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Interactive comment on "The open boundary equation" *by* D. Diederen et al.

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Reply to requests by Referee #2, 20 August 2015

(see also the attached file 'os-2015-29-supplement.pdf')

Referee #2: "I have read SC C558. I understand that the authors have now used a purely exponential width variation (Fig. 1), that is $B_\infty = 0$. Am I right?"

Reply: We used a very small asymptotic value of the width, $B_\infty = 10^{-3}$ m, in the numerical runs we presented. We also note that in these runs we approximated the hydraulic radius with the flow depth in the friction term to focus only on the effect of convergence.

Referee #2: "I have a number of questions/requests which are all related to the validity of $\zeta_t u_x = u_t \zeta_x$. The questions/requests regarding the case covered in SC C558 are: 1. How are the bottom panels of Fig. 2 obtained? After all, $u_t\zeta_x/\zeta_t u_x$ depends on time (see bottom panels Fig. 3 and 4)."

Reply: Indeed, the two sides of equation 22 vary with time. For this reason we averaged them following the procedure described below, which applies to the last simulated tidal cycle, when regime conditions are achieved.

- The products $u_t\zeta_x$ and $\zeta_t u_x$ are calculated in each computational step.
- Their mean, variance and covariance are computed referring to the tidal cycle.
- The correlation coefficient is calculated using mean, variance and covariance.
- The ratio between the two sides of the equation is calculated using the means.

Referee #2: "2. could the authors include plots of $|u_t\zeta_x/\zeta_t u_x - 1|$ and $^{10}\log|u_t\zeta_x/\zeta_t u_x|$ regarding the bottom panels in Fig. 2?"

Reply: Following the Referee's suggestions (see the last point below), we ran a set of new simulations. For each case, in figure 2 we added an additional row of plots. Now the second row shows $|R-1|$ and the third row $\log_{10}(|R-1|)$, where

$$R = \frac{\overline{u_t\zeta_x}}{\overline{\zeta_t u_x}} \tag{1}$$

is calculated as explained in the previous point (R in equation 1 refers to equation 22 in the manuscript, the other equations 26 and 27 are treated similarly).

Referee #2: "3. Regarding Figs. 3-8, please plot $|u_t\zeta_x/\zeta_t u_x - 1|$ together with $|u_t\zeta_x|$ and $|\zeta_t u_x - 1|$ (use double y axis for this, e.g. Matlab's plotyy). Also plot $^{10}\log|u_t\zeta_x/\zeta_t u_x|$."

Reply: Figures 3-8 are now grouped as figures 3-5. Now they show five lines instead of four. The last lines reports the temporal variations of the following quantities:

- $\log_{10}(|\zeta_t u_x|/M])$,
- $\log_{10}(|u_t\zeta_x|/M)$,
- $\log_{10}(|R(t)-1|)$,

where

$$R(t) = \frac{u_t\zeta_x}{\zeta_t u_x}\,, \tag{2}$$

$$M = \left(\overline{\zeta_t u_x} + \overline{u_t\zeta_x}\right)/2\,. \tag{3}$$

The scaling factor M is introduced in order to make the plotted quantities of similar order of magnitude.

Referee #2: "Next, I would like to obtain Figs. 2-11 (with the presentation modifications mentioned above) for the following variants to the case described in SC C558: 1. $b = 25$ km instead of $b = 100$ km, 2. $b = 300$ km instead of $b = 100$ km, 3. $K = 20$ m$^{1/3}$ s^{-1} instead of $K = 45$ m$^{1/3}$ s^{-1}, 4. $K = 80$ m$^{1/3}$ s^{-1} instead of $K = 45$ m$^{1/3}$ s^{-1}. By the way, are the authors able to run their case in SC C558 with linear bottom friction (i.e. $W = ru = h$, with r constant)? If so, please do!

Reply: We analysed the 5 variants as required by the Referee, together with the reference case and an additional variant of the strongly convergent case:

1. reference case: $b = 100$ km, $K = 45$ m$^{1/3}$ s^{-1}, nonlinear friction, $\gamma = 0.70$, $\chi = 3.17$;
2. variant 1: $b = 25$ km, $K = 45$ m$^{1/3}$ s^{-1}, nonlinear friction, $\gamma = 2.82$, $\chi = 3.17$;

3. variant 2: $b = 300$ km, $K = 45$ m$^{1/3}$ s^{-1}, nonlinear friction, $\gamma = 0.23$, $\chi = 3.17$;
4. variant 3: $b = 100$ km, $K = 20$ m$^{1/3}$ s^{-1}, nonlinear friction, $\gamma = 0.70$, $\chi = 16.0$;
5. variant 4: $b = 100$ km, $K = 80$ m$^{1/3}$ s^{-1}, nonlinear friction, $\gamma = 0.70$, $\chi = 1.00$;
6. variant 5: $b = 100$ km, $K = 45$ m$^{1/3}$ s^{-1}, linear friction, $\gamma = 0.70$, $\chi = 3.17$;
7. variant 6: $b = 25$ km, $K = 45$ m$^{1/3}$ s^{-1}, nonlinear friction, asymptotic width $B_\infty = 10$ m, $\gamma = 2.82$, $\chi = 3.17$.

In the list, we also report the value of the reference values of the dimensionless convergence and friction parameters, γ and χ (see, e.g., Toffolon and Savenije, 2011). The results are shown in the attached file 'os-2015-29-supplement.pdf'.

Concerning 'variant 5' (linear friction), we note that a slight deformation of the tidal wave is visible in the case with linear friction: this is obviously the result of the other nonlinearities that are present in the Saint Venant equations

The case 'variant 1' is strongly convergent, so the width becomes soon unrealistic small. To evaluate the effect of the asymptotic width, we added the case 'variant 6' with a larger B_∞. The Lagrangean analysis (figures 9-11) of these two cases was done considering the starting point $x_0 = 10$ km instead of $x_0 = 100$ km, which would have been in the region with asymptotic width.

The strongly convergent case 'variant 1' is particularly challenging for the validity of the Open Boundary Equation (OBE). In fact, 'variant 1' can be classified as an overamplified estuary (see figure 15 in Cai et al., 2012), which corresponds to the case of supercritical convergence $\gamma > 2$ defined in frictionless channels (e.g., Toffolon and Savenije, 2011). Tide propagation in this type of strongly convergent channels tends to respond as an apparent standing wave, so this is a sort of 'crash test' for the Lagrangean progressive character and the OBE. The analysis of wave propagation (see the attached document) reveals that the correlation is quite high for such a demanding test, although the correspondence of the two sides of OBE is not as good as in the other cases. However, better results are obtained in the more realistic case 'variant 6'. Finally, we remark that the progressive character of wave propagation in the Lagrangean framework is evident also in this strongly convergent case (figures 10 and 11), providing an additional element for the validity of the proposed approach.

References

Cai, H., Savenije, H. H. G., and Toffolon, M.: A new analytical framework for assessing the effect of sea-level rise and dredging on tidal damping in estuaries, J. Geophys. Res., 117, C09023, doi:10.1029/2012JC008000, 2012.

Toffolon, M. and Savenije, H. H. G.: Revisiting linearized one-dimensional tidal propagation, J. Geophys. Res., 116, C07007, doi:10.1029/2010JC006616, 2011.

Please also note the supplement to this comment:
http://www.ocean-sci-discuss.net/12/C588/2015/osd-12-C588-2015-supplement.pdf

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Interactive comment on Ocean Sci. Discuss., 12, 925, 2015.