Reply to the referee1 concerning 'Multifractal analysis of oceanic chlorophyll maps remotely sensed from space'

L. de Montera, M. Jouini, S. Verrier, S. Thiria and M. Crepon

Dear Editor,

Please find in this document the answers to the referee 1. The modifications of the original manuscript have been written in red color in order to facilitate the review.

Best Regards,

The authors.

Reply to Reviewer 1:

General comment:

"The paper is well written and easy to understand. Some of the discussions are very interesting, as for instance the introduction of the FIF model (although this is an old, well-known feature) and the discussion of biases in geophysical models. The application of the fractional cascade to assess the validity of the multifractal model, although not really novel, is an interesting approach. However, I have two (probably mild) objections to this paper."

Dear Referee,

Thank you for this careful review of the paper. We are happy that you appreciate the discussion concerning the biases in numerical models. Actually, originally we wanted to develop much more this part, and in particular we wanted to show that coupled physical-biogeochemical do not reproduce the observed scalings. However, the head of our laboratory was not pleased at all with this and ultimately refused us the access to the simulations outputs, arguing that we were not collaborating... This is very sad and clearly dishonest. Anyway, we hope that in the future the knowledge of multifractal properties will be used to improve these simulations and that the people working in this field will improve their ethical values.

Please find below our reply to your comments

Best regards,

The authors.

Comments:

"First, I do not feel that the results obtained are really new. Seuront et al. have many papers on the multifractal characterisitics of plankton concentration that arguably obtain essentially the same results, although in their case the range of scales studied is much smaller.

However, there are several papers in the literature (e.g., Nieves et al. given in the references, and others from Mahadevan et collaborators) which study the scaling properties of satellite chlorophyll maps; from my point of view, a comparative discussion with those previous studies would better put into context the findings of this paper."

Reply: As you say, the data studied by Seuront et al. are restricted to small scales. Moreover, they mainly use 1D data obtained by in situ Lagrangian measurements. This is why it is interesting to test their results with 2D remotely sensed Eulerian data. We agree that the results are not completely new, however the test of reproducibility is part of the scientific work, especially with different instruments and locations.

Even though the main purpose of the paper is to confirm previous studies, there is however some new points:

(i) based on the theory of turbulence and on a new technique (de Montera et al 2010), we propose an accurate prediction of chlorophyll fields second order exponent by estimating precisely all intermittency corrections

(ii) this prediction is tested with a large amount of high quality data (100 maps from L2 products, not from averaged and merged L3 products)

(iii) some implications concerning biases in numerical model are presented.

Concerning the paper you cite, it is difficult to compare our results with the one of Nieves et al. because this paper does not link directly numerical results and predictions from turbulence theory. It is also difficult to compare with Mahadevan & Campbell (2002) because there work was based on only 4 images. However, we added the following sentences in section 6:

"However, Nieves et al. (2007) performed a multi-scale analysis of SST data with a larger scale range (level L3 product) and found that the observed multifractal spectra was very similar to the one obtained with chlorophyll concentration data. This result provides an additional argument in favour of a link between phytoplankton patchiness and turbulent mixing at large scales, which will be developed in the next section."

The first sentence of the last paragraph in the introduction was also modified:

"In this context, to the best of our knowledge, the large volumes of data collected by remote sensing from space, over a period of more than two decades, have almost not been exploited in order to improve scientific understanding of the multi-scaling properties of phytoplankton fields (with the noticeable exception of Nieves et al., 2007)."

2- Second, the scaling quantity used for FIF is probably not the best suited one, particularly as the authors complain on the effects of noise at smaller scales. Wavelets were introduced long ago to deal with noise and discretization effects in multiscaling systems, and some old papers from Arneodo's group explain how to apply them to satellite images; other, more recent approaches (the authors mention several in the references) could probably used to overcome this problems. Although this does not restrain the validity of the results presented here, the use of these techniques could be considered to improve these results, possibly in future works.

Reply: We agree that wavelet analysis is a more precise technique to perform a multi-scale analysis. However, in the context of a publication in large scope journal, we think that the analysis technique has to remain simple (i.e., easy to understand and to reproduce). As you say, the use of a simple averagings does not reduce the validity of the results. Moreover, we have two additional reasons for this choice: (i) we need to work with the physical quantity because we compare it to a physical theory, (ii) the use of wavelets to filter the process is not straightforward because there is a part of arbitrary in it (like the choice of the threshold, etc.). Actually we also wanted to work with no prior assumption at all (the wavelet filtering may erase some interesting parts of the signal). Of course, in a different context and in future works, the use of wavelet decomposition will be considered.

Specific comments:

1- Errorbars are lacking, so we have no determination of the uncertainty in the estimated parameters.

Reply: Actually, the lack of error bars was a deliberate choice. The point is that errors depend on the scale range you choose to perform the fits, and is therefore largely arbitrary. The problem of providing errors is even more difficult concerning the estimation of cascade parameters because you have to choose not only the scale range, but also the orders of the moments. Therefore we renounced to give error bars. The text has been modified in the second paragraph of section 6:

"(here, we renounced to provide the standard deviations of the estimators because they would indicate an artificially high precision; the estimation error of the whole analysis technique has been tested with simulations and is found to be around 10% for both parameters)"

However, since the precision of the estimation of H is crucial because we make a theoretical prediction of this parameter, we included the error of H in the text with the corresponding scale range over which it was computed (end of first paragraph of section 6):

"Finally, over the scale range 4-128 km, the empirical first-order structure function is consistent with Eq. (17), and H is estimated to be around 0.4 (the numerical fit yields 0.402 with a standard deviation of the estimator equal to 0.005)."

2- On page 66 it is claimed that if chlorophyll concentration is the result of the application of a non-linear function f onto the reflectivity, the reflectivity itself does not need share the same scaling laws that chlorophyll. Well, in fact if the function f is locally invertible and smooth, it is guaranteed to be locally bi-Lipschiz, so over those neighborhoods were f is bi-Lipschitz the scaling laws are directly preserved (i.e., the Holder exponent of the chlorophill exponent and of the reflectivity are the same at the same point). The real problem with reflectivities is that in fact you need to combine several different channels to retrieve the chlorophyll concentrations and this gives rise to cancellations (e.g., the scaling exponent of channel 1 is cancelled by a similar contribution but of opposite sign of channel 2), so the scaling exponents of chlorophyll need not to coincide with those of the compounding channels. Cancellations are mainly due to the presence of yellow matter or suspended sediments; aparte from the places where those effects are important, the scaling exponents of chlorophyll concentration and the used reflectivity channels will usually coincide.

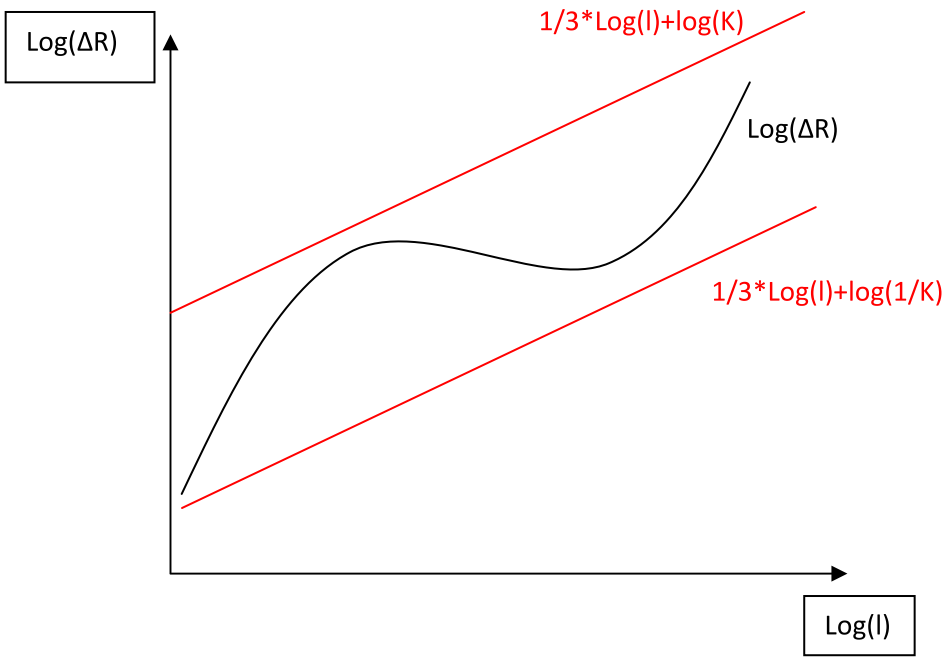
Reply: This comment is interesting and this point has to be clarified because many authors prefer to work with reflectivities rather than chlorophyll concentrations in order to avoid an additional error due to the retrieval algorithm. In this paper, we have a different opinion, because the tracer concentration is the only quantity that verifies a scale laws because of physical laws and that therefore, reflectivities have no reason to be scaling. The argument that reflectivities and chlorophyll concentrations have same exponent because the non-linear transformation that links them is bi-Lipschitz is not correct in our opinion. The bi-Lipschitz property only means that:

