Ocean Sci. Discuss., 7, 1289–1302, 2010
www.ocean-sci-discuss.net/7/1289/2010/
doi:10.5194/osd-7-1289-2010

Ocean Science Discussions

Mixed layer sub-mesoscale parameterization – Part 2: Results for coarse resolution OGCMs

V. M. Canuto[1,2] **and M. S. Dubovikov**[1,3]

[1]NASA, Goddard Institute for Space Studies, 2880 Broadway, New York, NY, 10025, USA
[2]Dept. Applied Physics and Math., Columbia University, New York, NY, 10027, USA
[3]Center for Climate Systems Research, Columbia University, New York, NY, 10025, USA

Received: 21 June 2010 – Accepted: 28 June 2010 – Published: 12 July 2010

Correspondence to: V. M. Canuto (vcanuto@giss.nasa.gov)

Published by Copernicus Publications on behalf of the European Geosciences Union.

Abstract

Recent studies have shown that sub-mesoscales (SM, ∼1 km) play an important role in mixed layer dynamics and have concluded that it has become necessary to include them in ocean global circulation models, OGCMs. In part A, we developed and assessed a parameterization of the vertical SM tracer flux for OGCMs that resolve mesoscales M but not SM. In the present paper, we derive a parameterization of the vertical SM tracer flux for OGCMs that do not resolve either M or SM, as those used in climate studies.

1 Introduction

Several studies have shown that sub-mesoscales (SM ∼1 km horizontal scale) play a significant role in mixed layer (ML) dynamics (Levy et al., 2001, 2009; Mahadevan and Tandon, 2006; Lapeyere et al., 2006; Mahadevan, 2006; Capet et al., 2008; Hosegood et al., 2008; Klein et al., 2008; Thomas et al., 2008; Mahadevan et al., 2010). In particular, high resolution simulations by Capet et al. (2008) have shown that the *re-stratification* induced by the SM is of the same order as the *de-stratification* induced by small scale turbulence, as well as that induced by the large scale velocity due to strong winds (Mahadevan et al., 2010). Because of these effects, it has become necessary to include SM in OGCMs that resolve mesoscales (M, sizes∼Rossby deformation radius) and in the OGCMs used in climate studies that do not resolve either M or SM. In a previous work (Canuto and Dubovikov, 2009, 2010; CD9, CD10), we developed and assessed a parameterization of the vertical SM tracer flux for OGCMs that resolve M but not SM. In order to derive a SM parameterization for OGCMs that do not resolve both M and SM, we need to average the results presented in CD9-CD10 over unresolved scales. The procedure, which is not trivial, is presented in Sects. 2 and 3. In Sect. 4, we present an interpretation of the results and in Sect. 5 we summarize

OSD
7, 1289–1302, 2010

Mixed layer sub-mesoscale parameterization

V. M. Canuto and M. S. Dubovikov

the results using a simplified notation. Readers not interested in the derivation can go directly to Sect. 5.

2 Averaging sub-mesoscale fluxes over unresolved scales

In CD9-CD10 we derived an expression for the z-derivative of the vertical SM tracer flux to be used in OGCMs that resolve M but not SM. Specifically, Eq. (4a and b) of CD10 yield the following vertical SM tracer flux $F_v=\overline{w'\tau'}$, where a prime stands to represent sub-mesoscale fields and an overbar denotes averages over intermediate scales smaller than mesoscales but larger than SM:

$$\partial_z F_V = \boldsymbol{u}_S^+ \cdot \nabla_H \overline{\tau}\,, \quad \boldsymbol{u}_S^+ = -(1+\gamma^2)^{-1}\left[\tilde{\boldsymbol{u}} - \gamma \frac{f}{|f|}\boldsymbol{e}_z \times \tilde{\boldsymbol{u}}\right] \tag{1a}$$

where $\boldsymbol{e}_z$ is the unit vertical vector, $\tilde{\boldsymbol{u}}$ is the ML baroclinic component of 2-D the mean velocity (h is the ML depth):

$$\tilde{\boldsymbol{u}} = \overline{\boldsymbol{u}} - h^{-1}\int_{-h}^{0} \overline{\boldsymbol{u}}(z)dz \tag{1b}$$

and the function γ is defined as follows:

$$\gamma = r_S|f|(2K_{SM})^{-1/2} = \mathrm{Ro}^{-1}\,, \quad r_S = \frac{Nh}{\pi|f|}\,, \quad K_{SM} = \frac{1}{2}\overline{\boldsymbol{u}'^2} \tag{1c}$$

where Ro is the Rossby number, r_S is the ML deformation radius and K_{SM} is the SM kinetic energy. The parameterization (Eq. 1a–c) also can be obtained from Eq. (7a) and (7b) of CD9 in the limit:

$$\tilde{K} \ll K_{SM}\,, \quad \tilde{K} = \frac{1}{2}|\tilde{\boldsymbol{u}}|^2 \tag{1d}$$

