelf sea dynamics

Obviously Stokes (1851) explained this problem, in what has become known as the Stokes "second problem" in fluid mechanics, it has been published in several textbooks, and it is applicable to any oscillatory case (i.e. waves and tides).

In a theoretical study of estuarine circulation, Ianniello (1977) showed that the inverse Stokes number (his parameter d_0^2) determines the profiles and residual currents in well-mixed tidal flow in closed estuaries.

Prandle (1982) and Prandle et al. (2011) show how the amplitude and phase of tidal currents vary as a function of Str. The amplitude structure increases asymptotically with increasing Str up to a value Str $\sim$ 350. Accompanying phase variations are maximum for this value of Str but decrease for both smaller and larger values. In meso- and macrotidal estuaries, the Strouhal number will be well in excess of 1000, indicating an amplitude structure close to the asymptotic solution for large Str but with a phase structure that reduces with increasing values of Str. They even defined this as the clockwise and anticlockwise components of the Srouhal number and suggest that the behaviour of each rotational component of velocity will behave following their individual Strouhal number.

Baumert and Radach (1992) identified the Strouhal number, as a characteristic parameter for the mixing associated with tidal flow. By demonstrating that apart from bottom and surface roughness lengths, Str is the only parameter defining the dynamics of the well-mixed irrotational pressure gradient driven tidal flow, they could show how the relative time lag of turbulent parameters with respect to the bed stress increases with Str. Burchard (2009), Burchard and Hetland (2010), Souza et al. (2012) have used this idea to define the behaviour of tidal and oscillatory flows in shelf seas. It needs to be mention that Prandle (1982), Baumert and Radach (1992), Burchard (2009) and Burchard and Hetland (2010) have defined the Strouhal wrongly as its inverse.

Winant (2007) used the Stokes number (his δ parameter) to define the tidal behaviour on elongated rotating estuaries, while Winant (2008) and Huijts et al. (2011)

Discussion Paper | Discussion Paper | Discussion Paper | Discussion Paper |
OSD
9, 3723–3738, 2012
Dynamics and water column structure on shelf seas
A. J. Souza
Title Page
Abstract Introduction
Conclusions References
Tables Figures
Back Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

used it to describe the transverse structure of residual flows in estuaries. Although both authors suggest that the Earth's rotation is important neither of them took it into account when defining the Stokes number, i.e. the frictional depth.

3 The rotational Stokes number

From now on we will only discuss the Stokes number as it looks at the actual dynamics that we are interested in, i.e. the ratio of the frictional to local accelerations. Due to the fact that any value larger than one would mean that the frictional layer is greater than the water depth, in practical terms it will be considered to cover the full water column.

To explain the elliptical behaviour of tidal currents, it is better to decompose the velocity in rotational components (Thorade, 1928; Godin 1972; Prandle, 1982): One rotating cyclonically (R_+ anti-clockwise) and another rotating anti-cyclonically (R_- clockwise).

Following Souza and Simpson (1996), the tidal ellipses' semi-major (U_M), semi-minor (U_m) and ellipticity (ε) are given by:

$$U_M = R_+ + R_- \tag{7}$$

$$U_m = R_+ - R_- \tag{8}$$

$$\varepsilon = \frac{U_m}{U_M} \tag{9}$$

Each of these rotational velocity components has a characteristic boundary layer thickness as explained in Souza and Simpson (1996). In analogy with Eq. (3) we follow here the boundary layer thickness as given by Soulsby (1983):

$$\delta_+ = \frac{c_2 U_*}{(\omega + f)} \tag{10}$$

for the cyclonic and

$$\delta_- = \frac{c_2 U_*}{(\omega - f)} \tag{11}$$

OSD
9, 3723–3738, 2012

for the anti-cyclonic, where f is the Coriolis frequency. In midlatitudes f and ω are close so that $\delta_- \gg \delta_+$, so that the anticyclonic component will cover more of the water column. Soulsby (1983) suggest that the effective combined boundary will be a weighted mean boundary layer δ_R made up of a combination of δ_+ and δ_-

$$\delta_R = \frac{R_+\delta_+ + R_-\delta_-}{R_+ + R_-} \tag{12}$$

which is equivalent to

$$\delta_R = C_2\sqrt{\mathrm{Cd}}\left(\frac{U_M\omega - U_m f}{\omega^2 - f^2}\right) \tag{13}$$