$$\exists K \geq 1, \quad \frac{\Delta Chl(l)}{K} \leq \Delta R(l) \leq K \Delta Chl(l)$$

Or, by using the chlorophyll concentration turbulent scale law and taking the logarithm:

$$\exists K \geq 1, \quad 1/3 * \mathrm{Log}(l) + \log\left(\frac{1}{K}\right) \leq \log(\Delta R(l)) \leq 1/3 * \mathrm{Log}(l) + \log(K)$$

Therefore reflectivities scales the same way as chlorophyll concentrations only if it is assumed that reflectivities are scaling. However, this assumption is not acceptable because the fact that reflectivities are scaling is not guaranteed by physical laws. It is possible to have the following situation (which actually appears with our data when taking the 3rd or 4th power):



The following text has been added in the paper (end of section 5):

"This statement may appear in contradiction with the fact that a bi-lipschitz transformation implies a similar scaling exponent. However, this argument is not correct here because we do not make the prior assumption that reflectivities are scaling."

Actually, originally, we wanted to develop much more this part due to the importance of biases, and in particular we wanted to show that coupled physical-biogeochemical do not reproduce the observed scaling; however, the head of our laboratory was not pleased at all with this and ultimately refused us the access to the simulations outputs... no comment)

Multifractal analysis of oceanic chlorophyll maps remotely sensed from space

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Abstract

Phytoplankton patchiness has been investigated with multifractal analysis techniques. We analyzed oceanic chlorophyll maps, measured by the SeaWiFS orbiting sensor, which are considered to be good proxies for phytoplankton. Multifractal properties are observed, from the sub-mesoscale up to the mesoscale, and are found to be consistent with the Corssin-Obukhov scale law of passive scalars. This result indicates that, within this scale range, turbulent mixing would be the dominant effect leading to the observed variability of phytoplankton fields. Finally, it is shown that multifractal patchiness can be responsible for significant biases in the nonlinear source and sink terms involved in biogeochemical numerical models.

1 Introduction

It is sometimes argued that turbulent mixing leads to homogeneous fields. However, Kolmogorov (1941), Obukhov (1949) and Corssin (1951) have shown that, on the contrary, turbulent mixing generates highly irregular structures that are heterogeneous at all scales. Their work was based on the hypothesis of scale invariance, which means, in simple words, that eddies can be expected to occur in a similar manner at all scales. In the case where the physical quantity is the concentration of a passive tracer, these authors demonstrated that its variability exhibits fractal properties which can be described statistically using scale laws. This result is often referred to as the theory of passive scalars.

Since the phytoplankton's ability to swim is very limited, its displacements are mainly due to the velocity of the fluid in which it evolves. Phytoplankton patchiness is thus strongly related to turbulence. This consequence has led numerous authors to study the scale invariance properties of phytoplankton patches, and to confront experimental data with phenomenological models derived from, or inspired by, the theory of passive scalars. Early studies remained confined to second-order moments, such as the slope of the power spectrum (see, e.g., Platt, 1972), whereas more recent research takes into account the intermittent transfer of conservative quantities in scale space, such as energy and scalar variance, which give rise to multifractal statistics through cascade processes

(Seuront et al., 1996a, b, 1999; Seuront & Schmitt, 2004, 2005a, b; Lovejoy et al., 2001a, b; Pottier et al, 2008).

At smaller scales, most of these studies found empirical proof for a passive scalar regime of phytoplankton patchiness, corresponding to the well-known '-5/3' power spectrum slope of homogenous and isotropic turbulence. This purely turbulent regime appears to be limited to spatial scales smaller than a particular scale of the order of 100m, called the 'planktonoscale' by Lovejoy et al. (2001b). In fact, although phytoplankton can reasonably be described as passively advected, in the sense that its retroaction on the turbulent flow is negligible, it cannot be considered to be totally passive, since it is biologically active. One important biological process is zooplankton grazing, and the 'planktonoscale' is currently interpreted as corresponding to the scale at which changes take place in the grazing regime. This modification of the grazing regime appears to be related to the zooplankton's ability to swim. Contrary to phytoplankton, zooplankton is able to swim, although its speed remains limited. Therefore, there exists a scale above which its displacements are dominated by turbulent mixing. This hypothesis is supported by the fact that the zooplankton's concentration power spectrum whitens at scales smaller than the 'planktonoscale' (Currie & Roff, 2006).

At scales larger than the 'planktonoscale', the phytoplankton patchiness description is more confused. Some authors found a power spectrum slope steeper than -5/3 (around -2, Currie & Roff, 2006; Seuront et al., 1999), interpreting this result as a transition from Eulerian to Lagrangian statistics, due to the inertia of the boat carrying the instruments (cf. Seuront et al., 1996b). On the other hand, the analysis of remotely sensed phytoplankton fields from aircraft led to a smoother slope (around -1.2, Lovejoy et al., 2001b). From a theoretical point of view, the situation is even less clear: some studies reached the conclusion that growth and trophic interactions should decrease the slope of the power spectrum (Denman & Platt, 1976; Fasham, 1978), whereas some others predict that it should increase (Steele & Henderson, 1979) or that the power spectrum has no specific regime (Horwood, 1978).

In this context, to the best of our knowledge, the large volumes of data collected by remote sensing from space, over a period of more than two decades, have almost not been exploited in order to improve scientific understanding of the multi-scaling properties of phytoplankton fields (with the noticeable exception of Nieves et al., 2007). The aim of the present study is to analyze oceanic chlorophyll maps obtained through the use of this type of sensor. The first part of the paper briefly recalls the passive scalar theory and the notion of multifractal intermittency. The second and third parts describe both the cascade model and the analysis technique. The fourth part presents the dataset and the pre-treatments. The fifth and sixth parts are dedicated to the results and their interpretation. Finally, the last part of the paper provides an example of the importance of multifractal patchiness in oceanic tracers, by assessing the biases it produces in biogeochemical numerical models.

2 Theoretical background: turbulence and multifractals

Richarsdon (1922) described turbulence as a cascade process that transfers kinetic energy from large scales to small scales by a hierarchy of imbricated eddies. The hypothesis of scale invariance relies on phenomenology and the invariance of the Navier-Stokes equations under dilatation or contraction of the reference system (see Schertzer & Lovejoy, 1987, Appendix A). On the basis of this hypothesis, and the conservation of energy in the inertial range, Kolmogorov (1941) used dimensionality and some general assumptions, such as homogeneity and isotropy, to derive his famous statistical scale law:

$$\Delta v_l \simeq \varepsilon^{1/3} l^{1/3} \tag{1}$$

In this equation, $\Delta v_l \equiv \left\langle \left| v(x+l) - v(x) \right| \right\rangle$ represents the mean shear of (longitudinal) velocity between two points separated by a distance *l,* and *ε* represents the mean density of the energy flux, which is equal to the rate of energy dissipation per unit mass. A similar scale law has been derived for the concentration *C* of a passive tracer (Obukhov, 1949; Corrsin, 1951):

$$\Delta C_l \simeq \varphi^{1/3} l^{1/3} \tag{2}$$

where $\varphi \equiv \chi^{3/2} \varepsilon^{-1/2}$ represents the non-linear coupling between velocity and concentration, with $\chi \equiv -\dfrac{\partial \left(\Delta C_l \right)^2}{\partial t}$ the mean density of the concentration variance flux (for a review concerning these early turbulent models, see, e.g., Panchev, 1971).

