**Ocean Science
Discussions**

Understanding mixing efficiency in the oceans: do the nonlinearities of the equation of state for seawater matter?

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Abstract

There exist two central measures of turbulent diffusive mixing in turbulent stratified fluids, which are both caused by molecular diffusion: 1) the dissipation rate $D(APE)$ of available potential energy APE; 2) the rate of change $W_{r,\mathrm{mixing}}$ of background gravitational potential energy GPE_r. So far, these two quantities have often been regarded as representing the same kind of energy conversion, i.e., the irreversible conversion of APE into GPE_r, owing to the well known result that $D(APE) \approx W_{r,\mathrm{mixing}}$ in a Boussinesq fluid with a linear equation of state. Here, this idea is challenged by showing that while $D(APE)$ remains largely unaffected by a nonlinear equation of state, $W_{r,\mathrm{mixing}}$ is in contrast strongly affected by the latter. This result is rationalized by using the recent results of Tailleux (2008), which argues that $D(APE)$ represents the dissipation of APE into one particular subcomponent of internal energy called the "dead" internal energy IE_0, whereas $W_{r,\mathrm{mixing}}$ represents the conversion between a different subcomponent of internal energy – called the "exergy" IE_{exergy} – and GPE_r. It follows that the concept of mixing efficiency, which represents the fraction of the stirring mechanical energy ultimately dissipated by molecular diffusion is related to $D(APE)$, not $W_{r,\mathrm{mixing}}$, which ensures that it should be largely unaffected by the nonlinear character of the equation of state, and therefore correctly described in the context of a Boussinesq fluid with a linear equation of state. The variations of GPE_r, on the other hand, are sensitive to the linear or nonlinear character of the equation of state.

1 Introduction

As is well known, turbulent diffusive mixing in the oceans is a physical process that it is crucially important to parameterize correctly in numerical ocean models. Indeed, it is the quality of the parameterized irreversible diabatic processes in such models that largely determines the realism of the simulated distribution of water mass properties, as well as of the behavior of the so-called meridional overturning circulation and

its associated heat transport (Gregg , 1987), which are two essential components of the large-scale ocean circulation that may interact with Earth climate. For this reason, much effort has been devoted over the past decades toward understanding the fundamental characteristics of turbulent diffusive mixing in stratified fluids, in order to

5   design physically-based parameterizations of irreversible mixing processes suitable for implementation in numerical ocean models used for climate change simulations.

At a fundamental level, turbulent diffusive mixing in stratified fluids is important for at least two distinct – although inter-related – reasons: 1) for its role as a mechanism responsible for a significant fraction – called the mixing efficiency – of the total irre-

10   versible decay of available mechanical energy (i.e., the sum of the kinetic energy KE and available potential energy APE); 2) for its role as a mechanism responsible for the diffusive mixing of temperature across isopycnal surfaces, called diapycnal mixing, thus permitting the downward transfer of heat from the surface that is required to counterbalance the effect of high-latitude cooling associated with deep water formation. In

15   the turbulence literature, these two distinct roles of turbulent diffusive mixing are often being regarded as being associated with a single physical process, whereby the diffusively dissipated available potential energy is converted irreversibly into the background gravitational potential energy GPE_r, e.g., Winters and al. (1995); Peltier and Caulfield (2003). Although there exists a considerable literature about how one should

20   parameterize turbulent diffusive mixing, the model that appears the most often used or cited appears to be that of Osborn (1980), viz.,

$$K_\rho = \frac{\gamma_{\mathrm{mixing}}\varepsilon}{N^2},\qquad(1)$$

where the turbulent diapycnal diffusivity K_ρ is expressed in terms of the kinetic energy dissipation ε, the so-called mixing efficiency γ_{mixing}, and squared buoyancy frequency

25   N^2.

To the extent that γ_{mixing} can be regarded as some kind of universal parameter,

