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Imbalance of energy and momentum source terms of the sea wave transfer equation for fully developed seas

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OSD

9, 2581–2619, 2012

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Abstract

Using a two-dimensional empirical sea wave spectral model at full development, this paper performs an assessment of the compatibility of the energy and momentum budgets of sea waves over the whole spectral range. Among the various combinations of model functions for wave breaking and wind source terms tested, no one fulfilled simultaneously the energy and momentum balance of the transfer equation. Based on experimental and theoretical grounds, wave breaking is known to contribute to frequency downshift of a narrow-banded wave spectrum, when the modulational instability is combined with wave breaking. On those grounds, it is assumed that, in addition to dissipation, wave breaking produces a spectral energy flux directed toward low wavenumbers. I show that it is then possible to remove the energy and momentum budget inconsistency, and correlatively the required strength of this spectral flux is estimated. Assuming such a downward spectral flux permits to fulfill both energy and momentum balance conditions, while slightly reducing the consistency between source terms and empirical spectra, as estimated by a cost function K . Other tests were performed in which it was further assumed that wave breaking would also be associated with azimuthal diffusion of the spectral energy. This could allow a slight improvement of the cost function K , and might correlatively reduce the required downward spectral flux by a factor of 4, although it would not be able to remove it entirely.

1 Introduction

The ocean wave prediction models are based on solving the energy conservation equation (Gelci et al., 1957; WAMDI, 1988; Banner and Morison, 2010). The evolution of the spectral wave-energy density is understood as being governed by the combination of several different physical processes: the propagation of energy, the energy input by wind forcing, the dissipation of energy through viscosity and wave breaking, and the exchange of energy between different wave components interacting weakly among

OSD

9, 2581–2619, 2012

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



themselves. An abundant literature has been devoted to finding relevant parameterizations for the different source terms of the transport equation. These have been incorporated efficiently into the wave prediction models, producing quite satisfactory operational results. In that context, a fully developed ocean wave spectrum would be describable as a nearly stationary solution, in which the different terms of the energy transfer equation tend to balance each other, leading to greatly reduced wave evolution.

In the context of a stationary (or quasi-stationary) state describing a fully developed sea, this paper explores the consistency of the energy and momentum budgets of the sea wave system. For this purpose, in Sect. 2 I will perform a test of the consistency of the integral energy and momentum equations in the framework of current parameterizations of the source and sink terms of the transport equation, and standard models of the fully developed directional spectrum. This will require taking into account not only the low wavenumber range for which the sea wave spectra are deduced from measurements by operational buoys, but also the high wavenumber range. To describe this high wavenumber region of the sea wave spectrum, empirical spectra were built based on other means, such as wave tank data (e.g. Jähne and Riemer, 1990), measurements based on arrays of height gauges (Donelan et al., 1985), stereophotogrammetric analysis (Banner et al., 1989), or radar reflectivity measurements at different radio frequencies. When the whole spectral range is considered, it will be shown that standard parameterization of the source terms does not allow to satisfy simultaneously the energy and momentum integral transport equations. In addition to producing wave dissipation, another effect of wave breaking will then be proposed in Sect. 3, involving downward flux of spectral energy in the wavenumber space. It will be shown that within this assumption it is possible to solve the energy and momentum budget inconsistency, and correlatively the required strength of this downward spectral flux will be estimated.

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



2 Energy and momentum budget of the sea wave spectral evolution

2.1 Spectral evolution equations integrated over wavenumber

Let $F(k, \varphi)$ be the surface elevation spectrum of sea waves as a function of wavenumber magnitude k and azimuth φ . The evolution of $F(k, \varphi)$ is governed by the basic transport equation involving a superposition of source terms, and is usually written in the following manner:

$$\frac{\partial F(k, \varphi)}{\partial t} + c_g \cdot \nabla F = \left(\frac{\partial F}{\partial t} \right)_w + \left(\frac{\partial F}{\partial t} \right)_{\text{visc}} + \left(\frac{\partial F}{\partial t} \right)_{\text{br}} + \left(\frac{\partial F}{\partial t} \right)_{\text{nl}} \quad (1)$$

where c_g is wave group velocity, $(\partial F/\partial t)_w$ represents the wind input term, $(\partial F/\partial t)_{\text{visc}}$ accounts for the damping of capillary waves due to water viscosity, but also includes a swell dissipation term related to friction with the atmosphere, as reported by several studies (Tolman, 2002; Ardhuin et al., 2009), as will be discussed below. The other terms of the right hand side of Eq. (1) are respectively the wave breaking dissipation term, and the resonant nonlinear wave-wave interaction term.

The case of a fully developed sea corresponds to a quasi-stationary situation in the open ocean where the wind has blown steadily over a sufficient time that the wave spectral density does not evolve any more. In that ideal case, for any wavenumber k and azimuth φ , both ∇F and $\partial F(k, \varphi)/\partial t$ are expected to be equal to zero, and the four source terms of the right hand side of Eq. (1) balance each other.

In reality, the existence of a fully developed sea describable as a perfect stationary state has not been clearly established, and sea waves might still be growing for an old sea, although the net growth has then to be weak. For example, the input and dissipation integrated energy source terms obtained by Banner and Morison (2010) cancel out almost perfectly as the asymptotic state is approached, as can be seen from their Fig. 4b, where the total integrated source term obtained for the oldest seas is reduced to less than 5% of the input and dissipation source terms taken separately.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Instead of Eq. (1), it is liable to express the transport equation in terms of the wave energy spectrum $\Phi(k, \varphi)$ that is related to F through:

$$\Phi(k, \varphi) = \rho(\omega^2/k)F(k, \varphi) \quad (2)$$

It is also possible to express it in terms of the momentum spectrum $\mathbf{M}(k, \varphi) = \rho\omega F(k, \varphi) (\mathbf{k}/k)$. In those expressions ρ is water density and ω is wave angular velocity. The momentum \mathbf{M} carried by the waves is a vector quantity, but if we limit ourselves to fully developed situations, symmetry of the 2-D-spectrum implies that only the alongwind component of the momentum will be different from zero after integration over wavenumber. Taking $\varphi = 0$ for the wind direction, that component may be written, at wavenumber (k, φ) :

$$M(k, \varphi) = \rho\omega F(k, \varphi) \cos \varphi \quad (3)$$

For a fully developed sea ($\partial F(k, \varphi)/\partial t \approx 0$), integrating the transport equation for the wave energy spectrum over wavenumber, we obtain:

$$\int_0^\infty \int_{-\pi}^\pi \left[\left(\frac{\partial \Phi}{\partial t} \right)_w + \left(\frac{\partial \Phi}{\partial t} \right)_{\text{visc}} \right] k dk d\varphi = - \int_0^\infty \int_{-\pi}^\pi \left(\frac{\partial \Phi}{\partial t} \right)_{\text{br}} k dk d\varphi - \int_0^\infty \int_{-\pi}^\pi \left(\frac{\partial \Phi}{\partial t} \right)_{\text{nl}} k dk d\varphi \quad (4)$$

The last term of the right-hand side of Eq. (4), which represents the nonlinear transfer of energy by resonant wave-wave interactions is described classically as resulting from four-wave interactions (Hasselmann, 1962, 1963) or three-wave interactions (Valenzuela and Laing, 1972). In a general discussion about interaction symmetries involving wave particle analogy, Hasselmann et al. (1994) report that conservation of energy and conservation of momentum both apply for all wave-wave resonant interaction processes, including four-wave and three-wave interactions (this is not the case for wave action conservation, which holds for four-wave but not three-wave interactions). As a

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



consequence, after integration over the wavenumber space the energy and momentum budget of resonant nonlinear interactions is zero. Therefore, the last term of the right hand side of Eq. (4) is equal to zero, and similarly the same property applies for the momentum transport equation. One then gets both equations:

$$5 \int_0^{\infty} \int_{-\pi}^{\pi} \left[\left(\frac{\partial \Phi}{\partial t} \right)_w + \left(\frac{\partial \Phi}{\partial t} \right)_{\text{visc}} \right] k dk d\varphi = - \int_0^{\infty} \int_{-\pi}^{\pi} \left(\frac{\partial \Phi}{\partial t} \right)_{\text{br}} k dk d\varphi \quad (5a)$$

$$\int_0^{\infty} \int_{-\pi}^{\pi} \left[\left(\frac{\partial M}{\partial t} \right)_w + \left(\frac{\partial M}{\partial t} \right)_{\text{visc}} \right] k dk d\varphi = - \int_0^{\infty} \int_{-\pi}^{\pi} \left(\frac{\partial M}{\partial t} \right)_{\text{br}} k dk d\varphi \quad (5b)$$

2.2 Tests based on standard source terms for fully developed seas

10 In order to check the integral energy and momentum budgets of Eqs. (5a) and (5b), it is necessary to quantify the different source terms involving wind input, viscosity and wave breaking. For each of them, we may define the rate of evolution of the sea wave spectrum $\beta_{\text{sub}}(k, \varphi)$ according to

$$15 \beta_{\text{sub}}(k, \varphi) = \frac{1}{F(k, \varphi)} \left(\frac{\partial F(k, \varphi)}{\partial t} \right)_{\text{sub}} \quad (6)$$

where the subscript “sub” refers to the type of source term involved (“visc” for viscous damping, “w” for wind input or “br” for wave breaking).