Landau (1944) pointed out that these fluxes have no reason to be homogeneous: although they are on average conserved during the cascade process, their transfer is a priori intermittent. This remark has led the transfer process to be described by stochastic multiplicative cascades (Novikov & Stewart, 1964; Yaglom, 1966). A multiplicative cascade can be constructed by iterating the following simple procedure: (i) distribute a quantity uniformly over an interval, (ii) divide this interval into sub-intervals, (iii) multiply these by a random variable in order to obtain the new quantity for each sub-interval, (iv) repeat steps (ii) and (iii) until the smallest scale of the cascade is reached. The important point here is that the distribution of the random variables, referred to as the multiplicative weights in the following, does not depend on the level of iteration of the construction algorithm. Thus, because the latter is not dependent on scale, the resulting mathematical object has fractal and even multifractal properties. It turns out that these properties can be described by the scaling of its statistical moments, of fractional order *q* (for more details, see Schertzer et al., 2002):

$$\left\langle \varepsilon_l^{\,q} \right\rangle \simeq \left(\frac{L}{l} \right)^{K_\varepsilon(q)} \tag{3}$$

$$\left\langle \chi_l^{\,q} \right\rangle \simeq \left(\frac{L}{l} \right)^{K_\chi(q)} \tag{4}$$

where ε_l and χ_l are the fluxes averaged at scale *l*, *L* is the largest scale of the cascade, and $K_\varepsilon(q)$ and $K_\chi(q)$ are the so-called moment scaling functions.

In order to obtain realistic fields, the discrete cascade model described above has been generalized to continuous cascades, obtained by scale densification (Schertzer & Lovejoy, 1987). This generalization was necessary because the ultra-metric distance is not homogeneous in a discrete cascade: two points separated by a given length in physical space do not always have the same distance to their closest common ancestor in the cascade (cf. Pecknold et al., 1993). An interesting property of continuous cascades is that their generators (i.e., the logarithm of the random multiplicative weights) converge towards infinitely divisible laws. However, there is still no consensus concerning the degree of convergence. Some authors proposed Poisson generators (She & Leveque, 1994; Dubrulle, 1994) whereas some others add an assumption of self-similar renormalization, so that the generator converges more accurately towards stable distributions (Schertzer & Lovejoy, 1987, 1997). Until this question finds a definitive answer, since the notion of self-similarity is at the root of the theory, the latter assumption seems plausible, and the decision was made to use stable generators which correspond to Gaussian distributions if the variance is finite or Levy distributions if the variance is infinite. In this type of case, the moment scaling functions take the simple form (Schertzer & Lovejoy, 1987):

$$K_{\varepsilon}(q) = \frac{C_{1\varepsilon}}{\alpha_{\varepsilon} - 1}\left(q^{\alpha_{\varepsilon}} - q\right) \tag{5}$$

$$K_{\chi}(q) = \frac{C_{1\chi}}{\alpha_{\chi} - 1}\left(q^{\alpha_{\chi}} - q\right) \tag{6}$$

where α_{ε} and α_{χ} are multifractality parameters varying between 0 and 2, and $C_{1\varepsilon}$ and $C_{1\chi}$ are intermittency parameters varying between 0 and the dimension of the embedding space, which here is equal to 2.

3 The FIF model

Concerning the chlorophyll concentration, these laws cannot be directly applied, the main reason being that biological activities may produce deviations from a purely passive scalar behaviour. Nevertheless, we expect that the variability of chlorophyll maps still presents some multifractal properties, and that it would be possible to use a cascade model similar to that presented above. We thus looked for a phenomenological model having the same form as Eq. (1), but in which the parameters are not known, i.e.:

$$\Delta Chl_{l} \simeq \langle \zeta^{a} \rangle l^{H} \tag{7}$$

where *Chl* is the chlorophyll concentration and $\Delta Chl_{l} \equiv \langle |Chl(x+l) - Chl(x)| \rangle$. ζ is a conserved flux and a and H are adjustable parameters. As described above, the conserved flux has to verify the basic multifractal relation:

$$\left\langle \zeta_l^{\,q} \right\rangle \simeq \left(\frac{L}{l} \right)^{K_\zeta(q)} \qquad (8)$$

and it is assumed that it converges towards a log-stable law:

$$K_\zeta(q) = \frac{C_{1\zeta}}{\alpha_\zeta - 1}\left(q^{\alpha_\zeta} - q \right) \qquad (9).$$

This model is described by four parameters: a, H, α_ζ and $C_{1\zeta}$. However, it is possible to reduce the number of parameters to three, since taking the a^{th} power of ζ in Eq. (7) is equivalent to a simple shift of H by $K_\zeta(a)$, and to the multiplication of $C_{1\zeta}$ by a factor a^{α_ζ} (Lavallée et al., 1993). The proof uses the q^{th}-order structure functions of chlorophyll maps defined by taking the q^{th} power of Eq. (7):

$$\Delta Chl_l^q \simeq \left\langle \zeta^{aq} \right\rangle l^{qh} \qquad (10).$$

By introducing Eq. (8), this equation simplifies to:

$$\Delta Chl_l^q \simeq l^{qH - K_\zeta(aq)} \qquad (11).$$

Then, the term $K_\zeta(aq)$ can be decomposed into conservative and non-conservative parts using the following identity, which can be straightforwardly derived from Eq. (9):

$$K(ap) = qK(a) + a^{\alpha} K(q) \qquad (12).$$

This operation yields:

$$\Delta Chl_l^q \simeq l^{q(H - K(a)) - a^{\alpha_\zeta} K_\zeta(q)} \qquad (13).$$

As expected, by defining:

$$\begin{cases} K'_\zeta(q) \equiv a^{\alpha_\zeta} K_\zeta(q) \\ H' \equiv H - K_\zeta(a) \end{cases} \qquad (14),$$

one obtains the usual form of the structure function corresponding to the case where $a = 1$:

$$\Delta Chl_l^q \simeq l^{qH' - K'_\zeta(q)} \qquad (15).$$

In the following, we thus use a simplified form of Eq. (7) requiring only three parameters, namely, H, α_ζ and $C_{1\zeta}$:

$$\Delta Chl_l \simeq \zeta l^H \qquad (16)$$

This model is called the fractionally integrated flux (FIF) and was first presented by Schertzer & Lovejoy (1987) in the framework of their study of rain fields. In this regard, it is interesting to note the similarities between marine biogeochemistry and the cycle of water in the atmosphere. Firstly, both are strongly dependent on ascending currents: these currents bring nutrients to the surface layers of the ocean and water vapour to the upper layers of the atmosphere. Then, the first phase transition to heavier particles generally occurs in thin layers in which physical conditions are appropriate: phytoplankton is produced near to the ocean's surface because it needs light to grow, whereas clouds are formed in the atmospheric layer in which water vapour condenses. The next phase transition to heavier particles occurs when phytoplankton feeds zooplankton, and when cloud droplets are incorporated into raindrops. Finally, zooplankton may die and sink (or return to a nutrient form by mineralization), whereas raindrops may fall (or return to water vapour by evaporation). Although these atmospheric and oceanic processes have very different space and time scales, with one involving biology and the other physics, the comparison is striking. The interesting point here is that, in both cases, the evolution cycle is an alternating composition of turbulent mixing and phase transition processes. This analogy leads phytoplankton patchiness to be thought of as 'clouds in the sea'. Besides, as will be shown below, the multifractal parameters obtained from chlorophyll maps are close to those obtained in the case of cloud or rain fields.