The value for c_2 was taken to be 0.075 following Soulsby (1983), who based it on observations of the measured mixed layer thickness from Pingree and Griffiths (1997). Cd is the drag coefficient used here as a constant of 2.5×10^{-3}, although we know that this might change due to bottom roughness (e.g. Burchard et al., 2011). So the rotational Stokes number Stk_R will be:

$$\mathrm{Stk}_R = \frac{\delta_R}{h} \tag{14}$$

4 Results

Lets assume that we are in a shelf sea of about 30 m depth at latitude of 54° N, with M_2 tidal current has a semi-major axis amplitude 1 ms^{-1} and an ellipticity that changes between −1 and 1. The results in table one show that the Stokes number varies between 0.5 for $\varepsilon = 1$ and 5.4 for $\varepsilon = -1$, with a value of 2.9 for the rectilinear currents.

In the Northwest European Shelf Sea, it will be more common to find values of the ellipticity between −0.5 and 0.5. Using results from the high resolution (1.8 km) POLCOMS (http://cobs.pol.ac.uk/modl/polcoms/) of the Irish Sea. The ellipticity in Fig. 1

OSD
9, 3723–3738, 2012

shows maximum values of 0.5 in Northern Liverpool Bay, Cardigan Bay, at the centre of the deep area west of the Isle of Man and in the Celtic Sea. A minimum of about −0.3 in the southern Liverpool Bay and small values anywhere else. This difference in the tidal ellipse rotation might result in a change of about 25 m between the rotational and none rotational bottom boundary layer ($\Delta\delta = \delta_R - \delta$). Figure 2 shows that the largest changes are in Liverpool Bay, Cardigan Bay and the Celtic Sea, where the bottom boundary layer gets overpredicted more than 25 m when using (regions in red) and underpredicted by about 10 m (cyan areas) when using the non rotational formulation, as expected this will be well correlated with the ellipticity.

If we then calculate the Stokes number for the rotational and non-rotational frictional depth (Fig. 3); and plot it between 0 and 1, i.e. the maximum number plotted is when the frictional depth is the same as the water column depth. We can observe again that the non-rotational Stokes number over estimates the values in the shallow parts of the eastern Irish Sea (i.e. Liverpool Bay and Cardigan Bay) where $\mathrm{Stk}_R \sim 1$. This is also apparent in the Celtic Sea and in the deep area east of the Isle of Man, although, here it is less critical as $\mathrm{Stk}_R \sim 0$.

5 Prediction of tidal mixing fronts

Soulsby (1983) hypothesise the boundary layer constraint is predominant and the positions of tidal mixing fronts are set by the outcropping of the tidal bottom boundary layer at the surface and that we should really take into account the Earth's rotation when we define the fictional depth, this hypothesis was also supported by results from Simpson and Sharples (1994) and Simpson and Tinker (2009). This is because if the boundary layer does not reach the surface, turbulent mixing would not be able to reach the surface thermal stratification. If Soulsby's hypothesis was true frontal positions will be when $\mathrm{Stk} \sim 1$ and better predictions will be achieved when Stk_R is used instead of Stk. To test this idea Stk and Stk_R are compared with a measure of stratification, in this

OSD
9, 3723–3738, 2012

case the non-dimensional buoyancy frequency is define as:

$$N^{2*} = N^2 \left(\frac{h^2}{B_s}\right)^{2/3} \tag{15}$$

Where B_s is the surface buoyancy flux due to solar heating and N^2 is the square of the buoyancy frequency, these have been calculated using the mean values for the warming half of the year (March to September).

Figure 5 shows the distribution of N^{2*} clearly showing the Irish Sea front east of the Isle of Man, the Celtic Sea front and a thermal front in Liverpool Bay. When we compare the values of the Stokes numbers with the stratification, it can be observe that the Stokes number is a good predictor of the tidal mixing front position. It can also be observed that the improvement in the predictability of the tidal mixing fronts is only obvious within Liverpool Bay, due to the fact that this is a shallow area of strong cyclonic tidal currents.

