

Interactive comment on “Dynamically constrained ensemble perturbations – application to tides on the West Florida Shelf” by A. Barth et al.

A. Barth et al.

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We thank the reviewer for the careful reading of the manuscript and the very helpful comments and constructive criticisms. Below is the list of issues raised by the reviewer and our proposed modifications to the manuscript.

1) *It would be helpful to mention in the introduction that the perturbations are drawn from a Gaussian distribution, since it is an essential assumption in the development of the method. In the present version of the introduction, this is only implicit in the 1st sentence of the 2nd paragraph. It is indeed because of the Gaussian assumption that uncertainty can be completely described by the covariance.*

We agree with the reviewer and the revised manuscript will state in the introduction that the method creates Gaussian distributed perturbations.

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2) *It would also be useful to make a clear distinction between the method that is proposed for constructing an appropriate covariance structure (which is the core of the paper) and the method for drawing a perturbation from a Gaussian probability distribution with given covariance (which is anecdotic in terms of method, even if certainly essential in terms of numerical cost, see next comments).*

We tried to clarify this distinction by stating that the dynamically constrained covariance function can be used directly in assimilation schemes that are based on the inverse of the background covariance matrix (such as variational assimilation) with the need to create an ensemble. Alternatively, different ways for creating an ensemble can be used such as the 2nd order re-sampling method used in the SEIK filter (Pham, 2001) or a Cholesky decomposition (as the reviewer brought to our attention in comment 4).

The main information that is lacking in the paper is certainly an information about the numerical cost of the method. As it is explained in the paper, the method requires explicit square root decomposition of the resulting covariance matrix, a computation which quickly becomes excessively expensive for large size systems. Please add an explicit mention of the cost of every elements of the algorithm as a function of the size of the perturbation vectors, the number of perturbations to draw,...

The manuscript will be extended by the following to address this issue:

To assess the numerical cost of this scheme and to test its feasibility with a high resolution ocean model, the method was applied to a square domain of 200 km length with different resolutions. In a first series of experiments only 50 eigenvectors are retained during the eigenvector decomposition. All other parameters were the same than in the perturbation for the WFS case.

The code is tested on a single core of an Intel Xeon E5420 CPU. The code is run

in Octave 3.0.5 compiled among others with SuiteSparse 3.4.0 (Davis, 2004a,b), GotoBLAS 1.26 and ARPACK 96 (Lehoucq et al., 1997). Those libraries are used in the eigenvector decomposition. The time in seconds of different steps in the algorithm are shown in table 1 for different grid sizes. A domain size of 512x512 was tested but the method required more than the available 16 GB of RAM. As expected, the creation of the matrix B^{-1} , and the ensemble creation increases essentially linearly with the number of grid points while the eigenvector decomposition increases faster than linearly with the number of grid points. The slope of a linear regression in log-log space is 1.3. This progression is still quite similar to a linear increase because the matrix B^{-1} is sparse and because a fixed number of eigenvectors are retained.

In a second series of tests, the domain size is fixed (128x128 grid points) and the number of eigenvectors are increased from 50 to 200 (table 2). The numerical cost of the eigenvector decomposition shows a linear progression relative to the number of eigenvectors. The numerical cost of the ensemble creation increases only slowly and is marginal in the overall cost.

By increasing the resolution of a domain, one can argue that the number of eigenvectors and ensemble members should also increase proportional to the number of grid points. In this case, the progression rate of both series have to be combined and the numerical cost scales approximately by the number of grid points elevated to 2.3.

The program code has been written such that it can run unmodified also on MATLAB. The benchmark was repeated on the same machine with MATLAB R2008a (64-bit version). The numerical cost in function of the number of grid points and the number of eigenvectors retained varied in a similar way than with Octave. Overall, Octave was 13% faster than MATLAB in completing the two series of tests. In summary, the CPU time of this method is acceptable since it is very small compared to the CPU time

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Size of domain	Create of matrix B^{-1}	Eigenvector decomposition	Ensemble
32x32	0.28	1.31	0.09
64x64	0.38	8.77	0.39
128x128	1.36	51.23	1.69
256x256	5.67	296.28	7.07
300x300	7.96	451.66	9.79
400x400	14.77	983.77	18.36

Table 1. Time in seconds as a function of the number of grid points of the different steps involved in the creation of the perturbation.

Number of eigenvectors	Eigenvector decomposition	Ensemble
50	51.23	1.69
100	112.87	2.47
150	151.90	3.42
200	227.19	4.60

Table 2. Time in seconds as a function of number of eigenvector retained for a domain of 128x128 grid points.

needed for the ensemble run. More limiting than the CPU time, can be the required amount of RAM memory for large model configurations.

4) Concerning the drawing of random perturbations from a Gaussian distribution with given covariance, please note that any square root of the covariance matrix can be used, as indicated for instance in the appendix of the paper: "Fukumori, I., A partitioned Kalman Filter and Smoother, Monthly Weather Review, 130, 1370-1383, 2002". Cholesky decomposition for instance would provide a cheaper square root.