4 Analysis technique

The first step of the analysis consists in verifying the scale law given by Eq. (16), and in estimating its exponent H. This is generally performed by using the first-order structure function. Since the flux ζ is assumed to be conserved in scale space, whatever the scale l, $\langle \zeta_l \rangle$ is constant. Therefore, Eq. (16) reduces to:

$$\Delta Chl_l \propto l^H \qquad (17).$$

This equation allows H to be estimated using the simple expression:

$$H = \log_l \left(\Delta Chl_l \right) \qquad (18).$$

The second step consists in quantifying the multifractal properties of the flux, and in estimating α_ζ and $C_{1\zeta}$. In order to do so, it is necessary to reconstruct the cascade and therefore to retrieve the flux at the finest available scale. According to Eq. (16), this requires a fractional derivation of order H. However, a simple derivation of integer order provides a good numerical approximation (Lavallée et al., 1993), such as taking the norm of the gradient of the field:

$$\zeta_{l\max} \approx \sqrt{\left(\frac{dChl}{dx}\right)^2 + \left(\frac{dChl}{dy}\right)^2} \qquad (19).$$

Note that, since the rest of the analysis is based on the gradient of the field, it is crucial to work with data affected by a low level of noise. Indeed, if the noise is strong, taking the gradient of the field will result in useless, noisy fields. Therefore the finest available scale does not necessarily correspond to the measurement scale: it is usually necessary to perform initial averaging of the data at a larger scale, before computing the gradient, in order to suppress the noisiest of the finest scales. The cut-off scale at which the fields have to be averaged is called the 'effective measurement scale' in the following. This scale is determined by computing the power spectrum, and then estimating the wave number above which it flattens out.

Once the flux has been obtained at the 'effective measurement scale', the stochastic multiplicative cascade can be reconstructed by averaging (or 'degrading') the flux at larger scales. The statistical moments are then computed for various orders and scales in order to test Eq. (8). If the scaling of the statistical moments is verified, $K_{\zeta}(q)$ can be estimated. Finally, the parameters α_{ζ} and $C_{1\zeta}$ are obtained by determining the least squares fit to this function.

5 Dataset

Particular attention was paid to the selection of chlorophyll maps, because multifractal analysis is very sensitive to the quality of the data. As explained above, the analysis technique is based on the gradient of the field. Therefore, if the signal is too noisy, the fluctuations due to turbulence or other processes will be hidden. Moreover, the estimation of higher-order statistical moments can easily be biased by the presence of a few unrealistic values in the data, such as isolated pixels having abnormally high chlorophyll concentrations.

Another difficulty is that areas below clouds or high aerosol concentrations cannot be observed, because the sensor cannot see the sea surface. As a consequence, chlorophyll maps remotely sensed from space present many 'holes' of different sizes (the set defined by the locations of these missing data may be fractal itself, because cloud and aerosol distributions are also fractal, see respectively (Lovejoy & Schertzer, 2006) and (Lilley et al., 2004)). We also noticed that the values around the periphery of these 'holes' were not reliable, presumably because of uncertainties in the correction of the atmospheric effect. Therefore, it was chosen to study only maps which had no missing values. This type of data is of course difficult to find, because of the abundance of clouds and aerosols in the atmosphere, such that a compromise needed to be found between the size of the maps and the sample size.

In order to optimize this compromise, the study area was carefully chosen. The most appropriate area was found to be the Senegalo-Mauritanian upwelling region, because it normally has a very low cloud cover (although this does not remain true during the summer months, when the InterTropical Convergence Zone (ITCZ) moves north). Another reason is the presence of upwelling, which provides high chlorophyll concentrations far from the coast, due to peculiar oceanic conditions (Aristegui et al., 2004; Lathuiliere et al., 2008). Therefore, the choice of this area reduced the measurement noise with respect to the coherent signal. Finally, the chosen location lies between 10°N-26°N and 14°E-

26°E, which corresponds to the area between the Cape Verde islands and the coast of West Africa, between Mauritania and Guinea-Bissau (see Fig. 1).

The choice of product level also has to be carefully considered. Classical 8-day composite maps could not be used, because they include a non-uniform time averaging in the data, depending on the amount of missing data for each pixel. Level L3 products (daily global maps mapped to a uniform scale grid) could not be used either, firstly because of projection effects, and secondly because this product is actually derived from sub-sampling of the original data: only one pixel is kept for each square of 4x4 pixels. This reduction in the amount of data is unavoidable, because SeaWiFS is positioned in Low Earth Orbit (LEO), meaning that the time it remains within the field of view of receiver stations it too short for full datasets to be transmitted to the ground. However, full resolution chlorophyll data was transmitted for some restricted areas, including the Senegalo-Mauritanian upwelling region. This dataset is called the local unmapped level L2 product, and is suitable for use in this study. In this product, the pixel resolution is around 1 km^2. However, the spot size over which the data are measured varies with elevation angle. Therefore, only the inner part of the scans was considered, in order to limit this effect. Note also that some authors (Lovejoy et al, 2001b) recommend using direct analysis of marine reflectivities (level L1 product) because the fields' heterogeneity may bias chlorophyll concentration retrieval algorithms. Indeed, since these algorithms are generally non-linear, it is not correct to extrapolate them directly to the measurement scale which is much larger than the scale of homogeneity. However, this problem should not affect our analysis because the retrieval of chlorophyll concentration was performed without any extrapolation in scale space (the retrieval algorithm is based on an empirical relation derived from the comparison between remotely sensed marine reflectivities and in-situ chlorophyll concentrations). Moreover, working directly with marine reflectivities is difficult because of its lack of physical interpretation. Actually, the only physical quantity that can be related to a theoretical scale law, such as that given in Eq. (16), is the chlorophyll concentration *Chl*. For example, consider a non-linear relationship of the form $Chl=f(R)$, where R denotes a marine reflectivity. If f is non-linear, then f^{1} is also non-linear. There is consequently no reason for the marine reflectivity $R=f^{1}(Chl)$ to verify a similar scaling, because non-linear transformations do not generally conserve scale laws. This statement may appear in contradiction with the fact that a bi-lipschitz transformation implies a similar scaling exponent. However, this argument is not correct here because we do not make the prior assumption that reflectivities are scaling.

Finally, 100 maps of 128x128 pixels of 1 km^2, with a minimum of 99.5% of available data, were extracted from SeaWiFS data over a period of one year running from July 2003 to June 2004 (a sample chlorophyll map is shown in Fig. 2). The few missing data were interpolated automatically by computing the mean of the surroundings pixels. All selected maps were checked manually. Some maps had to be rejected because of an offset affecting some parts of the field. The origin of this offset is not known. The gradient of the maps was also checked manually, in order to detect any isolated, unrealistically high values. Each of these unrealistic pixels was corrected using the mean value of the surrounding pixels.

6 Results

Fig. 3 shows the first-order structure function for the 100 SeaWiFS chlorophyll maps. The smaller scales (1-4 km) were not taken into account when determining the fit, because they present a deviation from the scaling observed throughout the remainder of the scale range (4-128 km). The cause of this deviation does not appear to be physical, because such a break in the scaling has never been observed in other studies (cf. Lovejoy et al. 2001b). This break was therefore associated with the scale below which the measurement noise becomes dominant, when compared with the coherent signal (here, the definition of the noise is very large: it includes not only the sensor's sensitivity, but also atmospheric corrections and retrieval algorithms errors). This hypothesis is confirmed by the power spectrum (Fig. 4), which flattens out beyond a wave number corresponding to 4 km in the physical space. Finally, over the scale range 4-128 km, the empirical first-order structure function is consistent with Eq. (17), and H is estimated to be around 0.4 (the numerical fit yields 0.402 with a standard deviation of the estimator equal to 0.005).