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as is often assumed[1], then Eq. (1) can be interpreted as stating that the amount of irreversible diffusive mixing is proportional to the sources of mechanical stirring. In the ocean modelling practice, however, the value of K_ρ is usually adjusted so that the simulated meridional heat transport in numerical ocean models agrees well with observational estimates. Such an approach was pioneered by Munk (1966), which led to the widespread idea that the canonical value $K_\rho = 10^{-4}\,\mathrm{m}^2/\mathrm{s}$ was apparently needed for achieving a meridional heat transport of the observed strength. Such an approach, however, is indirect, and does not address the issue of whether there is enough stirring in the oceans to achieve a value of turbulent diapycnal mixing that is about three orders of magnitude larger than the molecular diffusivity of heat, an issue that was only undertaken by Munk and Wunsch (1998) about a decade ago, as discussed in further details below. Whether this is the case was questioned in subsequent years, with several observational studies suggesting that K_ρ in the oceans interior was in general typically smaller by an order of magnitude than Munk (1966)'s value, stirring much debate in the ocean community as to correctness and accuracy of Munk (1966)'s estimate. On the other hand, it is also widely recognized that turbulent mixing in the oceans is highly variable, both spatially and temporally, and therefore certainly not well described by a single value independent of space and time. For that reason, Munk and Wunsch (1998) suggested to resolve that dilemma by regarding the value $K_\rho = 10^{-4}\,\mathrm{m}^2/\mathrm{s}$ as a bulk-averaged value to be interpreted as resulting from the overall effect of weak interior values combined with intense turbulent mixing occurring in coastal areas or over rough topography. In a second step, they also proposed to use Osborn (1980)'s model to see whether there is enough stirring energy available in the oceans.

Recently, these ideas have been used to examine whether the energetics of turbulent irreversible diffusive mixing impose constraints on the magnitude of the mechanical sources of stirring. To investigate this issue, Munk and Wunsch (1998) examined the

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[1]See Tailleux (2008), however, for physical arguments challenging this idea.

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budget of gravitational potential energy, which they argue must be a balance between the rate of GPE loss due to cooling and the rate of GPE increase due to turbulent diffusive mixing, i.e.,

$$\left| \frac{d(GPE)}{dt} \right|_{\text{cooling}} \approx \left| \frac{d(GPE)}{dt} \right|_{\text{mixing}} \tag{2}$$

Assuming the rate of GPE loss due to cooling to be known from an estimate of the North-Atlantic deep water formation rate, Munk and Wunsch (1998) invoke Osborn model to link the rate of GPE increase du to turbulent mixing to the work done by the mechanical sources of stirring $G(KE)$ by:

$$\left| \frac{d(GPE)}{dt} \right|_{\text{mixing}} \approx \gamma_{\text{mixing}} G(KE) \tag{3}$$

via the mixing efficiency γ_{mixing}. This approach, however, appears to rely on many classical ideas about turbulent mixing, most notably on the idea that the background GPE necessarily increases as the result of turbulent mixing, as well as on the idea that γ_{mixing} can be regarded as some kind of universal parameter with some fixed value. Yet, it is clear from the work by Fofonoff (1998, 2001) that there is no guarantee in general that the background GPE should necessarily increase as the result of turbulent mixing, as this depends on the particular vertical temperature profile and nonlinear character of the equation of state considered. To be specific, whether the background GPE should increase or decrease as the result of turbulent mixing appears to depend on the sign of the following parameter:

$$\frac{d}{dz} \left(\frac{\alpha P}{\rho C_p} \right) \tag{4}$$

where α is the thermal expansion coefficient, P is the pressure, ρ is the density, and C_p is the specific heat capacity at constant pressure. Thus, the classical regime for

which the background GPE increases as the result of turbulent irreversible mixing corresponds to the case where the sign of the above parameter is negative, as is the case for a hydrostatic fluid (so that $dP/dz=-\rho g<0$) for which $\alpha/(\rho C_p)$ can be regarded as a constant. This is the case, in particular, for a Boussinesq fluid with a linear equation of state, for which the increase of GPE due to turbulent mixing is accurately described by the following formula:

$$\left\{\frac{d(GPE)}{dt}\right\}_{\text{mixing}} = \int K_\rho \rho N^2 dV. \tag{5}$$