OSD
7, 1289–1302, 2010

which, as shown in CD10, holds true in SM resolving simulations and which is due to the fact that the baroclinic component of the mean kinetic energy $\tilde{K}$ is considerably smaller than the total mean kinetic energy $\overline{K}=|\overline{\boldsymbol{u}}|^2/2$. To complete the parameterization (Eq. 1a–c), one needs to parameterize K_{SM}, a result presented in Eq. (7d) of CD10 which can also be derived from Eq. (7j) of CD9 in the limit of Eq. (1d):

$$K_{\mathrm{SM}}^{3/2} = C^{3/2}(1+\gamma^2)^{-1} h r_{\mathrm{s}}[\boldsymbol{V} - \gamma(f/|f|)\boldsymbol{e}_z \times \boldsymbol{V}]\cdot\nabla_{\mathrm{H}}\overline{b}\,, \quad \boldsymbol{V} = h^{-2}\int_{-h}^{0} z\tilde{\boldsymbol{u}}(z)dz \tag{1e}$$

From the simulation data of Capet et al. (2080) we computed $C\approx 6$ and showed that although K_{SM} is sensitive to variations of C, the tracer flux is not. It is worth stressing that in Eq.(1e) and in the second relation of Eq. (1a), the second terms in the square bracket are vectors: in fact, although $\boldsymbol{e}_z\times\tilde{\boldsymbol{u}}$ and $\boldsymbol{e}_z\times\boldsymbol{V}$ are pseudo-vectors (cross products of the vectors $\boldsymbol{e}_z$ and $\tilde{\boldsymbol{u}}$ or $\boldsymbol{V}$), f is a pseudo-scalar which is the scalar product of the vector $\boldsymbol{e}_z$ and the pseudo-vector $2\boldsymbol{\Omega}$ and thus the full terms are vectors.

In relation (Eq. 1a), the velocity $\boldsymbol{u}_{\mathrm{S}}^{+}$ may be viewed as the SM induced velocity analogous to the mesoscale induced (bolus) velocity. As Killworth (2005) pointed out, to make the analogy with mesoscales more complete and since in the ML $\overline{\tau}_z$ is small due to the strong mixing, one may add to the rhs of the first relation (Eq. 1a) the term $w_{\mathrm{S}}^{+}\overline{\tau}_z$, where w_{S}^{+} is found from the continuity condition:

$$\partial_z w_{\mathrm{S}}^{+} + \nabla_{\mathrm{H}}\cdot\boldsymbol{u}_{\mathrm{S}}^{+} = 0 \tag{1f}$$

Next, we decompose the fields $\overline{A}$ as the sum of resolved $\overline{\overline{A}}$ and mesoscale A'' parts:

$$\overline{A} = \overline{\overline{A}} + A'' \tag{2a}$$

Thus, a generic field A is decomposed as follows[1]:

$$A = \overline{A} + A' = \overline{\overline{A}} + A'' + A' \tag{2b}$$

[1] Capet et al. (2008) notation maps into ours as follows: A', A'', $\overline{A}$, $\tilde{A} \rightarrow A''$, A', $\overline{\overline{A}}$, $\overline{A}$