As concerns the wind input and wave breaking terms, they have been the subjects of numerous parameterizations in the literature. I tested different combinations of wind input and wave breaking models. Basic types of models for source terms will be described below (Sects. 2.2.2 and 2.2.3), and will be used to check the energy and momentum budgets. Although not exhaustive, these illustrate the main assumptions taken by standard models. Other source functions tested did not lead to qualitatively different conclusions, and did not allow removing the systematic trends obtained and described below.

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



2.2.1 Viscous damping term

The viscous damping term is expressed as:

$$\beta_{\text{visc}} = \beta_{\text{capil}} + \beta_{\text{swell}} \quad (7)$$

In Eq. (7), β_{capil} is the damping term of capillary waves due to the viscosity of water, and is taken from Lamb (1932) as $\beta_{\text{capil}} = -4\nu_w k^2$, where $\nu_w = 1.3 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ is the kinematic viscosity of water. According to Dore (1978), however, the viscosity of the air gives a stronger dissipation for wavelength larger than 0.85 m. Swell dissipation was observed by Tolman (2002) and Ardhuin et al. (2009), who found it to be consistent with the effects of friction with the atmosphere. The term β_{swell} in Eq. (7) accounts for such damping of swells by friction with the air. Ardhuin et al. (2010) propose the following expression for this swell dissipation term:

$$\beta_{\text{swell}} = -1.2 \frac{\rho_a}{\rho_w} \left[2k \sqrt{2\nu_a \omega} \right] \text{ for } \text{Re} \leq \text{Re}_c$$

$$\beta_{\text{swell}} = -\frac{\rho_a}{\rho_w} \left[16f_e \omega^2 u_{\text{orb}} / g \right] \text{ for } \text{Re} > \text{Re}_c \quad (8)$$

In these expressions, $\text{Re} = 4u_{\text{orb}} a_{\text{orb}} / \nu_a$ is the boundary Reynolds number, where u_{orb} and a_{orb} are the significant surface orbital velocity and displacement amplitudes, and $\nu_a = 14 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ is the air viscosity. Following Ardhuin et al. (2010), the critical Reynolds number value Re_c is estimated as a function of significant wave height H_s according to $\text{Re}_c = (2 \times 10^5 \text{ m}) / H_s$, and the coefficient f_e is taken as:

$$f_e = 0.8 \left[0.003 + (0.015 - 0.018 \cos \varphi) \frac{u_*}{u_{\text{orb}}} \right] \quad (9)$$

2.2.2 Wind input source terms

Among the numerous parameterizations for the atmospheric wind-related growth rate β_w found in the literature, some are based on the friction velocity u^* , others are based

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



on the wind velocity at a given height, and still others are based on the velocity at a height scaled with the wavelength. Plant (1982) proposed:

$$\beta_w(k, \varphi) = 0.04 \cos \varphi \left(\frac{U^*}{c(k)} \right)^2 \omega \quad (10)$$

5 where $c(k)$ is the wave phase speed, and ω is wave angular velocity.

On the basis on previous studies (Snyder et al., 1981; Komen et al., 1984), the WAMDI group (1988), used a linear, rather than quadratic, dependence for β_w for their third generation wave prediction model,

$$\beta_w(k, \varphi) = \max \left\{ 0, 0.25 \frac{\rho_a}{\rho_w} \left(28 \frac{U^*}{c} \cos \varphi - 1 \right) \right\} \omega \quad (11)$$

10 where ρ_a and ρ_w are air and water densities, respectively.

Arguing that the appropriate reference wind is rather the wind at some height above the roughness elements that is related to their scale, Donelan and Pierson (1987) proposed a parameterization based upon the wind speed at a height π/k :

$$\beta_w = 0.194 \frac{\rho_a}{\rho_w} \omega \left[\max \left(\frac{U(\pi/k)}{c(k)} \cos \varphi - 1 \right), 0 \right]^2 \quad (12)$$

Following previous authors (Belcher and Hunt, 1993; Hara and Belcher, 2002), Kukulka and Hara (2005) proposed a wind input wave growth formulation under the sheltering assumption. According to this approach, the wave-induced stress of longer waves reduces the turbulent stress felt by shorter waves. The resulting β_w can be expressed as follows

$$\beta_w(k, \varphi) = \frac{c_\beta(k)\omega}{\rho_w c^2} h_\beta(\varphi) \tau_o \exp \left[- \int_{k'=0}^k \int_{\varphi'=-\pi/2}^{\pi/2} c_\beta(k') k'^3 h_\beta(\varphi') F(k', \varphi') \cos \varphi' dk' d\varphi' \right] \quad (13)$$

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



where $h_\beta(\varphi) = [\max(\cos \varphi, 0)]^2$, $c_\beta(k)$ is a coefficient which depends upon the boundary Reynolds number Re , and $\tau_o = \rho_a u^{*2}$ is the total momentum flux.

In the capillary-gravity range waves are steep and they may result in modification of the airflow which will reduce the momentum flux from air to water. This phenomenon was studied by Janssen (1991) who proposed a quasi-linear theory resulting in an effective high wave number cutoff. More recently, Banner and Morison (2010) and Ardhuin et al. (2010) proposed a wave prediction model based on this approach, including slight tuning modifications. The parameterization of the wind input term β_w based on the works by Janssen (1991), Banner and Morison (2010), and Ardhuin et al. (2010), will be referred to as the JBA model, and is described in more details in Appendix A.

I tested successively those 5 different wind input source terms, taken respectively from Plant (1982), Komen et al. (1984), Donelan and Pierson (1987), Kukulka and Hara (2005), and JBA model. In all cases the total atmospheric term (including wind input and swell dissipation) was set to 0 for $\cos \varphi < 0$.

2.2.3 Wave breaking source terms

Several approaches have been used for modeling β_{br} . The first one considers that, even though wave breaking is a nonlinear process, the phenomenon applied to the sea waves is weak in the mean, and as a consequence it leads to a quasi-linear behavior of the dissipation $(\partial\Phi/\partial t)_{br}$ with respect to wave spectrum Φ (Hasselmann, 1974). This approach was followed by Komen et al. (1984), who proposed:

$$\beta_{br}(k, \varphi) = -\gamma_K P_K(k, \varphi) \quad (14a)$$

with

$$P_K(k, \varphi) = \frac{\omega^2}{\langle \omega \rangle} \quad (14b)$$

where γ_K is a non-dimensional constant, and $\langle \omega \rangle$ is mean angular frequency.

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Another type of approach was proposed by Phillips (1985), who explored the consequences of the assumption that the processes of energy input from the wind, spectral flux divergence due to resonant nonlinear interactions, and loss by breaking are all important in the equilibrium range. These considerations led him to propose the following formulation for the variation of the wave action density $N(k, \varphi) = (g/\omega)F(k, \varphi)$ due to wave breaking:

$$\left(\frac{\partial N}{\partial t}\right)_{\text{br}} = -gk^{-4}aB^3(k, \varphi) \quad (15)$$

where g is acceleration of gravity, $B(k, \varphi) = k^4F(k, \varphi)$ is the saturation spectrum, and a is a non-dimensional constant. In terms of β_{br} , this can be rewritten as

$$\beta_{\text{br}}(k, \varphi) = -\gamma_P P_P(k, \varphi) \quad (16a)$$

with

$$P_P(k, \varphi) = \omega k^8 F^2(k, \varphi) \quad (16b)$$

and γ_P is a non-dimensional constant.

More recently, due to the observed strong threshold behaviour reported by Banner et al. (2002) for the wave breaking probability in the spectrum, threshold-based formulations for the breaking component of the dissipation term β_{br} were proposed (Banner and Morison, 2010; Ardhuin et al., 2010). The model results closely reproduced the observed breaking wave properties and wave spectral evolution. Ardhuin et al.'s (2010) approach will be expressed here as

$$\beta_{\text{br}}(k, \varphi) = -\gamma_a P_a(k, \varphi) \quad (17)$$

where γ_a is a non-dimensional constant, and $P_a(k, \varphi)$ is given in Appendix B.