In the literature, the idea according to which the background GPE should always increase as the result of turbulent mixing appears to be linked to the widespread belief that when the available potential energy (APE) is removed by molecular diffusion, it is necessarily irreversibly converted into background GPE, as proposed by Winters and al. (1995) for instance. Physically, this idea results from regarding the APE dissipation rate $D(APE)$ and the turbulent rate of GPE_r change $W_{r,\text{turbulent}}$ as being basically the same conversion[2], which is motivated by the fact that $D(APE)=W_{r,\text{turbulent}}$ for a Boussinesq fluid with a linear equation of state. This idea, however, was recently challenged by Tailleux (2008), who pointed out that the previous equality is actually a serendipitous feature of the Boussinesq approximation, but that $D(APE)$ and $W_{r,\text{turbulent}}$ can in fact be very different from each other for a real fluid, suggesting on the contrary that $D(APE)$ and $W_{r,\text{turbulent}}$ represent two distinct measures of turbulent irreversible diffusive mixing. Specifically, Tailleux (2008) argue that $D(APE)$ physically represents the dissipation rate of APE into a particular subcomponent of internal energy, called the "dead internal energy" IE_0, while $W_{r,\text{turbulent}}$, like $W_{r,\text{laminar}}$, physically represent the conversion rate between GPE_r and a different subcomponent of internal energy, called the

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[2]This assumes that the total GPEr rate of change $W_{r,\text{mixing}}=W_{r,\text{laminar}}+W_{r,\text{turbulent}}$ can be written as the sum of a background laminar rate $W_{r,\text{laminar}}$ plus a turbulent rate $W_{r,\text{turbulent}}$. For a turbulent fluid, $W_{r,\text{laminar}}$ can be neglected in general, so that $W_{r,\text{mixing}} \approx W_{r,\text{turbulent}}$, which is assumed in the rest of the paper.

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"exergy" IE_{exergy}. Physically, the "dead" and "exergy" components of internal energy can be regarded as being associated respectively with the equivalent thermodynamic equilibrium temperature $T_0(t)$ and vertical temperature stratification $T_r(z,t)$ of the fluid. Physically, it means that the APE dissipation mainly results in increasing the equivalent

5 thermodynamic temperature $T_0(t)$, whereas the conversion $W_{r,\text{turbulent}}$ associated with the GPE_r variations results in the smoothing out of the vertical temperature stratification $T_r(z,t)$.

Until now, most theoretical and numerical descriptions of turbulent mixing in stratified fluids have most often relied on the incompressible Navier-Stokes equations, often

10 in the context of the Boussinesq approximation. With regard to the thermodynamics of the fluid, it is generally considered to be unimportant at leading order, and as a result, a linear equation of state is generally considered to be accurate enough for the purposes of describing turbulent mixing. As is well known, however, the equation of state for seawater – which is the one appropriate for describing the oceans – is strongly

15 nonlinear in temperature, pressure, and salinity. The question arises, therefore, of which particular properties of turbulent mixing, if any, might be affected by the nonlinear nature of the equation of state for seawater. In order to examine this issue, this paper examines the particular case of a well-known property of turbulent mixing, called the mixing efficiency. Physically, mixing efficiency is often regarded as the fraction of the

20 total mechanical stirring energy that is eventually dissipated by molecular diffusion, and is a central quantity in the study of turbulent diffusive mixing. Section 3 provides a theoretical formulation of the issue discussed. Section 4 discusses the methodology, while the results are presented in Sect. 4. Finally, Sect. 5 summarizes and discusses the results.

25 2 Theoretical formulation of the problem

An important issue in the study of turbulence is to be able to distinguish the adiabatic and reversible process of stirring from the irreversible process of mixing. From a theo-

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retical viewpoint, the stirring and mixing processes are most easily distinguished if the gravitational potential energy is partitioned into its available and non-available components, as initially proposed by Lorenz (1955) to study atmospheric energetics, and more recently by Winters and al. (1995) to study stratified turbulence. This is because by construction, irreversible processes only affect the un-available part of the GPE, so that it allows for a natural quantification of the irreversible part of the density flux. In such an approach, one may write evolution equations for the volume-integrated kinetic energy (KE), available GPE (APE), and background GPE_r in the following form:

$$\frac{d(KE)}{dt} = -C(KE, APE) - D(KE) \tag{6}$$

$$\frac{d(APE)}{dt} = C(KE, APE) - D(APE) \tag{7}$$

$$\frac{d(GPE_r)}{dt} = W_{r,\text{mixing}} \tag{8}$$

where $C(KE, APE)$ is the reversible conversion of KE and APE, $D(KE)$ is the viscous rate of KE dissipation, $D(APE)$ is the diffusive rate of APE dissipation, and $W_{r,\text{mixing}}$ is the rate of change of background GPE due to molecular diffusion. As shown by Tailleux (2008), the above evolution equations appear to be generic, in the sense that they can be shown to apply to the general case of the fully compressible Navier-Stokes equations using a fully nonlinear equation of state, as well as to many different forms of the Boussinesq approximation using a linear or nonlinear equation of state. The explicit form of the conversion coefficients appearing in such equations, however, will depend on the particular set of equations considered. Some explicit expressions for $D(APE)$ and $W_{r,\text{mixing}}$ have been given by Tailleux (2008) in the case where a linear equation of state and the Boussinesq approximation is used. In such an approach, the process of stirring is associated with the reversible conversion of turbulent kinetic energy into APE via the energy conversion $C(KE, APE)$, whereas the irreversible process of mixing acts in two ways: 1) by dissipating APE via the term $D(APE)$; 2) by irreversibly altering the

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mean centre of gravity at the rate $W_{r,\text{mixing}}$. The link with internal energy were clarified by Tailleux (2008), who also showed that:

$$\frac{d(IE_0)}{dt} \approx D(KE) + D(APE) \tag{9}$$

$$\frac{d(IE_{\text{exergy}})}{dt} \approx -W_{r,\text{mixing}} \tag{10}$$

5　which establish that the viscously dissipated KE and diffusively dissipated APE both end up into the dead part of internal energy IE_0, whereas $W_{r,\text{mixing}}$ represent the conversion rate between GPE_r and the "exergy" component of internal energy IE_{exergy}.

　　The usefulness of the above energy equations is most easily illustrated by averaging the latter over a turbulent mixing event. Here, a turbulent mixing event is defined as
10　an episode of intense mixing followed and preceded by laminar conditions for which $APE=0$ to a very good approximation. In that case, denoting a time average by an overbar, one obtains:

$$\Delta KE = -\overline{C(KE, APE)} - \overline{D(KE)}, \tag{11}$$

$$0 = \overline{C(KE, APE)} - \overline{D(APE)}, \tag{12}$$

15　$$\Delta GPE_r = \overline{W}_{r,\text{mixing}}. \tag{13}$$

The first two equations can be summed to yield:

$$\Delta KE = -\overline{D(KE)} - \overline{D(APE)}, \tag{14}$$

which shows that both molecular viscous and diffusive processes can contribute to the irreversible dissipation of turbulent kinetic energy. The relative importance of the non-
20　viscous dissipation of kinetic energy over the total viscous and non-viscous dissipation is often measured in terms of the concept of mixing efficiency, defined here following

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Peltier and Caulfield (2003) (see also Staquet (2000) for a more thorough discussion of the origin of this definition, and its link to other measures of mixing efficiency such as the Bulk Richardson number) by:

$$\gamma_{\text{mixing}} = \frac{D(APE)}{D(KE) + D(APE)} \tag{15}$$

In the turbulent mixing literature, mixing efficiency is also often defined as follows:

$$\gamma_{\text{mixing}} = \frac{W_{r,\text{mixing}}}{D(KE) + D(APE)} \tag{16}$$

so that in order for the two definitions to be equivalent, we need $W_{r,\text{mixing}} \approx D(APE)$ to a good approximation. In the turbulence literature, the idea that the two definitions are generally equivalent appears to be widespread, and based on the idea that the APE dissipated by turbulent mixing is entirely converted into background GPE. As far as we understand it, the idea seems to originate in the result that in the Boussinesq approximation based on using a linear equation of state, then $D(APE)$ and $W_{r,\text{mixing}}$ are indeed nearly identical and therefore highly correlated. The fact that these two terms are nearly equal, however, is not sufficient to prove that the dissipated APE is necessarily converted into GPE_r. As discussed by Tailleux (2008), and further illustrated below, it is easy to show that $D(APE)$ and $W_{r,\text{mixing}}$ become very dissimilar when a nonlinear equation of state is used. As a consequence, it is not true that the above two definitions of mixing efficiency are equivalent, in contrast with popular belief. In fact, only the first definition Eq. (15) is valid as a measure of the relative importance of non-viscous dissipation over total dissipation of kinetic energy, and the only one able to produce a number between 0 and 1. The rate of change $W_{r,\text{mixing}}$ indeed, is not directly related to the dissipated APE, and can either decrease or increase depending on the stratification, so that definition (16) can in general produce a negative number, which conflicts with the properties generally attributed to mixing efficiency in the literature.