OSD
7, 1289–1302, 2010

When we substitute Eq. (2a) into Eqs. (1) and carry out the double bar averaging, terms such as $A''\overline{\overline{A}}$ vanish since $\overline{\overline{A''}}=0$. What remains is the sum of two contributions given by:

$$\partial_z \overline{\overline{F}}_\mathrm{V} = \partial_z F_\mathrm{L} + \partial_z F_\mathrm{M} \tag{3a}$$

$$\partial_z F_\mathrm{L} = -(1+\gamma^2)^{-1}(\hat{\boldsymbol{u}} - \gamma \boldsymbol{e}_z \times \hat{\boldsymbol{u}})\cdot\nabla_\mathrm{H}\overline{\overline{\tau}}\,, \quad \partial_z F_\mathrm{M} = -\overline{\overline{(1-\gamma^2)^{-1}(\tilde{\boldsymbol{u}}'' - \gamma \boldsymbol{e}_z \times \tilde{\boldsymbol{u}}'')\cdot\nabla_\mathrm{H}\tau''}} \tag{3b}$$

where:

$$\tilde{\boldsymbol{u}}'' = \boldsymbol{u}'' - h^{-1}\int_{-h}^{0} dz\,\boldsymbol{u}''(z)\,, \quad \hat{\boldsymbol{u}} = \overline{\overline{\boldsymbol{u}}} - h^{-1}\int_{-h}^{0}\overline{\overline{\boldsymbol{u}}}(z)dz \tag{3c}$$

In Eqs. (3a) and (3b) F_L and F_M represent the large scale (L) and mesoscale (M) contributions, respectively. Using the dynamical model for the mixed layer mesoscales (Canuto et al., 2010; C10), in the Appendix we show that:

$$\partial_z F_\mathrm{M} \approx -\gamma(1-\gamma^2)^{-1}|f|^{-1}\left(z+\frac{1}{2}h\right)\nabla_\mathrm{H}\overline{\overline{b}}\cdot\nabla_\mathrm{H}\overline{\overline{\tau}} \tag{3d}$$

Thus, the complete form of Eq. (3a–d) can be written as:

$$\partial_z \overline{\overline{F}}_\mathrm{V} = \boldsymbol{u}_*^S \cdot \nabla_\mathrm{H}\overline{\overline{\tau}} \tag{4a}$$

where:

$$\boldsymbol{u}_*^S = -(1+\gamma^2)^{-1}\left[\hat{\boldsymbol{u}} - \frac{f}{|f|}\gamma\boldsymbol{e}_z\times\hat{\boldsymbol{u}} + \frac{\gamma}{2|f|^{-1}}\left(z+\frac{1}{2}h\right)\nabla_\mathrm{H}\overline{\overline{b}}\right] \tag{4b}$$

together with the second of Eq. (3c). To complete the SM parameterization, we still need to carry out the above procedure on the function γ, a problem we study next.

OSD
7, 1289–1302, 2010

3 The function γ

As it follows from the first of Eq. (1c), in order to average γ over mesoscale fields, we need to average K_{SM}. Since the mesoscale velocity is geostrophic, its baroclinic component can be expressed through the horizontal gradient of the mesoscale buoyancy as follows (analogously to the first relations of Eq. 13a in CD9 and CD10):

$$\tilde{\boldsymbol{u}}'' = \left(z + \frac{1}{2}h\right) f^{-1}\boldsymbol{e}_z \times \nabla_{\mathrm{H}} b'' \tag{5}$$

Substituting relation (Eq. 2a) into Eq. (1e) and averaging over the mesoscale fields with the use of Eq. (5), we get:

$$K_{\mathrm{SM}}^{3/2} = 2C^{3/2}(1+\gamma^2)^{-1} r_{\mathrm{S}} h \left\{ \left[\hat{\boldsymbol{V}} - \gamma \frac{f}{|f|} \boldsymbol{e}_z \times \hat{\boldsymbol{V}} \right] \cdot \nabla_{\mathrm{H}} \overline{\overline{b}} + \frac{1}{12} \frac{\gamma}{|f|} h \overline{\overline{|\nabla_{\mathrm{H}} b''|^2}} \right\} \tag{6a}$$

where:

$$\hat{\boldsymbol{V}} = h^{-2} \int_{-h}^{0} z \hat{\boldsymbol{u}}(z) dz \tag{6b}$$

Using Eq. (A5) to express the last term in Eq. (6a), we finally obtain:

$$K_{\mathrm{SM}}^{3/2} = C^{3/2}(1+\gamma^2)^{-1} h r_{\mathrm{S}} \boldsymbol{V}^{+} \cdot \nabla_{\mathrm{H}} \overline{\overline{b}}, \quad \boldsymbol{V}^{+} = \hat{\boldsymbol{V}} - \gamma \frac{f}{|f|} \boldsymbol{e}_z \times \hat{\boldsymbol{V}} + \frac{\gamma}{12} h f^{-1} \nabla_{\mathrm{H}} \overline{\overline{b}} \tag{6c}$$