We thank the viewer for pointing this out. The revised manuscript will include also that Cholesky decomposition can be used to generate the perturbations. The difference

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with Fukumori here is that the covariance matrix \mathbf{B} is never formed explicitly; only the inverse of \mathbf{B} is explicitly formed since it can be expressed efficiently as a sparse matrix (given the constraints are discretized as finite differences). The inverse of the square root matrix of \mathbf{B}^{-1} has to be computed. But since the Cholesky decomposition returns a triangular matrix, the inverse times a given vector can be efficiently computed by back substitution.

The manuscript has also been updated to state more clearly that the method operates on \mathbf{B}^{-1} and not on \mathbf{B} .

5) Concerning the construction of the covariance matrix, you explain in detail why the dynamical constraint term is an essential component of the inverse covariance matrix (for instance, in equation 3), but you give little explanation as to why the smoothness component provides an appropriate reference covariance structure. This is the kind of parameterization that is assumed when little is known about the covariance structure (except a correlation length scale). A short explanation for this particular choice (which certainly does not fit any kind of purpose) would be welcome.

The following will be added to the manuscript:

“Boundary values are not constrained by the shallow water equations. If no smoothness constraint would be present, the resulting ensemble members would be discontinuous at the boundary in the direction parallel to the boundary. The explicit smoothness constraint (Laplacian with a zero gradient at the open boundary in a direction perpendicular to the boundary) is added to the cost-function to avoid those discontinuities at the boundary.”

In a follow-up experiment (in a different domain), satisfactory results are also obtained by enforcing the smoothness constraint only at the boundary points.

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6) In the "combined cost function" (equation 3), it is necessary that the subspaces defined by the dominant eigenvectors of the individual components (the longer principal axes of their respective covariance ellipsoids) are not orthogonal to each other. If they were, the method would only produce small perturbations not satisfying any of the required properties (because such perturbations would not exist). In the examples, the method works correctly because there exist perturbations which are both dynamically constrained and smooth (approximately at least). Maybe this is worth a word of caution.

We agree with the reviewer. In the revised manuscript we will mention that all constraints have to be compatible.

7) In the construction of an anisotropic covariance structure constrained by the land boundaries (section 3), little information is given about the numerical computation of the D matrix. How is it computed in practice? How are the boundary conditions applied? Moreover, the discussion would be more logical if you describe first the isotropic and homogeneous solution, and afterwards how this can be modified by taking into account the presence of land boundaries.

The following will be added:

"The derivatives in equation (10) and (11) are approximated by centered finite differences, yielding a sparse matrix representation of D . To facilitate the formulation of this sparse matrix, the operator is separated into sub-steps (calculating the vector field F , application of boundary conditions, calculating the divergence of the vector field). Each of the individual steps can be expressed as a sparse matrix in a more straightforward manner. The matrix D is then simply the product of all matrices corresponding to the sub-steps in the appropriate order. The same approach will also be used to create a sparse matrix representing the dynamical constraint."

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The isotropic and homogeneous perturbations are actually obtained by the Fourier based-method and its results are compared at the beginning of section 3 to the solution constrained by the land-sea boundaries. We don't think that the perturbations using the variable correlation length can be considered as isotropic since the correlation length increased in a particular direction. We placed this example after the impact of the land-sea mask because we think that information on how the correlation length varies in space might not always be available (while the land-sea mask certainly is) and might be less used in practice.

8) In the description of the model in section 4, the name of the region is missing (you only use the undefined acronym WFS): in the present version, it is only given in the title and at the end of the introduction. You could also give a few more elements about the model behaviour to help understanding the results of section 6.

In the revised manuscript the acronym WFS is defined as its first occurrence. The model description is also expanded:

The method is tested using the WFS ROMS model (Barth et al., 2008c) which is nested in the Atlantic HYCOM model (Chassignet et al., 2007). The nesting procedure is explained in Barth et al. (2008a). The model uses a curvilinear grid with a resolution of about 3.5 km near the coast and 10 km near the open boundary. The model is initialized on the 2005-01-01 using the elevation, velocity, temperature and salinity from HYCOM. The boundary conditions from HYCOM mainly imposes the path of the Gulf of Mexico Loop Current generating mesoscale eddies and filaments inside the model domain (Barth et al., 2008b). The tidal boundary conditions produce tidal wave propagating inside the model domain and increase their amplitude near the coast as predicted by linear tidal wave theory for wide continental shelves (Clarke, 1991) and by a numerical ocean model of the WFS (He and Weisberg, 2002).

9) The conclusion looks like an abstract of the paper, without any interpretations or

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perspectives. I think that it would be worthwhile to rewrite it carefully to give a better view of the output of this paper.

The conclusion was expanded to include more interpretations. We intend to use this technique to assimilate surface current observations. By including the boundary perturbations into the model state vector, the assimilation could provide also an improved estimation of the boundary values.

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