3 Methodology

In order to get insights into how the nonlinearities of the equation of state of seawater affects turbulent mixing, we computed the behaviour of $D(APE)$ and $W_{r,\text{mixing}}$ for a number of different stratifications possessing the same buoyancy frequency profile N, but different profiles of the parameter $\alpha P/(\rho C_p)$, as illustrated in Fig. 1. In order to compute $D(APE)$ and $W_{r,\text{mixing}}$, we used the explicit expressions derived by Tailleux (2008), which are given by:

$$W_{r,\text{mixing}} = \int_V \frac{\alpha_r P_r}{\rho_r C_{pr}} \nabla \cdot \left(\kappa \rho C_p \nabla T \right) dV, \qquad (17)$$

$$D(APE) = -\int_V \frac{T - T_r}{T} \nabla \cdot \left(\kappa \rho C_p \nabla T \right) dV \qquad (18)$$

These two expressions were estimated numerically for the case of a two-dimensional square domain discretized equally in the horizontal and vertical direction. In these expressions, κ is the molecular diffusivity, P is the pressure (assumed to be hydrostatic), ρ is the density, and C_p is the specific heat capacity at constant pressure. Also, the subscript "r" is used to refer to the quantities in their Lorenz (1955)'s reference state.

In total, 27 different types of stratification were considered, all possessing the same squared buoyancy frequency N^2 illustrated in the top panel of Fig. 1, but different mean temperature, salinity, and pressure resulting in different profiles for the $\alpha P/(\rho C_p)$ parameter illustrated in the bottom panel of Fig. 1. In all cases considered, the pressure varied from $P_{\min}$ to $P_{\max} + 10$ dbar, with $P_{\min}$ taking the three values (0 dbar, 1000 dbar, 2000 dbar). In all cases, the salinity was assumed to be constant, and taking one of the three possible values $S = (30\,\text{psu}, 35\,\text{psu}, 40\,\text{psu})$. With regard to the temperature profile, it was determined by imposing the particular value $T_{\max} = T(P_{\min})$ at the top of the fluid, and determining all remaining values by inversion of the buoyancy frequency N^2 common to all profiles by an iterative method. For each stratification, the thermodynamic properties of the fluid were estimated from the Gibbs function of Feistel (2003).

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We computed $D(APE)$ and $W_{r,\text{mixing}}$ in the case of a two-dimensional domain that was discretized into $N_{pi} \times N_{pj}$ points in the horizontal and vertical, with $N_{pi}=N_{pj}=100$. Mass conserving coordinates were chosen in the vertical, and regular spatial Cartesian coordinate in the horizontal. For practical purposes, the vertical mass conserving
coordinate can be regarded as standard height z, as the differences between the two types of coordinates were found to be insignificant in the present context, and thus chose $\Delta x = \Delta z$. In order to compute $D(APE)$ and $W_{r,\text{mixing}}$ for turbulent conditions, we modelled the stirring process by randomly shuffling the fluid parcels adiabatically from resting initial conditions. Shuffling the parcels in such a way requires a certain amount
of stirring energy, which is equal to the available potential energy APE of the randomly shuffled state.