Using the first and second relation in Eq. (1c), Eq. (6c) can be transformed into an equation for the function γ:

$$A_4 \gamma^4 + A_3 \gamma^3 - \gamma^2 - 1 = 0 \tag{6d}$$

$$A_4 = \pi^2 (2C)^{3/2} \left[\frac{f}{|f|} (\boldsymbol{e}_z \times \hat{\boldsymbol{V}}_*) \cdot \boldsymbol{s} + \frac{1}{12} \frac{N^2}{f^2} |\boldsymbol{s}|^2 \right], \quad A_3 = -\pi^2 (2C)^{3/2} \hat{\boldsymbol{V}}_* \cdot \boldsymbol{s} \tag{6e}$$

OSD
7, 1289–1302, 2010

V. M. Canuto and M. S. Dubovikov

$$\hat{\boldsymbol{V}}_* \equiv \frac{\hat{\boldsymbol{V}}}{h|f|}, \quad \boldsymbol{s} = -\frac{\nabla_{\mathrm{H}}\overline{b}}{N^2} \tag{6f}$$

the vector $\boldsymbol{s}$ being the slope of the isopycnals. In Sect. 5 we summarize the main results of the above parameterization using a simplified notation.

4 Interpretation of the results

As results Eq. (3a) and (3b) show, the vertical tracer flux has two components, the first one being due to the large scale fields while the second one is due to mesoscales. It is interesting to compare them. Let us begin with the no wind case when the large scale velocity is purely geostrophic. The salient feature of the SM, as it follows from Eq. (1), is that they are generated and governed by the baroclinic component of the mean velocity which, in the absence of wind, is given in Eq. (13a) of CD9 and CD10. In the present notation it is given by:

$$\hat{\boldsymbol{u}}|_{\text{no wind}} = \left(z + \frac{1}{2}h\right) f^{-1} \boldsymbol{e}_z \times \nabla_{\mathrm{H}} \overline{\overline{b}} \tag{7a}$$

Substituting Eq. (7a) into the first of Eq. (3b), we obtain:

$$\text{No wind}: \quad \partial_z F_{\mathrm{L}} = -(1+\gamma^2)^{-1} f^{-1} \left(z + \frac{1}{2}h\right) \left[\boldsymbol{e}_z \times \nabla_{\mathrm{H}} \overline{\overline{b}} + \gamma \frac{f}{|f|} \nabla_{\mathrm{H}} \overline{\overline{b}}\right] \cdot \nabla_{\mathrm{H}} \overline{\overline{\tau}} \tag{7b}$$

As one can see, for $\tau = b$, this result coincides with the contribution due to mesoscales Eq. (3d). This conclusion stems from the fact that while the barotropic component of the mesoscale kinetic energy exceeds that of the mean flow, for the baroclinic components $\tilde{K}_{\mathrm{M}} = |\tilde{\boldsymbol{u}}''|^2/2$ and $\hat{K} = |\hat{\boldsymbol{u}}|^2/2$ it is the other way around. Indeed, as it follows from Eqs. (5), (7a) and (A5), we have that:

$$\text{No wind}: \quad \hat{K} = \tilde{K}_{\mathrm{M}} \tag{7c}$$

OSD
7, 1289–1302, 2010

V. M. Canuto and M. S. Dubovikov

In the presence of a downfront wind, only the large scale velocity increases due to the Ekman component which is absent in mesoscale velocity. As a consequence,

$$\text{Downfront wind}: \quad \hat{K} > \tilde{K}_\mathrm{M} \tag{7d}$$

and an analogous relationship holds true for the buoyancy fluxes F_L and F_M:

$$\text{Down-front wind}: \quad |F_\mathrm{L}| > |F_\mathrm{M}| \tag{7e}$$

Inequalities (Eq. 7d) and (Eq. 7e) become stronger as the wind gets stronger. Next, we compare the contributions of M and SM vertical fluxes to the large scale tracer equation. The former has been recently parameterized in C10 with the following result:

$$\partial_z F_\mathrm{V}^\mathrm{M} = \boldsymbol{u}_*^\mathrm{M} \cdot \nabla_\mathrm{H} \overline{\overline{\tau}} \tag{7f}$$