Since the noise has to be removed before continuing the analysis, the data were averaged over 4x4 km^2 areas. Then, for each map, the norm of the gradient was computed and normalized in order to reconstruct the cascade. Fig. 5 shows the scaling of the statistical moments for various orders. This set of scale laws is found to be consistent with the basic multifractal relation given by Eq. (8). For each order q, the slope of the scale law provides an estimation of $K_\zeta(q)$. The moment scaling function retrieved by this method is shown in Fig. 6. The fit of this function according to Eq. (9) yields $C_{1\zeta} \approx$ 0.12 and $\alpha_\zeta \approx 1.92$ (here, we renounced to provide the standard deviations of the estimators because they would indicate an artificially high precision; the estimation error of the whole analysis technique has been tested with simulations and is found to be around 10% for both parameters). Note that the values of these parameters, as well as that of H, are close to those obtained for rain and clouds, which are respectively $H \approx 0.4$, $C_1 \approx 0.12$, $\alpha \approx 1.8$ (Verrier et al., 2010) and $H \approx 0.4$, $C_1 \approx 0.08$, $\alpha \approx 1.9$ (Lovejoy & Schertzer, 2006). We also tried to perform the same type of analysis using SST (Sea Surface Temperature), which is another useful, remotely sensed oceanic tracer. However, this attempt failed because the spectrum of the SST maps was found to flatten out at larger scales (around 32 km) than that of chlorophyll maps, and the available range of scales was thus insufficient. This whitening effect, which hides the small scale fluctuations, may be due to air-sea exchanges, which tend to spatially homogenize the SST. However, Nieves et al. (2007) performed a multi-scale analysis of SST data with a larger scale range (level L3 product) and found that the observed multifractal spectra was very similar to the one obtained with chlorophyll concentration data. This result provides an additional argument in favour of a link between phytoplankton patchiness and turbulent mixing at large scales, which will be developed in the next section.

The use of statistical moments is a very convenient way of estimating multifractal parameters. However, as it is not very intuitive, we propose here to demonstrate the existence of a cascade process, through the use of the more classical concept of probability density. The algorithm used in this method is the following: (i) compute the flux at the finest available scale, (ii) perform averages over 2x2 squares, (iii) compute the multiplicative weights that relate the values of the coarse-grained flux to the previous ones, (iv) plot the Probability Density Function (PDF) of the logarithm of these multiplicative weights, (v) iterate steps (ii), (iii) and (iv) until the largest scale of the cascade is reached. This method is straightforward to implement and does not require any prior assumption concerning the data. The results obtained with our selection of chlorophyll maps are given in Fig. 7.

The PDF of the logarithm of the multiplicative weights does not depend on the scale at which they are derived, thus confirming the use of a scale invariant cascade model. Fig. 8 shows the left tail of the total PDF, compared with a Gaussian distribution having the same mean and variance. The empirical PDF decays as a power law (producing a straight line on a log-log graphic), which is much slower than the Gaussian behaviour. This result supports the fact that the generator follows a Lévy law with infinite variance, and allows α to be estimated using a different approach, since the theoretical slope of the asymptote this distribution is equal to -(1+α). The resulting value of α is found to be 1.95, which is consistent with the value previously obtained using statistical moments.

7 Interpretation

Since the parameter H was found to be close to 1/3, it is tempting to relate it to the theory of passive scalars. This theory is based on the hypothesis of a 3D isotropic turbulence that does not hold for our selection of chlorophyll maps, because, in the considered scale range (1-128 km), the ocean is a stratified fluid with a horizontal dimension much larger than the vertical one. However, some recent studies (e.g., Lovejoy & Schertzer, 2010) suggest that the Corrsin-Obukhov scale law may still be valid in the horizontal. Therefore, if turbulent mixing is the dominant effect, we may expect that the horizontal variability of phytoplankton fields would verify the scale law given in Eq. (2). If this is correct, then, assuming the velocity and passive scalar fluctuations to be independent, Schmitt et al. (1996) have shown that the parameter H of the FIF model (Eq. (16)) should be equal to:

$$H = 1/3 + K_{\varepsilon}(1/6) - K_{\chi}(1/2) \qquad (20).$$

The deviation of H with respect to the value 1/3 is due to the intermittency of the energy and scalar variance fluxes, since a conserved flux raised to a power exponent, not equal to 1, is no longer a conserved quantity. The term $K_{\varepsilon}(1/6)$ depends only on the turbulence, and is well known; by assuming the parameters α_{ε}=1.5 and $C_{1\varepsilon}$=0.25 proposed by Schmitt et al. (1996), its value is expected to be around -0.05. However, the estimation of the term $K_{\chi}(1/2)$ is more delicate, since the multifractal parameters of χ are not known a priori, and have to be estimated. One possible solution consists in using the empirical multifractal parameters obtained for ζ in the previous section, because they have a simple relationship to those of χ (de Montera et al., 2010):

$$\begin{cases} \alpha_{\chi} \approx \alpha_{\zeta} \\ C_{1\chi} \approx 2^{\alpha_{\zeta}} C_{1\zeta} \end{cases} \qquad (21).$$

This yields $\alpha_{\chi} \approx 1.92$ and $C_{1\chi} \approx 0.45$, thus allowing $K_{\chi}(1/2)$ to be estimated at a value equal to -0.11. The (semi-)theoretical value of H is therefore 1/3-0.05+0.11 ≈ 0.39, which is consistent with its experimental value of 0.4 obtained with the SeaWiFS chlorophyll maps.

This coherency led us to the conclusion that phytoplankton behaves like a passive scalar within the studied scale range, which includes the mesoscale and the sub-mesoscale. This does not mean that phytoplankton is a purely passive scalar, however it implies that biological activity does not affect the scale law generated by turbulent mixing. This is consistent with the previous finding of (Currie & Roff, 2006), who showed that biological activity affected the scaling over a limited range only, between 30m and 500m, which is smaller than the resolution of remotely sensed satellite data.

8 Bias in biogeochemical numerical models

The forecasting of coupled turbulent/biogeochemical systems is currently performed by means of 3D numerical simulations. The main shortcoming of this technique is that it necessarily implies the use of high-pass filtering in scale space (or 'scale truncation'), which strongly affects the estimation of non-linear advection terms in the fluid mechanics equations. This truncation of scale space is unavoidable because of the limited power of computers. It means, for example, that a small length interval, considered as a differential element dx in the equations, has a value much larger than the scale of homogeneity in the numerical simulation (generally 10-100km for global models, whereas dissipation occurs at scales of the order of a millimeter). The impact of this drastic simplification remains unknown. Although it is generally believed that it can be compensated for, for example by increasing the viscosity (Boussinesq hypothesis), this remains to be demonstrated (for a test of the Boussinesq hypothesis, see Schmitt 2007).

If biogeochemical processes are involved, the situation is even worse, because the estimation of these interactions is also affected by the truncation error. Moreover, the parameters of biogeochemical models are often obtained by means of laboratory experiments performed at a typical scale of one meter. Therefore, since the relations in which these parameters are involved are generally non-linear, it is not correct to use them at larger scales if the real fields are heterogeneous. It can thus be useful to assess the bias generated by the assumption of homogeneity over larger scales. For this, we consider a global numerical model operating with a 1° grid scale (roughly corresponding to the 128 km^2 maps analyzed in the present paper), which includes a quadratic source term of the form βC^2, where C is the concentration of a tracer and β is a parameter assumed to be derived under stable conditions, at the scale of one meter in a laboratory. If it is assumed that the 100 SeaWiFS chlorophyll maps are realizations of the sub-grid heterogeneity of the tracer, then for each map we compute the source term at the finest available scale (which is 1 km in this case, whereas a 1 m scale would be needed!), and average these values over the whole 128 km^2 map. Finally, we estimate the value of this source term that would result from the hypothesis of homogeneity, by averaging the concentration over the whole 128 km^2 map and then computing the source term. The PDF distribution of the computed errors is shown in Fig. 9. The mean error, estimated to be approximately 22%, is far from being negligible. One possible approach for reducing this error would be to derive an analytic expression for the scale dependency of the biological parameters (such as β in the example above), using the multifractal parameters of the tracer patchiness, if available.

