Medhaug and Furevik (2011) discuss multi-decadal variability in coupled atmosphere-ocean general circulation models. They cite a lack of common phasing of warmer and cooler episodes as evidence of intrinsic climate variability. Their section 4 “Discussion”, in the two paragraphs after (7), suggests “fluctuating behaviour” in the overturning circulation. However, the following analysis shows that fluctuating behaviour is not a necessary consequence of the scenario described. [Equation numbers here correspond with those in Medhaug and Furevik (2011) and then continue].

Considering only temperature and abbreviating the notation in the remaining equations, (4) and (5) become

$$V_1 = V_2 \qquad (4)$$

for the volume transports V_1 northwards from the tropical region and through the northern region of sinking V_2 [figure 11 in Medhaug and Furevik (2011)], and

$$dT_2/dt + V_2(T_2 - T_1)/M' = - Q' \qquad (5)$$

where T_1, T_2 are the temperatures of the tropical and northern regions respectively and (in terms of the original notation)
$M' = M/\rho_0$ is the volume of the mixed layer in the sub-polar gyre region,
$Q' = Q_H/(Mc_p)$ is the heat flux from the sub-polar gyre relative to the gyre’s thermal capacity.

The discussion in Medhaug and Furevik (2011) then invokes a relationship between the large-scale north-south density gradient and the overturning circulation. Most simply, such a relation might be

$$V_2 = (T_1 - T_2)V' \qquad (8)$$

expressing an increased overturning circulation V_2 if the (depth-integrated) density gradient represented by $(T_1 - T_2)$ is increased. Here V' is a constant.

Equations (5) and (8) are easily combined to a single equation for T_2 (T_1 has no prognostic equation in Medhaug and Furevik (2011) and is implicitly assumed constant):

$$dT_2/dt - V'(T_2 - T_1)^2/M' = - Q' \qquad (9).$$

Equation (9) has a constant solution $(T_1 - T_2) = (Q'M'/V')^{1/2}$
and correspondingly $V_2 = (Q'M'V')^{1/2}$.

The time-varying form of (9) has the general analytic solution

$$T_1 - T_2 = (Q'M'/V')^{1/2} \{1 - \exp[-2(Q'V'/M')^{1/2} (t-t_0)]\}/\{1 + \exp[-2(Q'V'/M')^{1/2} (t-t_0)]\} \qquad (10).$$

Any initial value of T_2 may be accommodated by choice of t_0, e.g.
if $t_0 = 0$ then $T_2 = T_1$ at $t = 0$; if $t_0 > 0$ then $T_2 > T_1$ at $t = 0$; if $t_0 < 0$ then $T_2 < T_1$ at $t = 0$.

In all cases T_2 approaches the constant solution $T_1 - (Q'M'/V')^{1/2}$ as time t becomes large; the nature of the approach is exponential decay of the departure from the constant solution. If T_2 is close to the constant solution (i.e. the exponential term is small), then (10) becomes

$$T_2 \sim T_1 - (Q'M'/V')^{1/2} \{1 - 2\exp[-2(Q'V'/M')^{1/2} (t-t_0)]\}.$$

Hence an overturning – density relation in the form (8), combined with (5), does not admit fluctuating behaviour but only monotonic “decay” to a constant solution. This result does not deny intrinsic climate variability. However, in order to explain fluctuating behaviour in oceanic heat distributions, the result does suggest seeking a different mechanism. [The mechanism discussed here is direct advection, by an overturning circulation related to a meridional density gradient of thermal origin].

Reference

Medhaug, I. and T. Furevik, 2011. North Atlantic 20th century multidecadal variability in coupled climate models: sea surface temperature and ocean overturning circulation. Ocean Sci. Discuss., 8, 353-396.