Similarly to what has been done for wind input source terms, I tested successively those 3 different wave breaking source terms, taken respectively from Komen et al. (1984), Phillips (1985), and Ardhuin et al. (2010).

2.2.4 Empirical sea wave spectral model

In order to compute the integral quantities of Eqs. (5a) and (5b), one needs to use a model for the sea wave spectrum based on observations. As concerns the omnidirectional spectrum, I used the empirical sea wave spectral model by Elfouhaily et al. (1997) for different conditions of wind speed, assuming fully developed situations. An advantage of this model is that it describes the wave spectrum over the whole range of wavenumbers from the spectral peak to capillary waves, on a purely empirical basis. Also, while this model was tuned on in situ observations of wave spectra performed both in ocean and in laboratory, it is also consistent with the optical mean square slope measurements by Cox and Munk (1954).

As concerns the directional behavior of the spectrum, observations have shown that it exhibits distinct directionally bimodal peaks (Young et al., 1995; Long and Resio, 2007). Resio et al. (2011) have proposed wave-age dependent self-similar bi-modal model functions for the directional spreading consistent with recent observations. They showed that, with their model function, the nonlinear wave-wave interactions produce relatively constant fluxes of both energy and momentum through the equilibrium range. For those reasons I used Resio et al.'s (2011) directional spreading model function, assuming full development. This spreading function was used up to wavenumber $k = 25k_p$, i.e. frequency $f = 5f_p$, where k_p and f_p are the peak wavenumber and frequency, which corresponds roughly to the domain where Resio et al.'s model was validated against data. For higher wavenumbers, most directional observations are based upon remote sensing techniques (radar or optical observations including Banner et al.'s (1989) stereo-photogrammetric analysis), and give essentially information about the centrosymmetric (or folded) spreading function. Although this approach is relevant for electromagnetic modelers, it is not appropriate here, since the transport equations written above must involve directional, instead of folded spectra, and some hypothesis needs to be used to obtain a directional spectrum. For wavenumbers above $25k_p$, I used Elfouhaily's folded spreading function. Elfouhaily et al. define a "delta

OSD

9, 2581–2619, 2012

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



ratio” given by $\Delta(k) = (F(k, 0) - F(k, \pi/2)) / (F(k, 0) + F(k, \pi/2))$. Following Donelan et al. (1985), I assumed a directional spectrum given by an hyperbolic secant spreading function $F(k, \varphi) = F(k, 0)\text{sech}^2(h\varphi)$, and the value of h was adjusted in such a way that the resulting folded spectrum yields the delta ratio of Elfouhaily et al.’s model.

The momentum budget depends in a sensitive manner on the high wavenumber part of the spectrum, and it is therefore useful to estimate how much the momentum budget is sensitive to small errors in the high frequency part of the spectrum. For this purpose, tests were also performed using a composite spectrum where the model described above was replaced by Kudryavtsev et al.’s (2003) directional spectrum for high wavenumbers ($k \geq 25k_p$). Elfouhaily et al.’s (1997) spectrum in the high wavenumber range was designed by using only laboratory data, as well as optical data at sea, purposefully excluding radar data. On the contrary Kudryavtsev et al.’s (2003) spectrum was essentially based on normalized radar cross section (NRCS) measurements at sea, and more recent measurements of radar derived anisotropy of the sea surface mean square slope (Hauser et al., 2008) were found to be reasonably well fitted by Kudryavtsev et al.’s (2003) spectrum.

2.2.5 Energy and momentum budgets integrated over wavenumber

The wind input and wave breaking source terms described above, together with the viscous term (Eq. 7) were applied to the empirical sea wave spectrum in order to check the energy and momentum budgets expressed in Eqs. (5a) and (5b), which may be rewritten

$$\int_0^{\infty} \int_{-\pi}^{\pi} [\beta_w + \beta_{\text{visc}}] \Phi k dk d\varphi = - \int_0^{\infty} \int_{-\pi}^{\pi} \beta_{\text{br}} \Phi k dk d\varphi \quad (18a)$$

$$\int_0^{\infty} \int_{-\pi}^{\pi} [\beta_w + \beta_{\text{visc}}] M k dk d\varphi = - \int_0^{\infty} \int_{-\pi}^{\pi} \beta_{\text{br}} M k dk d\varphi \quad (18b)$$

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract Introduction

Conclusions References

Tables Figures

◀ ▶

◀ ▶

Back Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



The expressions for the wave breaking or wind input source terms given in Sects. 2.2.2 and 2.2.3 should be regarded as giving functional forms, but a fine tuning of their relative amplitudes might be needed in order to fulfill the energy and momentum balance Eqs. (18a) and (18b). To perform this fine tuning of the relative amplitudes of the wind input and wave breaking source terms, I considered coefficient γ of the wave breaking (referred as γ_k , γ_p , or γ_a in Eqs. 14a, 16a, or 17, respectively), as an adjustable coefficient. Since both energy and momentum balance (Eqs. 18a and 18b) must be fulfilled, there are indeed two ways to perform the tuning of the same coefficient γ .

$$\gamma_1 = \frac{\int_0^\infty \int_{-\pi}^\pi (\beta_w + \beta_{visc}) \Phi k dk d\varphi}{\int_0^\infty \int_{-\pi}^\pi P(k, \varphi) \Phi k dk d\varphi} \quad (19a)$$

$$\gamma_2 = \frac{\int_0^\infty \int_{-\pi}^\pi (\beta_w + \beta_{visc}) M k dk d\varphi}{\int_0^\infty \int_{-\pi}^\pi P(k, \varphi) M k dk d\varphi} \quad (19b)$$

where $P(k, \varphi)$ is equal to $P_k(k, \varphi)$, $P_p(k, \varphi)$, or $P_a(k, \varphi)$ according to whether Eqs. (14a), (16a), or (17) is used. Of course, one expects both obtained values γ_1 and γ_2 to be close to each other, since they represent two estimates of the same quantity γ . Table 1 displays the ratio $R = \gamma_2/\gamma_1$ for the various combinations of the wind and breaking source terms described above. While the expected value of R is $R = 1$, one can see that there is a considerable scatter between the values of R obtained, with a mean of $R = 3.5$, and extreme values from 2.66 to 5.11 depending on the functional form chosen for β_w and β_{br} .

The computations of coefficients γ_1 and γ_2 depend not only on the functional forms of the source terms β_w and β_{br} , but also on the assumed shape of the sea wave spectrum in the (k, φ) space. The ratio R (not shown) computed when replacing Resio

et al.'s (2011) spreading function by other spreading functions (Elfouhaily et al., 1997; Donelan and Pierson, 1987; Banner, 1990) at low wavenumbers, did not allow to reduce the ratio R to a value close to 1.

The ratio R was also computed with the composite model mentioned in Sect. 2.2.4, where the spectrum based on Elfouhaily et al. (1997) and Resio et al. (2011) is replaced by Kudryavtsev et al.'s (2003) spectrum at high wavenumbers. The results obtained with this composite spectral model are given in Table 2. It can be seen that the new ratio R is modified, but still remains within 35% of the values listed in Table 1 for each combination of the β_w and β_{br} models. The smallest value for R ($R = 1.72$) is obtained with Komen et al.'s (1984) model for β_{br} , WAMDI's (1988) model for β_w , and the composite spectral model used for Table 2. However, no combination of the current models of sea wave spectrum, wind input source term, and wave breaking source term, was indeed found to be able to produce a ratio R equal (or at least close) to 1.

Assuming for example that the wave breaking coefficient γ (i.e., γ_k , γ_p , or γ_a of Eqs. 14a, 16a, or 17) is determined by the energy balance equation (i.e., $\gamma = \gamma_1$), the preceding results indicate that the wave momentum removed from the waves by wave breaking and viscous dissipation is unable to balance the momentum brought by the wind (this would require $\gamma = \gamma_2$ while instead $\gamma = \gamma_1 < \gamma_2$). Thus, within the classical expressions for wave growth terms of wind input and wave breaking, and for fully developed situations, a proportion of about one half or more of the momentum brought by the wind to the waves would accumulate within the wave system. This is not a tenable assumption for a fully developed sea which is supposed to be nearly stationary.