6 Conclusions

First of all it is suggested that the correct number to be used when discussing the ratio of the frictional depth to total depth is the Stokes number, as it is the ratio of the frictional to local accelerations.

The results presented here suggest that the rotational Stokes number should be used when working in shallow shelf seas or estuaries where rotation is important. This could be important in places like Liverpool Bay where your frictional layer can be overestimated or under estimated by more than 10 m, this should be considered when doing classifications like those proposed by Burchard (2009) and Burchard and Hetland (2010) for estuaries and ROFIs. This is not unique of Liverpool Bay as there are other shelf seas that have shallow areas with strong cyclonic and anticyclonic currents, e.g. the English Channel and the southern North Sea.

OSD
9, 3723–3738, 2012

The Stokes number is a good predictor for the position of tidal mixing fronts, with the thermal front position occurring at vales between 0.8 and 1. The use of the rotational Stokes number appears to improve the predictions of tidal mixing front in shallow strongly cyclonic regions. This is in accordance with results found by Simpson and Tinker (2009), but it highlight that it is even more important in shallow areas such as Liverpool Bay, in contrast with the deep areas of the Celtic Sea.

It is obvious from Prandle (1982) and Soulsby (1983) that we should take into account the Earth's rotation when describing the vertical distribution of tidal currents. This is evident when explaining the modification of tidal ellipses by stratification as described by Souza and Simpson (1996) and Palmer (2011). For this process to be active it appears to be necessary for $\mathrm{Stk}_- = \frac{\delta_-}{h} \geq 1$; so that the anticyclonic frictional layer reaches the surface, so in the presence of a pycnocline the surface layer will be decoupled.

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OSD

9, 3723–3738, 2012

Dynamics and water column structure on shelf seas

A. J. Souza

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OSD
9, 3723–3738, 2012
Dynamics and water column structure on shelf seas
A. J. Souza

Table 1. Frictional layer depth and stokes number for a 1 ms^{-1} tidal current with different ellipticities at 54° N and 30 m water column depth.

ε	−1	−0.5	−0.3	0	0.3	0.5	1
δ_R	162	125	110	88	66	51	15
Stk_R	5.4	4.2	3.7	2.9	2.2	1.7	0.5

OSD
9, 3723–3738, 2012
Dynamics and water column structure on shelf seas
A. J. Souza

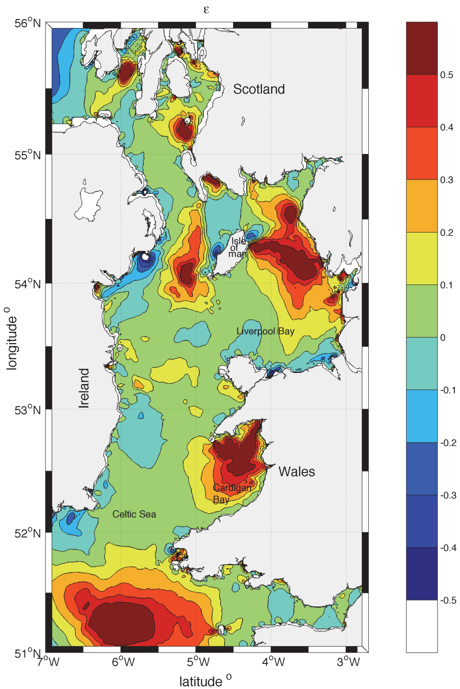

Fig. 1. Ellipticity values in the Irish Sea, positive values are cyclonic negative values are anti-cyclonic.