where superscript M stands to represent mesoscale variables and $\boldsymbol{u}_*^\mathrm{M}$ is parameterized in Eq. (8b) of C10. In contrast to its SM counterpart (Eq. 4b), $\boldsymbol{u}_*^\mathrm{M}$ depends not only on the fields in the ML but also on those in the ocean interior, a manifestation of the fact that while SM are trapped in the ML, mesoscale eddies form coherent structures which extend through the whole ocean. For this reason, we compare the baroclinic components of the SM and M vertical fluxes which contribute to the restratification of the ML. If we neglect the first term in Eq. (8b) of C10 which is due to the contribution of $\overline{\overline{w''\tau_z''}}$ whose counterpart was neglected in CD9,10 for SM, the baroclinic component of the M induced velocity $\hat{\boldsymbol{u}}_*^\mathrm{M}$ is given by:

$$\hat{\boldsymbol{u}}_*^\mathrm{M} = \mathrm{Ro}_\mathrm{M} \boldsymbol{e}_z \times \hat{\boldsymbol{u}}\,, \quad \mathrm{Ro}_\mathrm{M} = (2K_\mathrm{M})^{1/2}/(r_\mathrm{d}|f|)\,, \tag{7g}$$

where Ro_M is the mesoscale Rossby number and the baroclinic mean velocity $\hat{\boldsymbol{u}}$ is defined in Eq. (3c). We recall that in C10 relations (Eq. 8a and b) were derived under the condition $\mathrm{Ro}_\mathrm{M} \ll 1$ which is amply satisfied for mesoscales. If in Eq. (1a) we formally impose the analogous condition $\mathrm{Ro} = \gamma^{-1} \ll 1$ (which is not satisfied for SM),

OSD
7, 1289–1302, 2010

Mixed layer sub-mesoscale parameterization

V. M. Canuto and M. S. Dubovikov

then we obtain a relation analogous to Eq. (7g). Notice that in the no-wind case for the buoyancy flux, in the second of Eq. (1a), the first term vanishes and only the second one remains. Thus, the formulae for M and SM are formally similar, the difference is that while $\mathrm{Ro}_M \ll 1$, in the SM case, $\mathrm{Ro} \gtrsim 1$. Therefore, *the SM vertical flux exceeds its mesoscale counterpart by an order of magnitude.* A similar situation occurs in the case of a strong wind provided its direction is favorable for the generation of SM eddies. For example, in the case of a strong down-front wind when $\gamma \ll 1$, from Eq. (9a) of CD10 we obtain:

$$v_*^S \approx -A[\cos(z/\delta_E) - \sin(z/\delta_E)]\exp(z/\delta_E), \quad A = \sqrt{2}u_*^2/(\delta_E|f|) \tag{7h}$$

$$v_*^{\hat{M}} \approx -\mathrm{Ro}_M A[\cos(z/\delta_E) + \sin(z/\delta_E)]\exp(z/\delta_E) \tag{7i}$$

where ρu_*^2 is the surface stress and δ_E is the depth of the Ekman layer. These results differ by the factor $\mathrm{Ro}_M \lesssim 0.1$. However this does not mean that the parameterization of the mesoscale vertical flux is unimportant. First, mesoscales are necessary to compute the surface mesoscale kinetic energy, as it follows from Eq. (10) of C10. Second, when the wind direction is not favorable for generating SM, it may still be favorable for generating mesoscales. For example, as we showed in CD9,10, a strong wind in the direction opposite to the horizontal mean buoyancy gradient (down gradient wind) cannot generate vigorous SM eddies while it is favorable for generating mesoscale eddies.

Finally, we compare the mesoscale and sub-mesoscale horizontal fluxes:

$$\boldsymbol{F}_H^M = -\kappa_M \nabla_H \tau, \quad \boldsymbol{F}_H^{SM} = -\kappa_{SM} \nabla_H \tau, \quad \kappa_M = K_M^{1/2} r_d, \quad \kappa_{SM} = K_{SM}^{1/2} r_S \tag{8a}$$

where r_d and r_S are the deformation radii, K_M is the mesoscale kinetic energy which is mostly barotropic and K_{SM} is the SM eddy kinetic energy parameterized in Eq. (6c)–(6f). The ratio of the corresponding divergences that enter the mean tracer equation:

$$\nabla_H \cdot \boldsymbol{F}_H^M = -\kappa_M \nabla_H^2 \overline{\tau}, \quad \nabla_H \cdot \boldsymbol{F}_H^{SM} = -\kappa_{SM} \nabla_H^2 \overline{\tau} \tag{8b}$$

OSD
7, 1289–1302, 2010

Mixed layer sub-mesoscale parameterization

V. M. Canuto and M. S. Dubovikov

equals $K_{\mathrm{M}}^{1/2} r_{\mathrm{d}} / K_{\mathrm{SM}}^{1/2} r_{\mathrm{SM}} \sim 100$, and therefore the contribution of the horizontal SM flux may be neglected in comparison with its mesoscale counterpart.