3 A modified formulation of the breaking source term restoring energy and momentum balance

Due to the difficulty reported in the preceding section to fulfill both energy and momentum budgets, it appears that present parameterizations of the source terms are inconsistent with empirical spectral shapes. Does this mean that the chosen spectral

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



shapes are unrealistic, or else that the source term parameterizations are unrealistic? It is unfortunately not possible to answer this question. However, the spectral models were built by direct confrontation to the observations, taking advantage of numerous kinds of instruments (in situ as well as remote sensing data) operating in the various sea state conditions, and it would be uncomfortable to modify those purely observation-based wave spectra, just in order to conform to the energy and momentum budget requirements. In contrast, the source terms of the transport equation are quantities determined in a more indirect manner (and for that reason, as seen in Sect. 2, numerous different approaches have been followed to parameterize those source terms). In view of these considerations, in this paper I choose to consider the empirical spectra as the observational reference. Basically, a way to restore energy and momentum balance would be to add an extra source term at low wavenumbers, and/or to add an extra sink term at high wavenumbers, but obviously there is not a unique way to modify the source terms in order to fulfill energy and momentum balance, and also such a modification should not alter significantly the resulting sea wave spectrum.

Among the source or sink terms, the one which is the least well established is the wave breaking term, because the processes represented by this unique term are indeed highly nonlinear, involving spilling of the crests of large gravity waves, formation and dissipation of whitecaps, spray, entrainment of air bubbles within the water upper layers, production of turbulence at shorter scales. Also, since the wave breaking events occurring near the steep crests are localized features in the spatial domain, duality of the spectral analysis implies that their signature cannot be local in the wavenumber space. Thus expressing β_{br} as a mere function of $F(k, \varphi)$ and k may be considered as an oversimplification, even if it is scaled by integral quantities of the wave spectrum (as in Eq. 14b for instance).

From the discussion above, it follows that one approach to restore energy and momentum balance would be to decrease the amplitude of the wave breaking sink term at low wavenumbers and to increase it at high wavenumbers. Note however that a similar result would be obtained without modifying the wave breaking term, but assuming

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



that wave breaking is producing an additional flux of spectral energy from high to low wavenumbers. Such downshifting of spectral energy resulting from wave breaking in presence of modulational instability has been highlighted both theoretically and experimentally, as will be discussed in detail below. In this paper we will show that following this latter approach permits to restore the energy and momentum balance, without the need to modify the wave breaking sink term itself.

3.1 Frequency downshift related to the modulational instability in presence of wave breaking

It is well known that a uniform train of Stokes waves is unstable to the so-called modulational instability, as shown theoretically and experimentally by Benjamin and Feir (1967). Following Benjamin and Feir's pioneering work, numerical simulations of the evolution of gravity wave spectra of fairly narrow bandwidth were performed (Alber, 1978; Dysthe et al., 2003), and a number of experimental investigations were conducted on the long time evolution of nonlinear wave trains (Lake et al., 1977; Melville, 1982; Tulin and Waseda, 1999). The modulational instability is an interaction among three monochromatic wave trains: carrier (ω_0), upper ($\omega_0 + \delta\omega_0$), and lower ($\omega_0 - \delta\omega_0$) sideband waves. An asymmetric growth of the sidebands makes the lower to grow at the expense of the upper. In the absence of wave breaking, the evolution occurs in a recurrent fashion, where the modulation periodically increases and decreases, the wave form returning periodically to its previous form. The effect of breaking dissipation is to increase and render irreversible the energy difference of the upper and the lower sidebands after peak modulation. The end state of the evolution following breaking is an effective downshifting of the spectral energy (Tulin and Waseda, 1999). Although these studies were performed in the framework of monochromatic wave trains, we will assume here that a frequency downshift associated with wave breaking is still occurring in the case of a broad-banded wave spectrum. Since the frequency downshift effectively reduces the wave momentum when the wave energy is conserved, it may provide

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



the mechanism required to solve the problem of imbalance between momentum and energy reported in Sect. 2.

It should be stressed that the modulational instability without wave breaking would produce no downshifting on the long term, and would not be associated with a loss of energy and momentum. In that sense it is a conservative process. Similarly, as reported by Hasselmann et al. (1994), conservation of energy and momentum both apply for all resonant wave-wave interaction processes, including four-wave and three-wave interactions. On the contrary, the process invoked here assumes a further downshifting specifically associated with wave breaking, which violates energy and momentum conservation.

3.2 New formulation of the wave breaking source term

Due to the preceding discussion, we will assume henceforth that, in addition to reducing the spectral energy, wave breaking also produces a transfer of energy to lower frequencies. The detailed mechanism for the frequency change is rather unknown. It might be a continuous and gradual process, or result from a superposition of local, abrupt, and discrete processes (Huang et al., 1996). Whatever the detailed process, we will describe the frequency downshift by means of a downward energy flux $f(k, \varphi)$ in the wavenumber space. In a first time, we will assume that $f(k, \varphi)$ is radially oriented in the k -space, and therefore that $f_\varphi = 0$. As discussed above, the radial component f_k should be related to the wave breaking dissipation. As a simplest approach, we will assume that f_k is merely proportional to the dissipation term. Dimensional consideration then leads to the following expression of the energy flux:

$$f(k, \varphi) = \mu \left(\frac{\partial \Phi}{\partial t} \right)_{\text{br}} \mathbf{k} \quad (20)$$

where μ is a non-dimensional coefficient.

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



The divergence of this energy flux in the wavenumber space gives rise to a supplementary source term $(\partial\Phi/\partial t)_S$ according to the conservation equation:

$$\left(\frac{\partial\Phi}{\partial t}\right)_S = -\text{div}_k(\mathbf{f}) \quad (21)$$

5 where div_k stands for the divergence operator in the wavenumber space. According to Eq. (20), this may be rewritten:

$$\left(\frac{\partial\Phi}{\partial t}\right)_S = -\frac{\mu}{k} \frac{\partial}{\partial k} \left[k^2 \left(\frac{\partial\Phi}{\partial t}\right)_{br} \right] \quad (22)$$

10 The energy and momentum budgets can now be performed exactly as in Sect. 2, except that β_{br} should now be replaced by $\beta_{br} + \beta_S$. Equations (19a) and (19b) thus become:

$$\gamma_1 = \frac{\int_0^\infty \int_{-\pi}^\pi (\beta_w + \beta_{visc}) \Phi k dk d\varphi}{\int_0^\infty \int_{-\pi}^\pi \left[P\Phi - \frac{\mu}{k} \frac{\partial}{\partial k} (k^2 P\Phi) \right] k dk d\varphi} \quad (23a)$$

$$\gamma_2 = \frac{\int_0^\infty \int_{-\pi}^\pi (\beta_w + \beta_{visc}) M k dk d\varphi}{\int_0^\infty \int_{-\pi}^\pi \left[PM - \frac{\mu}{k} \frac{\partial}{\partial k} (k^2 PM) \right] k dk d\varphi} \quad (23b)$$

15 The value of coefficient μ is then adjusted in such a way that $\gamma_1 = \gamma_2$, thus ensuring that the energy and momentum budgets are balanced.

Although any of the different combinations of wind and wave breaking growth rate models mentioned in Sect. 2 may be used to perform those calculations, henceforth we report only the results obtained using β_{br} from Ardhuin et al. (2010) and β_w from

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



the JBA model, since those model functions include the most recent developments of sea wave modeling studies. Figure 1 displays the value of coefficient μ adjusted in such a way that $\gamma_1 = \gamma_2$ in Eqs. (23a) and (23b). The adjusted value of μ is seen to be of the order of 3 to 6. It is slightly increasing with wind speed over the 5–15 ms^{-1} range.

Considering Eq. (20), one may notice that, since the wave breaking term $(\partial\Phi/\partial t)_{\text{br}}$ is negative, a positive value of μ implies that the flux f is directed toward decreasing wavenumbers, which is consistent with a frequency downshift. As concerns the high frequency (HF) part of the sea wave spectra, both Elfouhaily et al.'s (1997) and Kudryavtsev et al.'s (2003) models were tested, and both results are displayed in Fig. 1. It can be seen that the value of coefficient μ tends to be slightly smaller when Kudryavtsev et al.'s HF spectrum is used instead of Elfouhaily et al.'s HF spectrum.

3.3 Comparison with previous studies on downshifting associated with wave breaking

The experiments performed by Tulin and Waseda (1999) permitted them to quantify the downward flux associated with wave breaking. Although these experiments were carried out in laboratory with discrete waves, it may give order of magnitude estimates allowing comparisons with our results.