4 Results

For any particular reference stratification constructed, several hundreds of random permutations were generated in order to shuffle the fluid parcels adiabatically, each yield-
ing a particular value of $D(APE)$ and $W_{r,\text{mixing}}$, associated with a particular value of APE, that were used to plot the former two quantities as a function of the latter one in Fig. 2. The main result is that for all 27 different particular reference stratifications considered, and corresponding hundred of randomly generated permuations of the parcels, the diffusive APE dissipation term $D(APE)$ was found to be well approximated
by a linear function of APE represented in the top panel of Fig. 2 as a thick red line. With regard to $W_{r,\text{mixing}}$, i.e., the rate of change of GPE_r, it is depicted in both panels of Fig. 2 as crosses, with the top panel being restricted to the particular stratifications for which $W_{r,\text{mixing}}$ is positive, and with the bottom panels including all cases considered. What these results show is that for each particular stratification considered, $W_{r,\text{mixing}}$
appears to be either an increasing or decreasing function of APE, depending on the overall behavior of the parameter $\alpha P/(\rho C_p)$ depicted in Fig. 1. Unlike $D(APE)$, which appears to be a linear function of APE, the dependence of $W_{r,\text{mixing}}$ upon APE appears

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to be nonlinear in general. As discussed in Tailleux (2008), $W_{r,\mathrm{mixing}}$ is expected to be positive only when the vertical gradient of $\alpha P/(\rho C_p)$ is negative, which is obviously the case when the vertical variations of $\alpha/(\rho C_p)$ are negligible compared to the vertical variations of pressure if the latter is hydrostatic.

5   The case when the vertical gradient of $\alpha P/(\rho C_p)$ is positive was extensively discussed by Fofonoff (1962, 1998, 2001), and can be easily encountered in the oceans. Based on Fig. 2, $W_{r,\mathrm{mixing}}$ appears to be always smaller than $D(APE)$, which was already pointed out by Tailleux (2008) as implying that values of mixing efficiency based on measuring GPE change are likely to underestimate the latter.

10   One of the most important points of these results is that there exist stratifications for which the GPE_r turbulent rate of change $W_{r,\mathrm{turbulent}}$ is indeed very close to the APE dissipation rate $D(APE)$, as is expected from the study of the Boussinesq fluid with a linear equation of state, but that there also exist stratifications for which $W_{r,\mathrm{turbulent}}$ and $D(APE)$ are radically different. The existence of the first case makes it acceptable
15 to measure the mixing efficiency by measuring the rate of change of GPE_r over a turbulent mixing event, as is often done in laboratory experiments. Note, however, that in order for this to be physically meaningful, one needs to make sure that the particular profile chosen is expected to correlate well with the dissipation rate of APE. So far, such checks have not been done, for the general expectation, based on the use of a
20 linear equation of state, was that the rate of GPE_r increase and APE dissipation were always correlated. Again, we need to stress that this will not generally be the case except in very special circumstances, as illustrated by the results of Fig. 2.

5   Conclusions

The main result of this paper is to suggest that the nonlinearities of the equation of state
25 are critically important to understand the behavior of the rate of GPE_r change $W_{r,\mathrm{mixing}}$, while playing only a minor role to understand that of the APE dissipation rate $D(APE)$. This further confirms the conclusions of Tailleux (2008) that $D(APE)$ and $W_{r,\mathrm{mixing}}$ represent fundamentally two distinct measures of irreversible diffusive mixing, with the

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former representing the dissipation rate of APE into IE_0, and the latter representing the conversion rate between GPE_r and IE_{exergy}, and not the same kind of conversion associated with the irreversible conversion of APE into GPE_r, in contrast to what is usually assumed in the turbulent mixing literature, e.g., Winters and al. (1995); Peltier and Caulfield (2003). Because $D(APE)$ and $W_{r,mixing}$ represent fundamentally two distinct types of energy conversion, it follows that the fraction of the stirring mechanical energy that is eventually dissipated by molecular diffusion is associated with $D(APE)$, and not with $W_{r,mixing}$, so that the correct definition of mixing efficiency is given by Eq. (15) and not by Eq. (16).

If one agrees to adopt the definition Eq. (15) above, then the present results suggest that the nonlinearities of the equation of state of seawater only weakly affect mixing efficiency, whereas they are a leading order factor on the rate of change of GPE_r, in line with the results of Fofonoff (1998, 2001). This result is very important from the viewpoint of the observational determination of mixing efficiency based on definition Eq. (16). This is because, according to the present results and those of Tailleux (2008), it appears that $W_{r,mixing}$ always systematically underestimates $D(APE)$. As a result, any observational determination of mixing efficiency based on measuring the rate of change of mean GPE variations is bound to underestimate, possibly by a large factor, the actual value of mixing efficiency as determined from definition 1). As shown here, this approach is meaningful only in the cases for which it can be established that $D(APE)$ and $W_{r,mixing}$ are strongly correlated, as can be sometimes the case, see the top panel of Fig. 2 for particular examples. The present results motivate a re-examination of published values of mixing efficiency based on definition Eq. (16), which we hope to report in a subsequent paper. So far, such test has never been carried out as far as we can judge, for the assumption of equivalence between the definitions Eqs. (15) and (16) has never been questioned before. Hopefully, the present results will provide a sounder physical basis for understanding the very important concept of mixing efficiency, and how to measure it in the laboratory and in the oceans.