9 Conclusions

Multifractal properties of oceanic chlorophyll maps have been investigated with remotely sensed data recorded from space. The FIF model has been validated, showing that chlorophyll maps can be modelled statistically, through the use of a fractionally integrated multiplicative cascade. The scale law exponent H was found to be consistent with passive scalar behaviour, indicating that phytoplankton variability is dominated by turbulent mixing over the studied scale range (4-128km), and that biological activity do not modify this scaling. Finally, it has been shown that, as a consequence of this multifractal patchiness, the non-linear source and sink of biogeochemical numerical models could be strongly biased.

Acknowledgements

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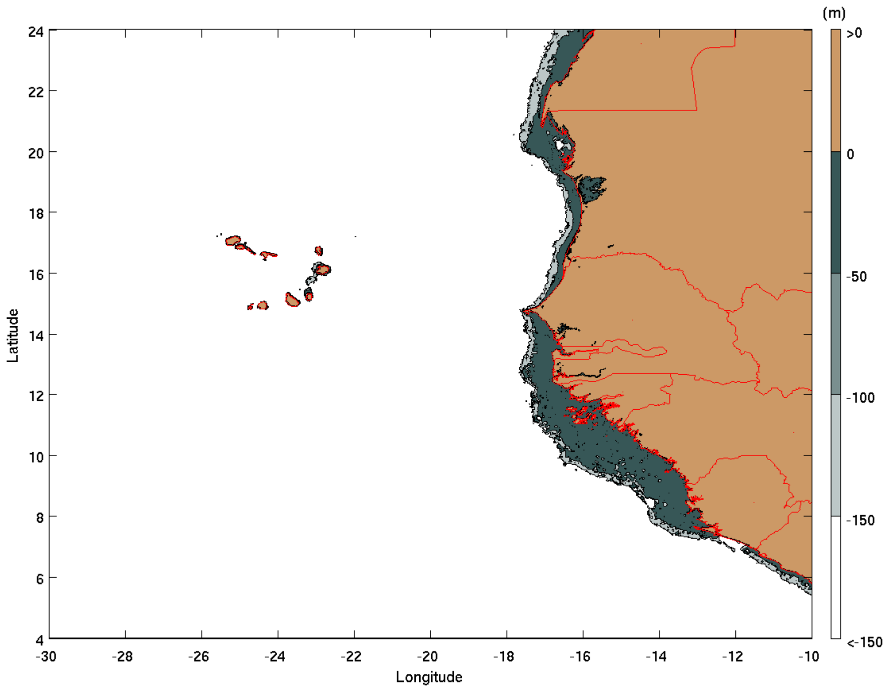

Figure 1. Geographic map of the Senegalo-Mauritanian upwelling region.

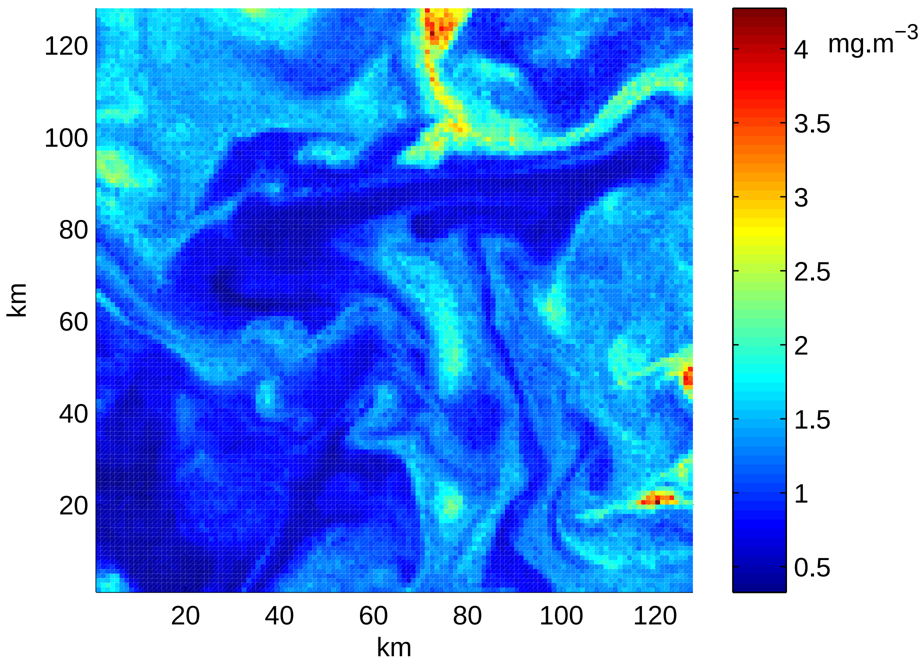

Figure 2. Example of a 128 km^2 horizontal chlorophyll map (resolution 1 km^2) extracted from the SeaWiFS local L2 product.

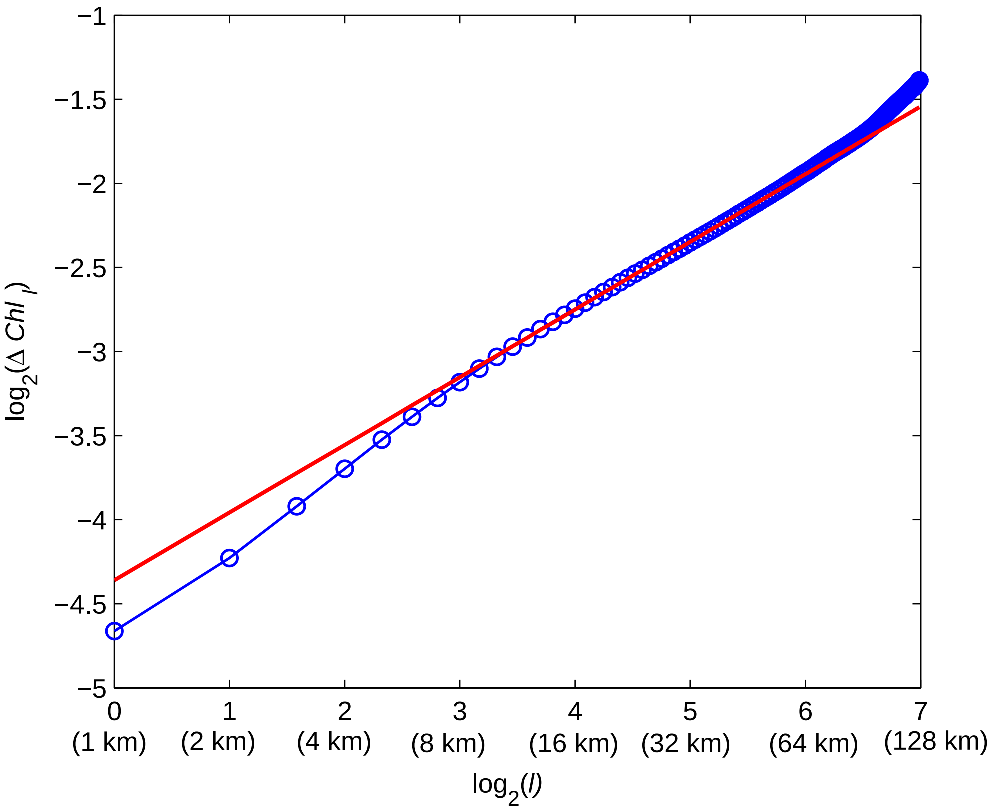

Figure 3. First-order structure function of SeaWiFS chlorophyll maps compared with a linear curve of slope equal to 0.4. The departure from the theorical fit observed at the finest scales is attributed to measurement noise, and corresponds to flattening of the power spectra at high wave numbers (see Fig. 4).