The evolution of the lower and upper sidebands of a system of discrete waves in presence of wave breaking was studied by Tulin (1996), and is given by Eqs. (5–10) from Tulin and Waseda (1999):

$$\frac{\partial}{\partial t} (E_{-1} - E_{+1}) = \eta D_b / (\delta\omega / \omega) + (\text{higher harmonic terms}) \quad (24)$$

The influence of the higher harmonic terms is negligible and will be ignored henceforth. E_0 , E_{-1} and E_{+1} are the energies of the wavetrain in the carrier, lower and upper sidebands, with angular velocities ω , $\omega - \delta\omega$, $\omega + \delta\omega$, and wavenumbers k , $k - \delta k$, $k + \delta k$, respectively. $D_b = -\partial E_3 / \partial t$ is the energy dissipation by wave breaking, where $E_3 = E_0 + E_{-1} + E_{+1}$, and η is a nondimensional coefficient.

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



The time derivative of $(E_{-1} - E_{+1})$ can also be described as an energy flux f_{Ek} in the wavenumber space:

$$f_{Ek} = -2\delta k \frac{\partial}{\partial t} (E_{-1} - E_{+1}) \quad (25)$$

In the domain of gravity waves, the dispersion relation $\omega = \sqrt{gk}$ yields $\delta k/k = 2\delta\omega/\omega$.
 5 Therefore from Eqs. (24) and (25) one gets:

$$\frac{f_{Ek}}{(\partial E_3/\partial t)} = 4\eta k \quad (26)$$

Equation (26) expresses the ratio between the k -component of energy flux in wavenumber space, and energy dissipation rate due to wave breaking, in the case of a discrete wavetrain in the vicinity of wavenumber k . Equation (20) proposed a similar
 10 relation, but involving this time a broadband spectrum instead of a discrete wavetrain. Equation (20) may be rewritten:

$$\frac{f_k(k, \varphi)}{(\partial\Phi/\partial t)_{br}} = \mu k \quad (27)$$

Assuming that the results of discrete wavetrains may be extrapolated to a broadband spectrum, consistency between Eqs. (26) and (27) would be obtained by taking $\mu = 4\eta$.
 15 Tulin and Waseda's (1999) experimental value of η is $\eta = 0.4$, while the theoretical estimate by Tulin (1996) is $\eta \approx 0.4$ for a strong breaker to $\eta \approx 0.7$ for a weak breaker. The value of μ expected to be consistent with those studies would thus be $\mu \approx 1.6$ to 2.8.

Such a comparison between broadband spectra at sea with studies involving discrete
 20 wave systems should not be overinterpreted, and only orders of magnitude estimates are relevant here. It appears, however, that the values we obtained in Fig. 1 (μ between 3 and 6) are higher than the ones expected from Tulin's (1996) and Tulin and

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page	
Abstract	Introduction
Conclusions	References
Tables	Figures
◀	▶
◀	▶
Back	Close
Full Screen / Esc	
Printer-friendly Version	
Interactive Discussion	



Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Waseda's (1999) studies. It may be noted that when, instead of using β_{br} from Ardhuin et al. (2010) and β_w from the JBA model as done in Fig. 1, one uses β_{br} from Komen et al. (1984) and β_w from WAMDI (1988) (the combination of source terms leading to the smallest value of the ratio γ_2/γ_1 in Tables 1 and 2), the coefficient μ is reduced to values between 2.3 and 3, much closer to the results by Tulin (1996) and Tulin and Waseda (1999). Also, we will see in Sect. 4 that introducing directional diffusion reduces significantly the value of parameter μ required, leading to values closer to Tulin and Waseda's (1999) results.

4 Assessment of the ability of the model to reproduce the empirically determined sea wave spectrum

4.1 Methodology

The requirement for stationarity of the sea wave spectrum, which is expected for fully developed seas, implies that the source and sink terms balance each other at every wavenumber k and direction φ . Therefore, for any k and φ , the total source term $(\partial F/\partial t)_T$ should vanish, which reads:

$$\left(\frac{\partial F(k, \varphi)}{\partial t}\right)_T = \left(\frac{\partial F}{\partial t}\right)_w + \left(\frac{\partial F}{\partial t}\right)_{visc} + \left(\frac{\partial F}{\partial t}\right)_{br} + \left(\frac{\partial F}{\partial t}\right)_S + \left(\frac{\partial F}{\partial t}\right)_{nl} = 0 \quad (28)$$

Note that, while in the preceding sections integration of the balance equations over wavenumber space allowed us to ignore the resonant nonlinear interactions which conserve energy and momentum, these need however to be included as soon as the balance equation is written at a given wavenumber.

Of course, due to the inaccuracies of the empirically modeled sea wave spectrum, as well as the simplistic nature of the modeled source and sink terms used, one cannot expect Eq. (28) to be fulfilled at every location (k, φ) within the spectral plane. At most one can expect $(\partial F(k, \varphi)/\partial t)_T$ to be close to zero on the average. In order to assess

the efficiency with which the different source terms of Eq. (28) cancel out each other, we define a cost function K . On dimensional grounds, this cost function is taken as the average of $1/\omega|\partial(k^4 F(k, \varphi))/\partial t|_T$ over the spectral plane. Also, a similar weight is given to the different wavenumber intervals of the sea wave spectrum in logarithmic scale. This leads to the following cost function:

$$K = \frac{\int_{k=k_{\min}}^{k_{\max}} \int_{-\pi}^{\pi} \frac{1}{\omega} \left| \frac{\partial(k^4 F)}{\partial t} \right|_T d(\ln k) d\varphi}{\int_{k=k_{\min}}^{k_{\max}} \int_{-\pi}^{\pi} d(\ln k) d\varphi} \quad (29)$$

The lower limit of the integration is taken as $k_{\min} = 0.1k_p$, where $k_p = g(0.84/U)^2$ is the spectral peak, g is acceleration of gravity and U is the 10 m wind speed. The determination of the upper limit k_{\max} will be discussed below. While ideally K should be zero, comparison of the values of K obtained with different approaches will allow to compare the ability of the source terms to reproduce the empirical spectra over the spectral range $[k_{\min}, k_{\max}]$.

4.2 Resonant nonlinear term

The resonant nonlinear term $(\partial F/\partial t)_{nl}$ in Eq. (28) was estimated by the discrete interaction operator parameterization proposed by Hasselmann et al. (1985) to account for the four-wave interactions. This parameterization permits to avoid the computation of the exact nonlinear transfer expression of the term $(\partial F/\partial t)_{nl}$ involving a six-dimensional integral, and has been tested with satisfactory results by those authors on several types of situations. Hasselmann et al.'s (1985) discrete interaction operator method expresses the increments δS_{nl} to the source function $S_{nl}(f, \varphi) = \frac{\partial \Psi(f, \varphi)}{\partial t}$, where $\Psi(f, \varphi) = 2\pi k F(k, \varphi)/(d\omega/dk)$ is the wave height spectrum expressed as a function of frequency $f = \omega/2\pi$ and azimuth φ . Hasselmann et al. (1985) give the expressions for

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



the increments δS_{nl} for three interacting wavenumbers (f, φ) , $(f+, \varphi+)$, $(f-, \varphi-)$ filling a resonance condition, as follows:

$$\begin{bmatrix} \delta S_{nl} \\ \delta S_{nl}^+ \\ \delta S_{nl}^- \end{bmatrix} = \begin{bmatrix} -2 \\ (1+\lambda)\Delta f/\Delta f^+ \\ (1-\lambda)\Delta f/\Delta f^- \end{bmatrix} Cg^{-4}f^{11} \left[\Psi^2 \left(\frac{\Psi^+}{(1+\lambda)^4} + \frac{\Psi^-}{(1-\lambda)^4} \right) - 2 \frac{\Psi\Psi^+\Psi^-}{(1-\lambda^2)^4} \right] \quad (30)$$

where $f^+ = f(1+\lambda)$, $f^- = f(1-\lambda)$, with coefficients $\lambda = 0.25$ and $C = 3 \times 10^7$. With respect to component (f, φ) , the azimuths of the two other interacting components are $\varphi^+ = \varphi + \varepsilon \times 11.5^\circ$ and $\varphi^- = \varphi - \varepsilon \times 33.6^\circ$, where $\varepsilon = \pm 1$. Δf , Δf^+ , Δf^- denote the discrete resolution of the spectrum at frequencies f , f^+ , f^- , respectively, and the angular resolution $\Delta\varphi$ is assumed the same everywhere. Equation (30) is summed over all frequencies, directions and interaction configurations to yield the net source function S_{nl} .