5 Sub-mesoscale parameterization: summary

To summarize the sub-mesoscale parameterization just derived, we simplify the notation and denote by u, b and τ *the resolved fields*. The z-derivative of the SM vertical tracer flux can be presented as follows:

$$\text{Sub-mesoscales:} \quad \partial_z F_{\mathrm{V}} = \boldsymbol{u}_* \cdot \nabla_{\mathrm{H}} \tau \tag{9a}$$

where:

$$\boldsymbol{u}_* = -(1+\gamma^2)^{-1} \left[\hat{\boldsymbol{u}} - \frac{f}{|f|} \gamma \boldsymbol{e}_z \times \hat{\boldsymbol{u}} + \frac{1}{|f|} \gamma \left(z + \frac{1}{2} h \right) \nabla_{\mathrm{H}} b \right] \tag{9b}$$

$$\hat{\boldsymbol{u}} = \boldsymbol{u}(z) - h^{-1} \int_{-h}^{0} \boldsymbol{u}(z) dz \tag{9c}$$

The function γ is solution of the following equation:

$$A_4 \gamma^4 + A_3 \gamma^3 - \gamma^2 - 1 = 0 \tag{9d}$$

$$A_4 = \pi^2 (2C)^{3/2} \left[\frac{f}{|f|} (\boldsymbol{e}_z \times \boldsymbol{V}) \cdot \boldsymbol{s} + \frac{N^2 |\boldsymbol{s}|^2}{12 f^2} \right], \quad A_3 = -\pi^2 (2C)^{3/2} \boldsymbol{V} \cdot \boldsymbol{s} \tag{9e}$$

$$\boldsymbol{V} = (h^3 |f|)^{-1} \int_{-h}^{0} z \hat{\boldsymbol{u}}(z) dz, \quad \boldsymbol{s} = -\frac{\nabla_{\mathrm{H}} \overline{b}}{N^2}, \quad C = 6 \tag{9f}$$

OSD
7, 1289–1302, 2010

V. M. Canuto and M. S. Dubovikov

6 Conclusions

We recall that the equation for a mean, arbitrary tracer, e.g., active tracers such as T, S and passive tracers such as CO_2, to be used in OGCMs that do not resolve either M or SM has the following form:

$$\partial_t \tau + \overline{\boldsymbol{U}} \cdot \nabla \tau + \underbrace{\nabla_H \cdot \boldsymbol{F}_H + \partial_z F_V}_{\text{M}} + \underbrace{\nabla_H \cdot \boldsymbol{F}_H + \partial_z F_V}_{\text{SM}} = \partial_z (k_V \tau) + G \tag{10a}$$

where the two terms on the right hand side are the contribution of small scale turbulence with a diapycnal diffusivity k_v and the sink-sources term G.

The penultimate term on the lhs of Eq. (10a) is the horizontal flux due to SM which however can be neglected since it is far smaller than the one due to mesoscales:

$$\boldsymbol{F}_H(SM) \ll \boldsymbol{F}_H(M) \tag{10b}$$

where $\boldsymbol{F}_H(M) = -\kappa_M \nabla_H \overline{\tau}$ is given by Eq. (4h) of C10. As for the two vertical fluxes due to M and SM, the latter, specifically, the function $\partial_z F_{SM}$ is given by Eqs. (9) above which, due to their algebraic nature, are easy to implement in OGCMs. The parameterization of the M vertical flux is given by Eqs. (9) of C10.

In the case of a *strong down-front wind*, the SM vertical flux F_{SM} considerably exceeds that of M. However, this does not imply that the vertical M flux may be neglected since the re-stratifying effect of SM is largely cancelled by the de-stratifying effect of the mean flow (Mahadevan et al., 2010; CD9-CD10a) and therefore the mesoscale vertical flux becomes important. The latter is also true in the case of a strong up-front wind when SM are not generated (CD9-CD10). In addition, a parameterization of the vertical M flux in the mixed layer is indispensable for the parameterization of the mesoscale kinetic energy that enters the M diffusivity, as seen in Eq. (8b).