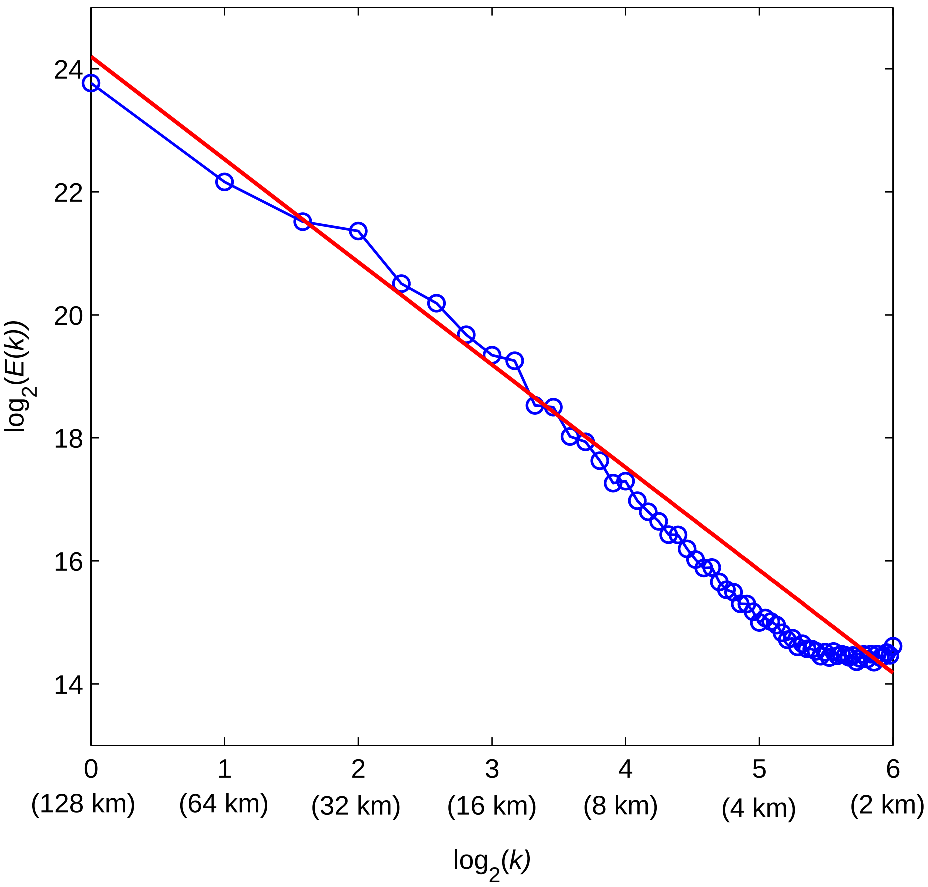

Figure 4. Angle-integrated power spectrum of SeaWiFS chlorophyll maps compared with a linear curve of slope equal to -1.67. The power spectrum flattens out at the fifth octave, corresponding to wavelengths smaller than 4km.

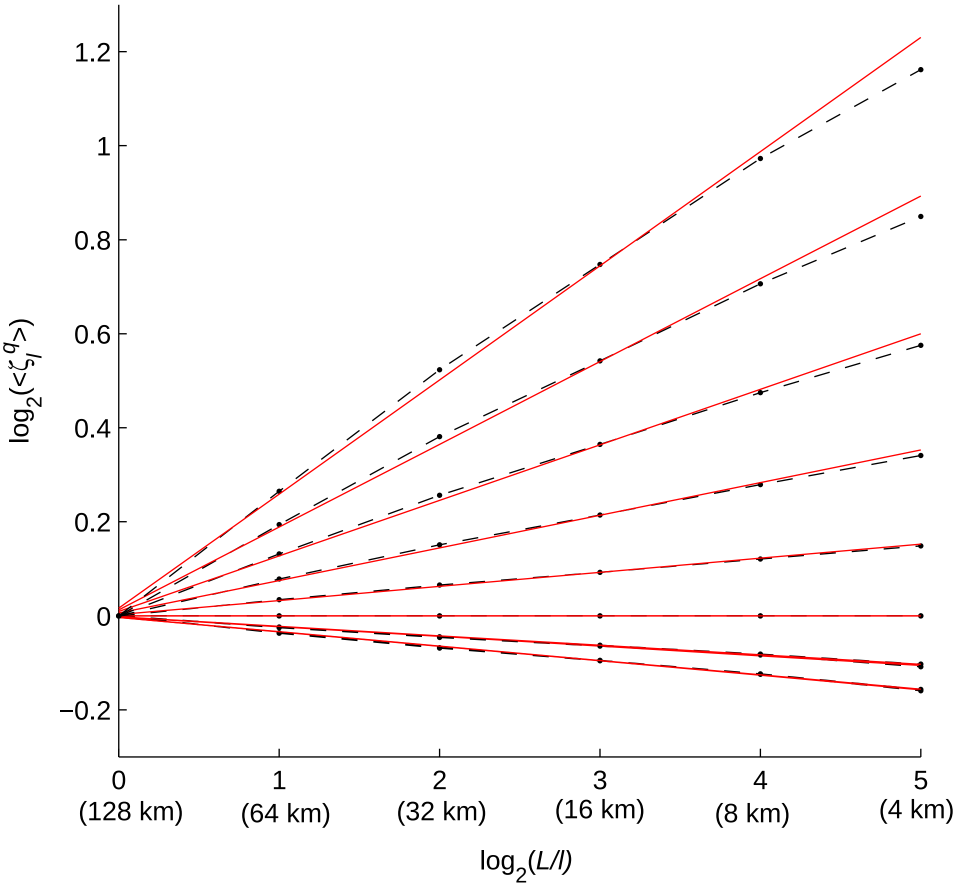

Figure 5. Scaling of the statistical moments of the flux *ζ* for the orders *q*=0, 0.1, 0.2,… , 2, with corresponding theoretical fits. Here, *L* corresponds to the largest scale of the SeaWiFS chlorophyll maps, i.e. 128 km. For each map, the flux was normalized to a mean value of 1.

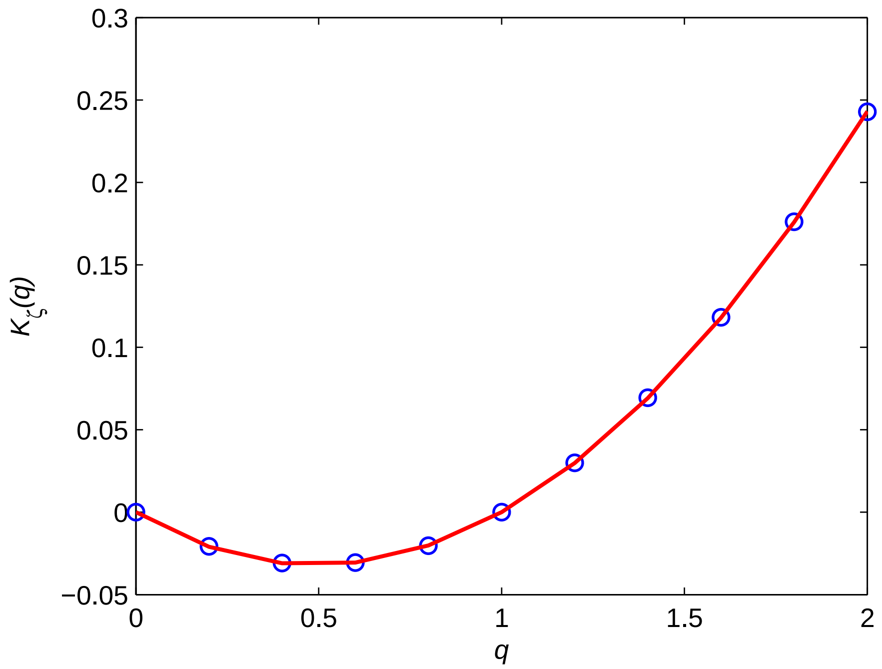

Figure 6. Moment scaling function $K_\zeta(q)$ of the flux ζ, with theoretical fit.