As the gravity-capillary transition is approached, for wavelengths in the neighborhood of 1.7 cm, another type of resonant nonlinear interactions involving three-wave, rather than four-wave resonance conditions occur (Valenzuela and Laing, 1972). Those three-wave interactions also conserve energy and momentum (Hasselmann et al., 1994). They are not accounted for in this study, and as a consequence that region close to the gravity-capillary transition will not be used to estimate the cost function K . The gravity-capillary transition occurs at wavenumber $k_m = (g/T)^{1/2} = 364 \text{ rad m}^{-1}$, where $T = 74 \times 10^{-6} \text{ m}^3 \text{ s}^{-2}$ is the surface tension/density ratio for water. In the figures displayed below, the upper limit of integration in Eq. (29) will be taken as $k_{\max} = 0.2k_m$. Even if this choice may seem rather arbitrary, the behaviour of K is not qualitatively different if other choices are made for k_{\max} between $0.05k_m$ and k_m , although K would increase by a factor of up to 4 if $k_{\max} = k_m$ rather than $k_{\max} = 0.2k_m$ were used. This is related to the fact that close to $k = k_m$ the spectrum is modified by processes not taken into account in this study, such as three-wave resonant interactions.

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



4.3 Comparison with the classical approach

The values of the cost function K obtained here may be compared to the ones obtained when the frequency downshift introduced hereabove is ignored. In that case, the supplementary term $(\partial F/\partial t)_S$ vanishes. This is the classical way of dealing with wave breaking, and it can be obtained simply by ignoring the momentum balance equation. In that case we just take $\gamma = \gamma_1$ as given by the energy Eq. (19a), and we ignore Eq. (19b). In Fig. 2, the cost function K obtained with both approaches is displayed. As expected, the cost function is seen to increase with wind speed, since the net residual obtained after algebraic summation of the different growth rates scales with the wind input growth rate, and thus increases with wind speed. The version using Kudryavtsev et al.'s (2003) HF spectrum gives systematically greater cost function K , except for the strongest winds ($U \geq 14 \text{ ms}^{-1}$). When Elfouhaily et al.'s (1997) HF spectrum is used, the cost function obtained with the approach of this paper is slightly greater than the one obtained with the classical approach. When Kudryavtsev et al.'s (2003) HF spectrum is used, the cost functions obtained with both approaches are virtually identical. However, the increase of cost functions obtained with the approach of this paper is much smaller than the difference of cost functions related to the choice of the reference spectral model (solid and dashed lines in Fig. 2).

Other tests (not shown) were also performed using other model functions for the source terms β_{br} and β_w . For example, it has been shown from Tables 1 and 2 that the lowest ratio R ($R = 1.72$) was obtained by using β_w from WAMDI (1988) and β_{br} from Komen et al. (1984), together with Kudryavtsev et al.'s (2003) HF spectrum. If those model functions are used, then the cost function obtained with the approach of this paper is slightly lower than the cost function obtained in the classical approach.

4.4 Effect of azimuthal diffusion of spectral energy

Up to now, the downward energy flux $f(k, \varphi)$ associated with wave breaking was assumed to be radially oriented, and thus its azimuthal component f_φ was assumed to

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



be equal to zero. In order to relax that constraint, I tested an approach in which, in addition to inward flux, wave breaking was also associated with azimuthal diffusion of the spectral energy. Such azimuthal diffusion should be somehow related to the inward energy flux and its azimuthal gradient. The following expression is chosen here for the azimuthal energy flux:

$$f_{\varphi} = \nu \frac{\partial f_k}{\partial \varphi} \quad (31)$$

where the non-dimensional constant ν plays the role of a diffusion coefficient.

The supplementary source term $(\partial\Phi/\partial t)_S$ can be computed again from the conservation Eq. (21), in which both radial and azimuthal components of \mathbf{f} are now considered, and the resulting values of μ and K can thus be obtained for any value of ν . Figure 3 displays the difference $K(\nu) - K(0)$ of the cost function K , as a function of diffusion parameter ν . As ν increases from $\nu = 0$, K first decreases, until ν reaches a value of the order of 0.3 to 0.5. As ν increases further, the variation of K becomes much weaker, and K saturates toward some asymptotic value. It can be seen, however, that the relative difference of cost function K is rather weak as ν varies (less than 4%). Except for the low wind case ($U = 5 \text{ m s}^{-1}$), the inclusion of azimuthal diffusion of spectral energy always produces a decrease of K , and thus slightly improves the consistency with empirical spectra.

Figure 4 displays the parameter μ characterizing the strength of inward flux corresponding to those values of diffusion parameter ν . It is seen that μ decreases with increasing values of ν , while still remaining positive. For ν of the order of 0.5, which is the value beyond which ΔK was found to become virtually constant in Fig. 3, the flux parameter $\mu(\nu)$ is about a factor of 2 smaller than $\mu(0)$. As ν increases further, $\mu(\nu)$ still decreases down to about $1/4\mu(0)$. However, in any case, it is seen that μ remains positive, which means that a downward flux of spectral energy is still required in order to fulfill simultaneously the balance of both energy and momentum budgets.

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



5 Conclusions

Using a unified two-dimensional sea wave spectral model over the whole range of wavenumbers at full development, I attempted to assess the compatibility of the energy and momentum budgets over the whole spectral range. For fully developed situations, among the various combinations of model functions for wave breaking and wind source terms tested, no one allowed to fulfill together the integral energy and momentum balance equations. For example, assuming that the integral energy balance equation is fulfilled, a proportion of about one half or more of the momentum brought by the wind to the waves would accumulate within the wave system. This is in contradiction with the fact that a fully developed sea is expected to be nearly stationary. This indicates that either the chosen spectral shapes are unrealistic, or else that the source term parameterizations are unrealistic. In this paper I choose to consider the empirical spectra as the observational reference, and I propose a correction to the source terms, based on physical grounds, permitting to exhibit a solution where the energy and momentum balance is restored.

The following approach is thus proposed in order to reconcile both energy and momentum budgets. It is known, both experimentally and theoretically, that wave breaking may contribute to frequency downshift of a narrow-banded wave spectrum, when the modulational instability is combined with wave breaking. In this paper it is assumed that such a frequency downshift also occurs for the sea-wave spectrum, despite the fact that it is a broad-banded spectrum. This is expressed by assuming that, in addition to dissipation, wave breaking is also associated with an energy flux f directed toward low wavenumbers, and quantified by means of a non-dimensional coefficient μ according to Eq. (20). Coefficient μ is estimated to be of the order of 3 to 6 for wind speeds within the 5–15 ms⁻¹ range. A cost function K is computed allowing to assess the ability of the source functions to reproduce the empirically determined two-dimensional sea wave spectrum at full development. For the model source functions used to obtain the results displayed in this paper, the introduction of this inward spectral flux f slightly

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



alters the consistency between source terms and empirical spectra, as estimated by the cost function K . Other tests were performed in which it was assumed that, in addition to inward flux, wave breaking was also associated with azimuthal diffusion of the spectral energy. Incorporating such azimuthal diffusion could allow a slight improvement of the cost function K , and would correlatively reduce the inward flux parameter μ by a factor of up to 4, although it would not be able to remove entirely the inward flux.

These studies involving the spectral flux f were performed using the most recent wave breaking and wind energy transfer rates (Ardhuin et al.'s, 2010, and JBA model, respectively). Similar results were obtained by using other other models of wave breaking and wind input energy transfer rates, although with some combinations of transfer rate models the approach of this paper gave a slightly smaller (instead of slightly higher) cost function than the classical approach. As concerns the sea wave spectral model, two kinds of reference spectra were taken at high wavenumbers ($k > 25k_p$): the spectrum by Elfouhaily et al. (1997), and the one by Kudryavtsev et al. (2003). The same tendencies are obtained here with those two kinds of wavenumber spectra. However, since the momentum of waves is quite sensitive to the directional spectrum at high wavenumbers, further improvements of the models for directional spectrum should be highly valuable especially at high wave numbers to establish more firmly the tendencies reported in this paper.

Appendix A

JBA model for the atmospheric term for growth rate

The JBA model mentioned in this paper accounts for the wind input term β_w of wave growth rate, as proposed by Janssen's (1991) quasi-linear theory, including tuning modifications by Banner and Morison (2010) and Ardhuin et al. (2010). According to this

OSD

9, 2581–2619, 2012

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



model, the dimensionless critical height, μ_1 , is defined as:

$$\mu_1(k, \varphi) = (u_*/c)^2 (gz_o/u_*^2) \exp(J_1 \kappa / [\cos \varphi (u_*/c + 0.006)]) \quad (A1)$$

where $\kappa = 0.4$ is Van Karman constant, c is the phase speed, and $J_1 = 0.99$. The roughness length z_o is given as:

$$z_o = \frac{0.01 u_*^2}{g \sqrt{1 - c_o (\tau_w / \tau)}} \quad (A2)$$

where $c_o = 0.8$, and the ratio of wave-induced stress τ_w to the total stress τ is taken from Janssen's (1989) Eq. (7).