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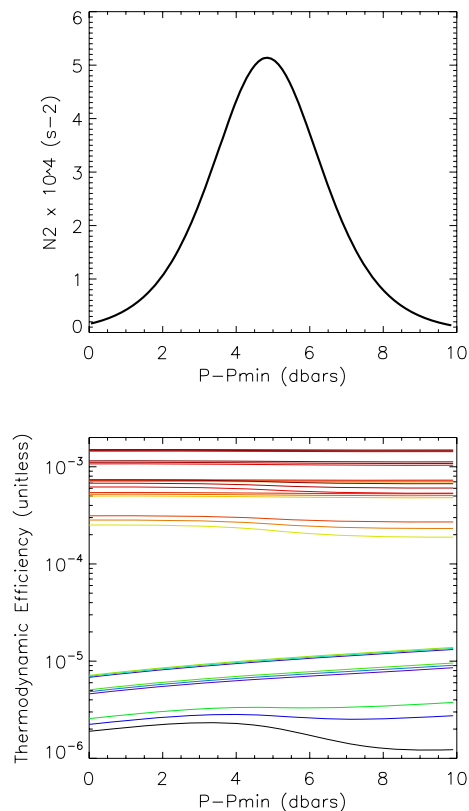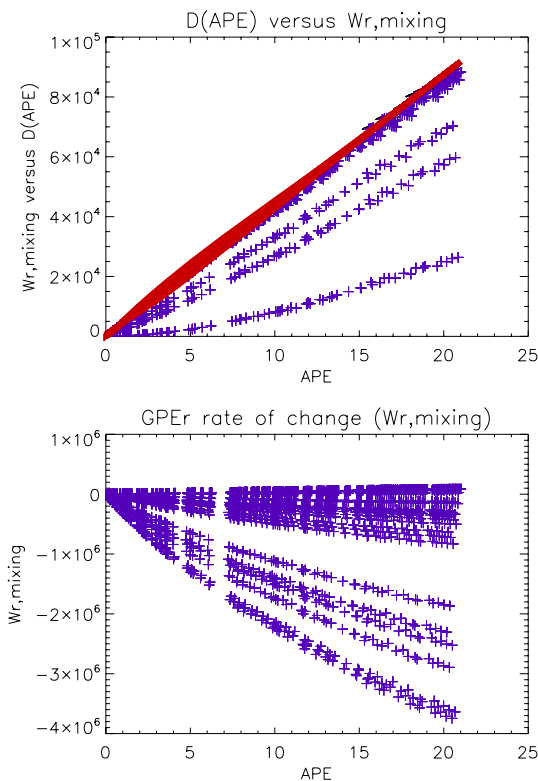
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Fig. 1. (Top panel) The squared buoyancy frequency N^2 common to all stratifications considered. (Bottom panel) The thermodynamic efficiency-like quantity $\alpha P/(\rho C_p)$ corresponding to the 27 different cases considered. Note that the Fofonoff regime, i.e., the case for which GPE decreases as the result of mixing, is expected whenever the latter quantity decreases for increasing pressure. The classical case considered by the literature, i.e., the case for which GPE increases as the result of mixing corresponds to the case where the latter quantity increases with increasing pressure.

Fig. 2. (Top panel) The dissipation rate of APE as a function of APE. Since all the points were lying along a straight line, a linear regression was superimposed (thick red line). The crosses represent the rate of change $W_{r,\text{mixing}}$ of GPE for the particular experiments for which the rate of change of GPE was positive. (Bottom panel) The rate of change $W_{r,\text{mixing}}$ of GPE as a function of APE for all cases considered. Units are arbitrary but the same in both panels.