OSD
9, 3723–3738, 2012

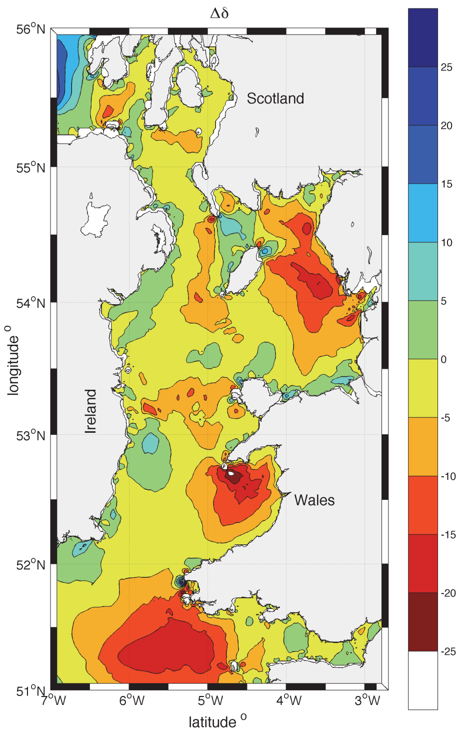

Fig. 2. Difference between the rotational–oscillating and oscillating bottom boundary layer in the Irish Sea. Negative values suggest that the boundary layer is overpredicted by the non-rotational boundary layer.

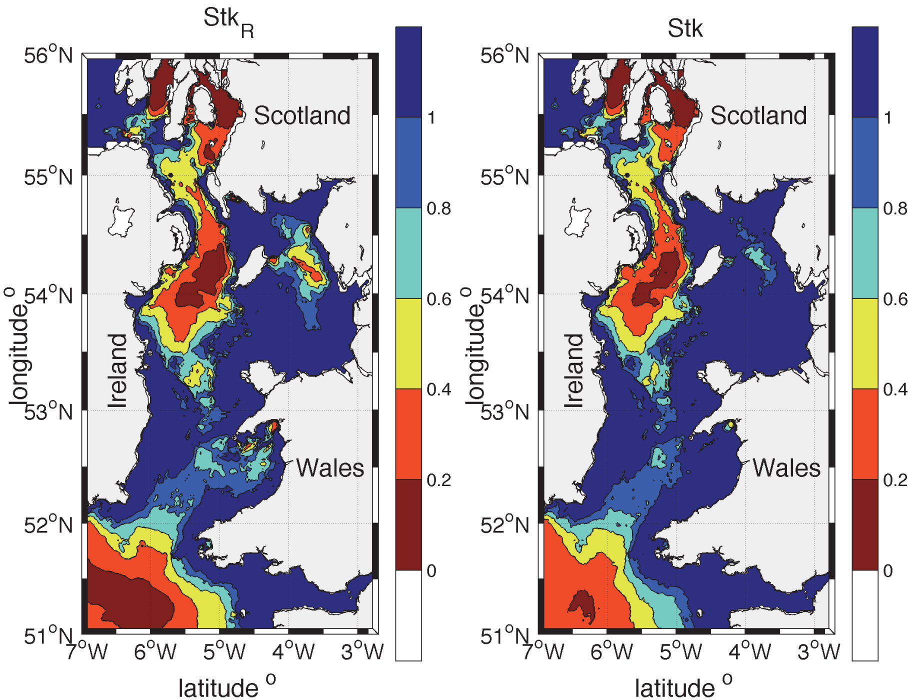

Fig. 3. Rotational **(a)** and non-rotational **(b)** Stokes number for the Irish Sea.

OSD

9, 3723–3738, 2012

Dynamics and water column structure on shelf seas

A. J. Souza

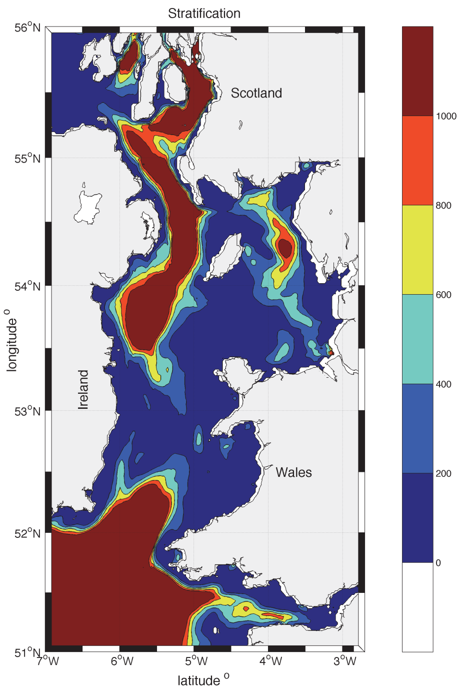

Fig. 4. Non-dimensional stratification of the Irish Sea as defined by the non-dimensional buoyancy frequency.