The Miles parameter b is given by:

$$b(k, \varphi) = J_2 \mu_1 (\ln(\mu_1))^4 / \kappa^2 \text{ for } \mu_1 \leq 1, \text{ where } J_2 = 1.6$$

$$b(k, \varphi) = 0 \text{ for } \mu_1 > 1 \quad (A3)$$

The spectral growth rate is then

$$\beta_w(k, \varphi) = \frac{\rho_a}{\rho_w} b(k, \varphi) \omega \left[\frac{u_*^{\text{red}}(k)}{c} \right]^2 \cdot [\max(\cos \varphi, 0)]^2 \quad (A4)$$

Here, u_*^{red} is the reduced friction velocity which is felt by wavenumber k , as a consequence of the sheltering of the short waves by the longer waves. We express it by removing the integrated waveform drag due to waves with wavenumbers between 0 and k , which gives:

$$(u_*^{\text{red}})^2 = u_*^2 - s_u \int_0^k \int_{-\pi}^{\pi} \frac{\rho_w}{\rho_a} \beta_w(k', \varphi) \omega(k') F(k', \varphi) \cos \varphi k' dk' d\varphi \quad (A5)$$

where the sheltering coefficient s_u is taken as $s_u = 1$ following Ardhuin et al. (2010).

Equations (A4) and (A5) are solved iteratively.

Appendix B

Expression of the wave breaking source term by Ardhuin et al. (2010)

The threshold-based formulation for the wave breaking source term $P_a(k, \varphi)$ introduced in Eq. (17) is described by Ardhuin et al. (2010) as the sum of a saturation-based term P_{sat} , a cumulative term P_{cu} , and a term related to wave-turbulence interaction P_{turb} , as follows:

$$P_a(k, \varphi) = P_{\text{sat}}(k, \varphi) + P_{\text{cu}}(k, \varphi) + P_{\text{turb}}(k, \varphi) \quad (\text{B1})$$

The last term of the right hand side P_{turb} due to wave-turbulence interactions is expected to be much weaker than both other terms and is neglected by Ardhuin et al. (2010). The saturation-based term P_{sat} is given as:

$$P_{\text{sat}}(k, \varphi) = \omega \frac{C_{\text{sat}}}{B_r^2} \left\{ \delta \cdot \max[B_1(k) - B_r, 0]^2 + (1 - \delta) \cdot \max[B'(k, \varphi) - B_r, 0]^2 \right\} \quad (\text{B2})$$

In this expression, $B'(k, \varphi)$ is a direction-integrated spectral saturation, with a restriction of the integration over a $\pm\Delta\varphi$ interval (with $\Delta\varphi = 80^\circ$) in order to maintain a dependence of dissipation rate upon direction. It is expressed here as:

$$B'(k, \varphi) = \frac{\int_{\varphi-\Delta\varphi}^{\varphi+\Delta\varphi} \cos^{s_B}(\varphi - \varphi') B(k, \varphi') d\varphi'}{\int_{\varphi-\Delta\varphi}^{\varphi+\Delta\varphi} \cos^{s_B}(\varphi - \varphi') d\varphi'} \quad (\text{B3})$$

where $B(k, \varphi) = k^4 F(k, \varphi)$, and

$$B_1(k) = \max[B'(k, \varphi), 0 \leq \varphi < 2\pi] \quad (\text{B4})$$

Following Ardhuin et al. (2010), in Eq. (B2) the threshold for the onset of breaking is taken as $B_r = 0.0009$, and the other constants are $C_{\text{sat}} = 2.2 \times 10^{-4}$, $\delta = 0$, and $s_B = 2$.

The cumulative term P_{cu} represents the smoothing of the surface by big breakers with celerity C' that wipe out smaller waves of phase speed C . It is expressed by Ardhuin et al. (2010) as follows:

$$P_{\text{cu}}(k, \varphi) = -1.44 C_{\text{cu}} \int_0^{r_{\text{cu}} f} \int_0^{2\pi} \max \left[\sqrt{B(k', \varphi')} - \sqrt{B_r, 0} \right]^2 \Delta C d\varphi' dk' \quad (\text{B5})$$

where f is sea wave frequency, $\Delta C = \|C - C'\|$, and the constants are taken as $C_{\text{cu}} = -0.4$ and $r_{\text{cu}} = 0.5$.



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Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



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**Imbalance of energy
and momentum of
sea waves**

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



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Imbalance of energy and momentum of sea wavesG. V. Caudal

[Title Page](#)[Abstract](#)[Introduction](#)[Conclusions](#)[References](#)[Tables](#)[Figures](#)[⏪](#)[⏩](#)[◀](#)[▶](#)[Back](#)[Close](#)[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)

Imbalance of energy and momentum of sea waves

G. V. Caudal

Table 1. Ratio $R = \gamma_2/\gamma_1$, where γ_1 and γ_2 are the coefficients of the wave breaking dissipation term required to fulfill the energy and momentum balance, respectively, for different combinations of the expressions of β_w and β_{br} assumed. The sea wave spectral model for $k < 25k_p$ is from Elfouhaily et al. (1997), with directional spreading function by Resio et al. (2011). Above $k = 25k_p$, the sea wave directional spectral model is from Elfouhaily et al. (1997). A wind speed $U = 10 \text{ m s}^{-1}$ is assumed.

Ratio $R = \gamma_2/\gamma_1$	β_{br} Komen et al. (1984)	β_{br} Phillips (1985)	β_{br} Ardhuin et al. (2010)
β_w Plant (1982)	2.97	2.55	2.67
β_w WAMDI (1988)	2.66	2.29	2.40
β_w Donelan-Pierson (1987)	5.11	4.39	4.60
β_w Kukulka-Hara (2005)	4.78	4.10	4.30
β_w JBA model	3.68	3.16	3.31

[Title Page](#)
[Abstract](#)
[Introduction](#)
[Conclusions](#)
[References](#)
[Tables](#)
[Figures](#)
[Back](#)
[Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)


Imbalance of energy and momentum of sea waves

G. V. Caudal

Table 2. Same as Table 1, but Elfouhaily et al.'s (1997) wave spectrum is replaced by Kudryavtsev et al.'s (2003) spectrum at high wavenumbers ($k \geq 25k_p$).

Ratio $R = \gamma_2/\gamma_1$	β_{br} Komen et al. (1984)	β_{br} Phillips (1985)	β_{br} Ardhuin et al. (2010)
β_w Plant (1982)	2.58	3.32	3.45
β_w WAMDI (1988)	1.72	2.21	2.30
β_w Donelan-Pierson (1987)	4.00	5.15	5.36
β_w Kukulka-Hara (2005)	3.38	4.35	4.53
β_w JBA model	2.76	3.55	3.69

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



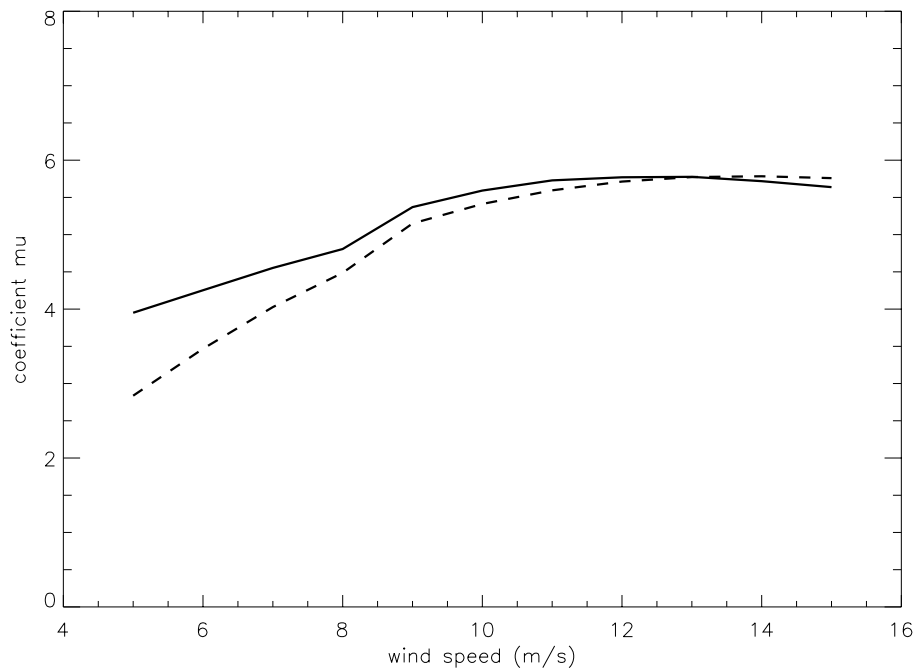


Fig. 1. Coefficient μ quantifying the downward spectral energy flux (as defined by Eq. 20), allowing to balance the energy and momentum budgets, as a function of wind speed. The wave breaking term β_{br} is from Ardhuin et al. (2010), and the wind input term β_w is from the JBA model. The sea wave spectral model for $k < 25k_p$ is from Elfouhaily et al. (1997), with directional spreading function by Resio et al. (2011). Above $k = 25k_p$, the sea wave spectral model is either from Elfouhaily et al. (1997) (solid line), or from Kudryavtsev et al. (2003) (dashed line).

