Estimation of friction parameters in gravity currents by data assimilation in a model hierarchy

by **A. Wirth**

Referee Comment by **M. Ghil**

General. This is the third in a series of papers that study the modelling of gravity currents across a hierarchy of models. The previous two dealt, respectively, with the fundamental issues of oceanic gravity currents (Wirth, 2009) and with the estimation of friction parameters in a reduced-gravity, shallow-water equation (SWE) model (Wirth & Verron, 2008). The present paper applies the data assimilation methodology of the second paper to the same SWE model, but uses a more complete and detailed Navier-Stokes model in the Boussinesq approximation as well. The latter model is implemented according to the HAROMOD formulation of Wirth (2004) and is used to produce the “control” or “nature” runs that need to be approximated as well as possible by the SWE model, given the paucity of actual ocean observations that would be needed to fully determine (or “constrain”) the friction parameters of this model.

The paper at hand demonstrates convincingly that the sequential-estimation (“Kalman filtering”) approach to parameter estimation is well adapted to produce both qualitative and quantitative results on friction laws in such a fluid model. The key result is that the friction law changes from linear to quadratic at a value of the Ekman-layer–based Reynolds number of about 800, in rough agreement with laboratory experiments (Nikuradse, 1933; Schlichting & Gersten, 2000). The numerical experiments are quite careful and the numerical values of the parameters so obtained are clearly useful.

The paper brings in considerable expertise on relevant literature from widely differing areas: SWE models in rivers (Gerbeau & Perthame, 2001), engineering studies on boundary layers (e.g., Schlichting & Gersten, 2000), and even pseudo-random number generation (Matsumoto & Nishimura, 1998). The constant interplay between linear theory and nonlinear computation makes the results not only more convincing but also more easily understood.

Semi-major comments

1. It is a bit odd to read that Eqns. (11) and (12) describe the ensemble Kalman filter (EnKF). In fact, they describe quite generally any type of Kalman-filter–like sequential estimator and were presented first in the geophysical literature by Ghil et al. (1981), who applied them to a one-dimensional SWE model with rotation. It is only Eq. (13) that provides the particular EnKF approximation to the general problem of nonlinear sequential estimation.

2. The paper does mention in passing the results of Sun et al. (2002) and of Kondrashov et al. (2008) using another such approach, the extended Kalman filter (EKF). It would be worth stating explicitly that the model of these authors is a coupled ocean-atmosphere model of a physical and numerical complexity that is at least comparable to the SWE model treated here, and probably quite a bit larger. While there is no finite-time method that can solve completely the nonlinear sequential-estimation problem for partial differential equations, the EKF not only preceded in time but is actually quite competitive with the EnKF both computationally and in terms of accuracy. Furthermore, the EKF results on state-and-parameter estimation of

Sun et al. (2002) and of Kondrashov et al. (2008) were first announced by Ghil (1987) and extended to a highly nonlinear problem in solid mechanics by Kao et al. (2006).

Truly minor comments. These include the presence of a fairly high number of typos and misprints.

1. The Stull (1988) reference is missing.

2. The German-and-French–speaking author has no excuse for misspelling "ForschungsCheft" in Nikuradse (1933) or "permaMnent" in Saint-Venant (1871).

3. Errors of agreement in number between the noun and verb, and similar grammatical ones, include, in the Abstract alone:
last-but-one sentence – "The drag coefficient […] compare […]"
last sentence – "[…] systematically connection models […]

Additional references

1. Ghil, M., S. Cohn, J. Tavantzis, K. Bube, and E. Isaacson, 1981: Applications of estimation theory to numerical weather prediction, in *Dynamic Meteorology: Data Assimilation Methods*, L. Bengtsson, M. Ghil and E. Källén (Eds.), Springer Verlag, pp. 139–224.

2. Ghil, M., S. Cohn, J. Tavantzis, K. Bube, and E. Isaacson, 1981: Applications of estimation theory to numerical weather prediction, *Dynamic Meteorology: Data Assimilation Methods*, L. Bengtsson, M. Ghil and E. Källén (Eds.), Springer Verlag, pp. 139–224.

3. Kao, J., D. Flicker, K. Ide and M. Ghil, 2006: Estimating model parameters for an impact-produced shock-wave simulation: Optimal use of partial data with the extended Kalman filter, *J. Comput. Phys.*, **214** (2), 725–737, doi: 10.1016/j.jcp.2005.10.022.