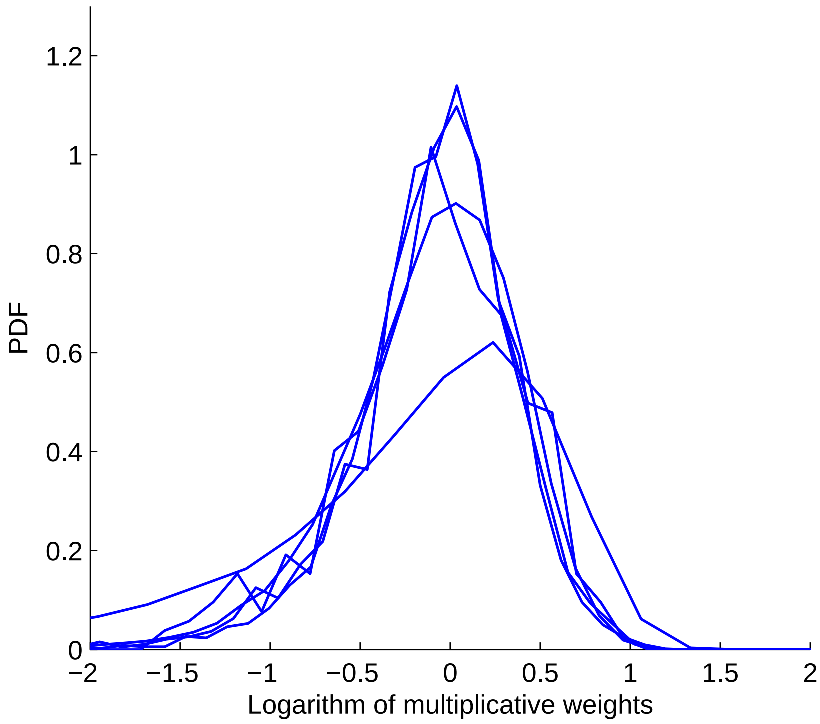

Figure 7. PDFs of the logarithm of the multiplicative weights for each level of the cascade (corresponding to contractions of the averaging area by a factor 2^2, from 128 km^2 until 4 km^2). The PDFs are very similar, with the exception of the function corresponding to the last scale contraction (from 8 km^2 to 4 km^2), which is flatter. This may be due to the presence of noise.

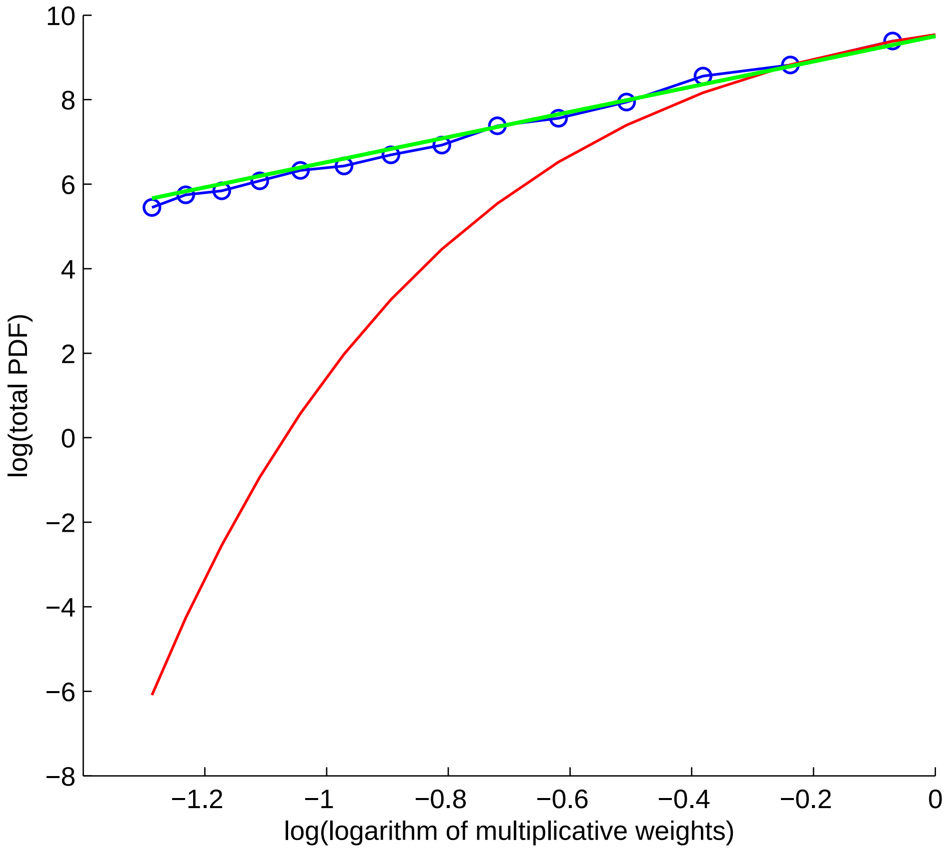

Figure 8. Log-log graph of the left tail of the total PDF of the logarithm of multiplicative weights (blue), compared with a Gaussian having the same mean and variance (red). The PDF decays as a power law, with a slope -2.95 (green fit), corresponding to a Lévy law of index α=1.95. The Gaussian function decays much faster, and would therefore be inappropriate for cascade generation.

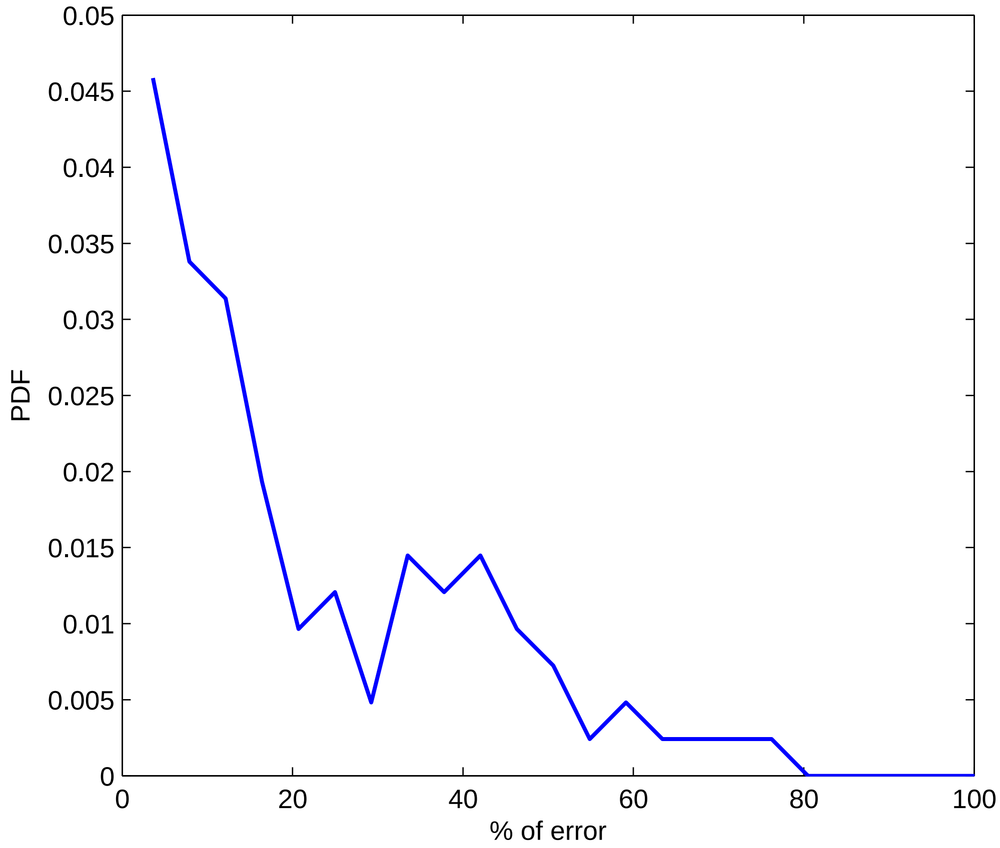

Figure 9. Assessment of the distribution of the error percentage resulting from the hypothesis of homogeneity over 128 km^2 areas, for a quadratic source term in a biogeochemical numerical model.