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



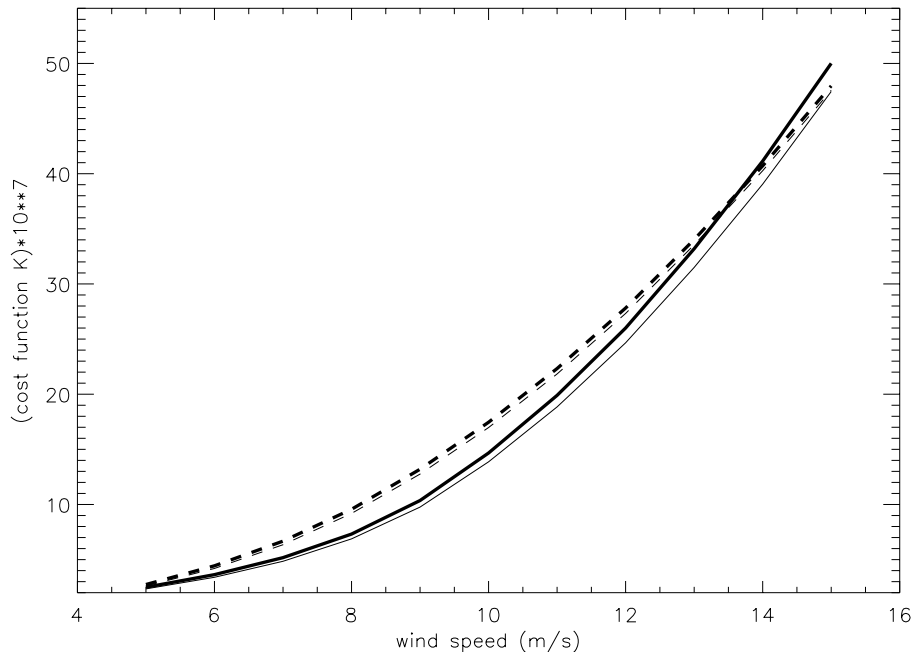


Fig. 2. Cost function K times 10^7 , as defined by Eq. (29), corresponding to the model of this paper (thick lines), compared with the cost function K obtained in the classical approach (thin lines), as a function of wind speed. The sea wave spectral models are the same as in Fig. 1, involving high wavenumber spectrum from Elfouhaily et al. (1997) (solid lines), or from Kudryavtsev et al. (2003) (dashed lines).

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



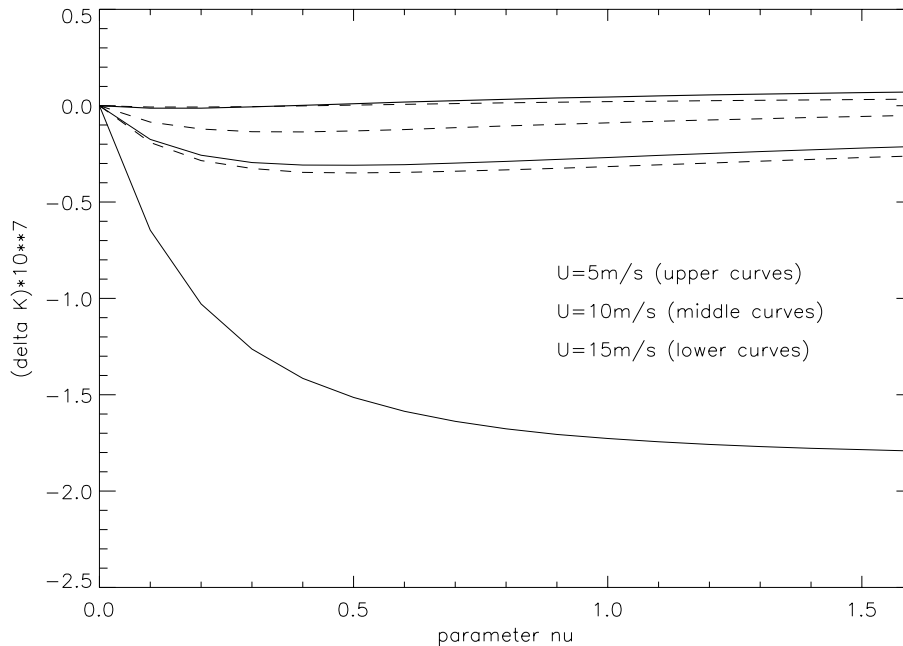


Fig. 3. Difference $\Delta K = (K(\nu) - K(0))$ between the cost function K obtained when a diffusion parameter ν is assumed, and cost function obtained without azimuthal diffusion ($\nu = 0$). The quantity $\Delta K \times 10^7$ is displayed as a function of parameter ν for 5, 10, and 15 m s^{-1} wind speeds. The sea wave spectral models are the same as in Fig. 1, involving high wavenumber spectrum from Elfouhaily et al. (1997) (solid lines), or from Kudryavtsev et al. (2003) (dashed lines).

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



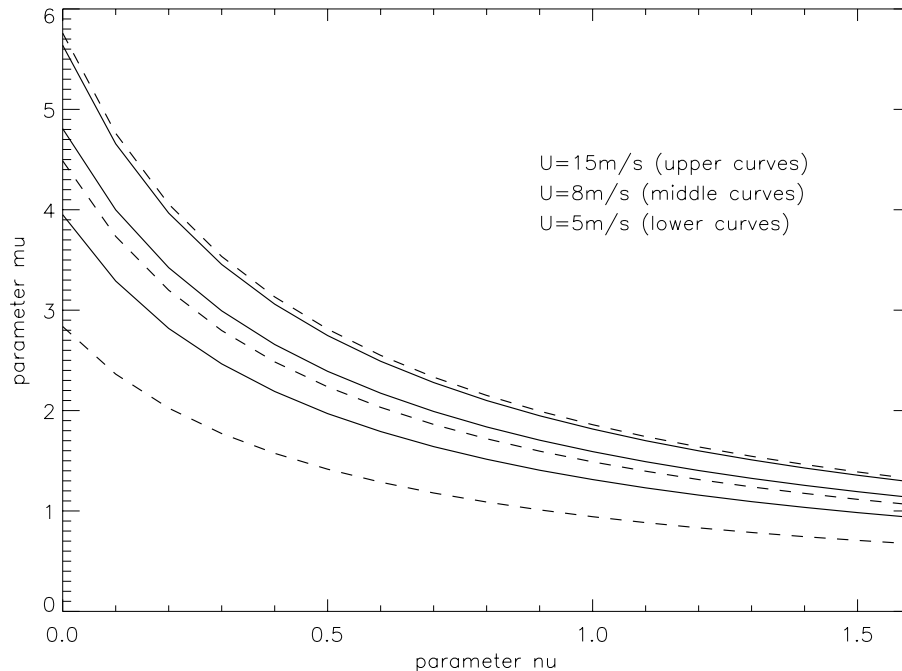


Fig. 4. Spectral flux parameter μ as a function of diffusion parameter ν , displayed for 5, 8, and 15 m s^{-1} wind speeds. The sea wave spectral models are the same as in Fig. 1, involving high wavenumber spectrum from Elfouhaily et al. (1997) (solid lines), or from Kudryavtsev et al. (2003) (dashed lines).

Imbalance of energy and momentum of sea waves

G. V. Caudal

Title Page

Abstract Introduction

Conclusions References

Tables Figures

◀ ▶

◀ ▶

Back Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