Finally, it must be stressed that SM and M affect each other, for example, the last term in Eq. (9b) is contributed by M, a term that becomes less relevant as the down-front wind becomes stronger. On the other hand, as shown in C10, SM affect the

OSD
7, 1289–1302, 2010

V. M. Canuto and M. S. Dubovikov

mesoscale fluxes and the corresponding effect increases when a down-front wind becomes stronger.

Appendix A

Derivation of Eq. (3d)

Substituting Eq. (5) into the second of Eq. (3b), we derive the relation:

$$\partial_z F_{\mathrm{M}} = -\left(z+\frac{1}{2}h\right) f^{-1}(1+\gamma^2)^{-1}\left(\overline{\overline{\nabla_{\mathrm{H}} b''\times\nabla_{\mathrm{H}}\tau''}}\cdot\boldsymbol{e}_z + \gamma\,\overline{\overline{\nabla_{\mathrm{H}} b''\cdot\nabla_{\mathrm{H}}\tau''}}\right) \quad \text{(A1)}$$

To compute the two terms in Eq. (A1), we note that mesoscale eddies are almost axi-symmetric so that both $\nabla_{\mathrm{H}} b''$ and $\nabla_{\mathrm{H}}\tau''$ are directed toward the eddy center (or opposite). Therefore, averaging mesoscale fields yields the following results:

$$\overline{\overline{\nabla_{\mathrm{H}} b''\times\nabla_{\mathrm{H}}\tau''}} = 0\,, \quad \overline{\overline{\nabla_{\mathrm{H}} b''\cdot\nabla_{\mathrm{H}}\tau''}} \approx r_{\mathrm{d}}^{-2}\,\overline{\overline{b''\tau''}} \quad \text{(A2)}$$

where r_{d} is the Rossby deformation radius not to be confused with the deformation radius of the ML r_{S} given in Eq. (1c). To parameterize the correlation functions in the second of (A2), we use the following relations (K_{M} is the mesoscale kinetic energy)

$$\tau'' \sim -t_{\mathrm{M}}\boldsymbol{u}''\cdot\nabla_{\mathrm{H}}\overline{\overline{\tau}}\,, \quad b'' \sim -t_{\mathrm{M}}\boldsymbol{u}''\cdot\nabla_{\mathrm{H}}\overline{\overline{b}}\,, \quad t_{\mathrm{M}} \sim r_{\mathrm{d}} K_{\mathrm{M}}^{-1/2} \quad \text{(A3)}$$

which we now derive. To that end, the recall the relation between the Fourier components of the mesoscale tracer and velocity fields in the vicinity of the maximum of the mesoscale kinetic energy spectrum at $|\boldsymbol{k}|{\sim}r_{\mathrm{d}}^{-1}$, Eq. (3f) of C10, which in the present notation reads as follows:

$$\tau''(\boldsymbol{k}) = -\frac{\boldsymbol{u}''(\boldsymbol{k})\cdot\nabla_{\mathrm{H}}\overline{\overline{\tau}}}{\chi_{\mathrm{M}} + i\boldsymbol{k}\cdot\left(\overline{\overline{\boldsymbol{u}}} - \boldsymbol{u}_{\mathrm{d}}\right)}\,, \quad \chi_{\mathrm{M}} = r_{\mathrm{d}}^{-1} K_{\mathrm{M}}^{1/2} \quad \text{(A4)}$$

OSD

7, 1289–1302, 2010

Mixed layer sub-mesoscale parameterization

V. M. Canuto and M. S. Dubovikov

where $\boldsymbol{u}_\mathrm{d}$ is the drift velocity of mesoscale eddies. Assuming that $K_\mathrm{M}^{1/2} > |\overline{\overline{\boldsymbol{u}}}|$, $K_\mathrm{M}^{1/2} > |\boldsymbol{u}_\mathrm{d}|$ we may keep only the first term χ in the denominator of Eq. (A4). Finally, assuming that the shapes of the spectra are similar in the vicinity of $|\boldsymbol{k}| \sim r_\mathrm{d}^{-1}$, we arrive at Eq. (A3). Substituting it into the second relation (Eq. A2), we obtain:

$$\overline{\overline{\nabla_\mathrm{H} b'' \cdot \nabla_\mathrm{H} \tau''}} \approx r_\mathrm{d}^{-2} \overline{\overline{b''\tau''}} \approx r_\mathrm{d}^{-2} \left(r_\mathrm{d} K_\mathrm{M}^{-1/2} \right)^2 \overline{\overline{(\boldsymbol{u}'' \cdot \nabla_\mathrm{h} b)(\boldsymbol{u}'' \cdot \nabla_\mathrm{h} \tau)}}$$

$$\approx \frac{1}{2} K_\mathrm{M}^{-1} |\overline{\overline{\boldsymbol{u}''}}|^2 \nabla_\mathrm{h} \overline{\overline{b}} \cdot \nabla_\mathrm{h} \overline{\overline{\tau}} = \nabla_\mathrm{H} \overline{\overline{b}} \cdot \nabla_\mathrm{H} \overline{\overline{\tau}} \tag{A5}$$

Substituting this result, together with the first of Eq. (A2), into Eq. (A1), we arrive at relation (Eq. 3d).

References

Canuto, V. M. and Dubovikov, M. S.: Derivation and assessment of a mixed layer sub-mesoscale model, Ocean Sci. Discuss., 6, 2157–2192, doi:10.5194/osd-6-2157-2009, 2009.

Canuto, V. M., Dubovikov, M. S., Luneva, M., Clayson, C. A., and Leboissetier, A.: Mixed layer mesoscales: a parameterization for OGCMs, Ocean Sci. Discuss., 7, 873–917, doi:10.5194/osd-7-873-2010, 2010.

Canuto, V. M. and Dubovikov, M. S.: Mixed layer sub-mesoscale parameterization – Part 1: Derivation and assessment, Ocean Sci. 6, 1–15, doi:10.5194/os-6-1-2010, 2010.

Capet, X., McWilliams, J. C., Molemaker, M. J., Shchepetkin, A. F.: Mesoscale to submesoscale transition in the California current system, Part I: flow structure, eddy flux, and observational tests, J. Phys. Oceanogr., 38, 29–43, 2008.

Hosegood, P. J., Gregg, M. C., and Alford, M. H.: Restratification of the surface mixed layer with submesoscale lateral density gradients: diagnosing the importance of the horizontal dimension, J. Phys. Oceanogr., 38, 2438–2460, 2008.

Killworth, P. D.: On parameterization of eddy transport, J. Mar. Res., 55, 1171–1197, 2005.

Klein, P., Hua, B. L., Lapeyre, G., Capet, X., Le Gentil, S., and Sasaki, H.: Upper ocean turbulence from high resolution 3-D simulation, J. Phys. Oceanogr., 38, 1748–1763, 2008.

Lapeyere, G., Klein, P., and Hua, B. C.: Oceanic re-stratification forced surface frontogenesis, J. Phys. Oceanogr., 36, 1577–1590, 2006.
Levy, M., Klein, P., and Treguier, A. M.: Impacts of sub-mesoscale physics on production and subduction of phytoplankton in an oligotrophic regime, J. Mar. Res., 59, 535–565, 2001.
Levy, M., Klein, P., Treguier, A. M., Iovino, D., Madec, G., Masson, S., and Takahashi, K.: Modification of gyre circulation by submesoscale physics, Ocean Modell., 34, 1–16, 2010.
Mahadevan, A.: Modeling vertical motion at ocean fronts: are non-hydrostatic effects relevant to sub-mesoscales?, Ocean Modell., 14, 222–240, 2006.
Mahadevan, A. and Tandon, A.: An analysis of mechanisms for sub-mesoscale vertical motion at ocean fronts, Ocean Modell., 14, 241–256, 2006.
Mahadevan, A., Tandon, A., and Ferrari, R.: Rapid changes in mixed layer stratification driven by sub-mesoscales instabilities and wind, J. Geophys. Res., 115, C03017, doi:10.1029/2008JC005203, 2009
Thomas, L. N., Tandon, A., and Mahadevan, A.: Submesoscale processes and dynamics, in: Eddy Resolving Ocean Modeling, edited by: Hecht, M. and Hasumi, H., AGU Monograph, American Geophysical Union, Washington, DC, 17–38, 2008.

OSD
7, 1289–1302, 2010

Mixed layer sub-mesoscale parameterization

V. M. Canuto and M. S. Dubovikov