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Interactive comment on "The open boundary equation" *by* D. Diederen et al.

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Some qualitative considerations on the effect of the boundary condition and on the tendency towards the proposed equation

This reply aims at integrating the previous response to Referees by providing an analogy to a simpler problem (advection-diffusion of a tracer) in which the forcing imposed by the boundary condition is progressively altered by the governing differential equation and eventually tends to a well-known solution.

Nothing is novel in the following considerations, but they might support the interpretation of the proposed 'open boundary equation' in the correct conceptual framework.

On the effect of the seaward boundary condition

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In general, the boundary conditions determine the actual solution of a differential problem. In the case of the Saint Venant equations, the forcing imposed at the seaward boundary of the computational domain transfers information from that boundary to the interior. In particular, the tidal wave originated by the water level oscillations at the sea propagates along the channel and modifies its shape according to the governing differential equations. Hence, the wave functions of flow depth h (or water level) and velocity u are the result of the interaction between the information coming from the boundary condition and the modifications actually produced by the mass and momentum balances.

If the water level is imposed at $x = 0$, the wave function $h(x,t)$ will be initially affected by the boundary condition, but after some time (and space) h will be modified and the wave distorted, e.g. with the generation of overtides due to the non-linear terms in the governing equations, the damping of high-frequency variations, or the steepening of the front. The velocity is obtained using the information coming from h, so the shape of wave function $u(x,t)$ is initially affected by the boundary condition as well.

After some time and space (which we could term ‘adaptation’ time and length, as usually done in other contexts), the direct influence of the boundary condition on the wave's shape disappears and the wave will attain a sort of ‘dynamic equilibrium’ shape that is primarily determined by the governing equations. This is the point, in our view, where the ‘open boundary condition’ becomes valid.

Analogy with the advection-diffusion process

An analogy with the advection-diffusion equation (ADE) can help in understanding the process. Let us consider a simple one-dimensional, constant coefficient ADE:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = D\frac{\partial^2 f}{\partial x^2}\,, \tag{1}$$

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where f is the concentration of a passive tracer, u is the flow velocity, and D the diffusion coefficient.

The one-dimensional solution for a instantaneous point source of a mass M is

$$f = \frac{M}{\sqrt{4\pi Dt}} \exp\left[-\frac{(x-ut)^2}{4Dt}\right] . \tag{2}$$

If the same mass is initially injected in a reach of the channel having length L_0, then the solution is

$$f = \frac{f_0}{2}\left[\text{erfc}\left(\frac{x-ut-L_0/2}{\sqrt{4Dt}}\right) - \text{erfc}\left(\frac{x-ut+L_0/2}{\sqrt{4Dt}}\right)\right] , \tag{3}$$

where $f_0 = M/L_0$ is the initial concentration of the tracer, which is distributed as a square wave ($f = f_0$ is constant along L_0, and null elsewhere).

Figure 1 in this reply shows the spatial distribution of the concentration $f(x)$ at different times, as originated by the two initial conditions (the Dirac function for the point source, and the square wave), for $M = 1$, $u = 1$, $D = 0.1$, and $L_0 = 2$ (in dimensionless terms). It is clear that the initial difference between the two solutions (2) and (3) vanishes after some time, eventually leading to the same Gaussian shape.

A similar result can be obtained using a square wave of duration $T_0 = L_0/u$ as boundary condition at $x = 0$ (instead of as an initial condition as in the previous case), for which the solution is only slightly different from equation (3). Figure 2 shows the temporal behaviour $f(t)$ in different positions.

Conclusions

Extending the qualitative behaviour of the examined case to the hydrodynamic equations, we argue that the tidal wave similarly tends to an asymptotic condition, in which

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the propagation of the h and u signals will adapt to a more general equation,

$$\frac{h_x}{h_t} = \frac{u_x}{u_t} \tag{4}$$

that we termed ‘open boundary equation’, in an analogous way as the distribution of a tracer tends to become normally (Gaussian) distributed. For the tidal wave the shape is not always the same (there is not a ‘normal’ tidal wave), of course, but the celerities of the propagation of h and u become intrinsically correlated if there is no reflection from a landward boundary, as in the case of an infinite channel.

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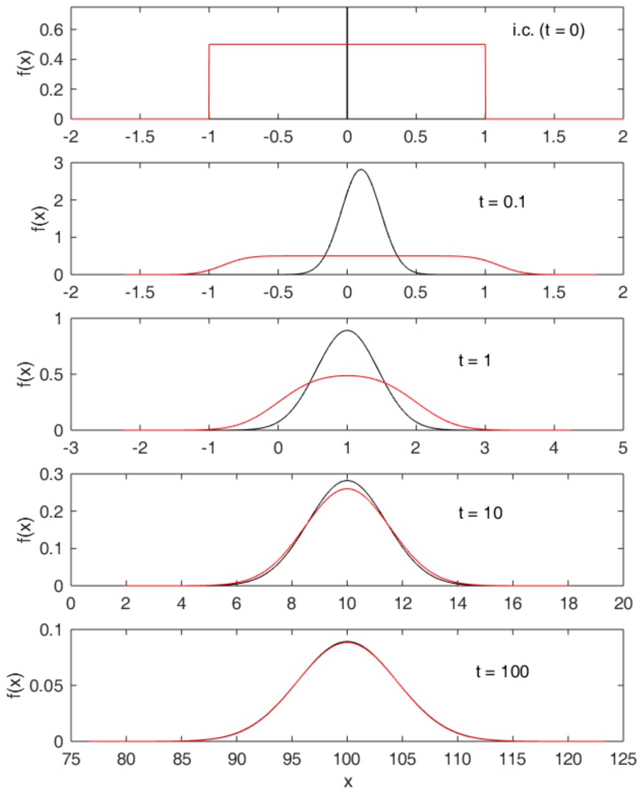

Fig. 1. Spatial distribution at different times.

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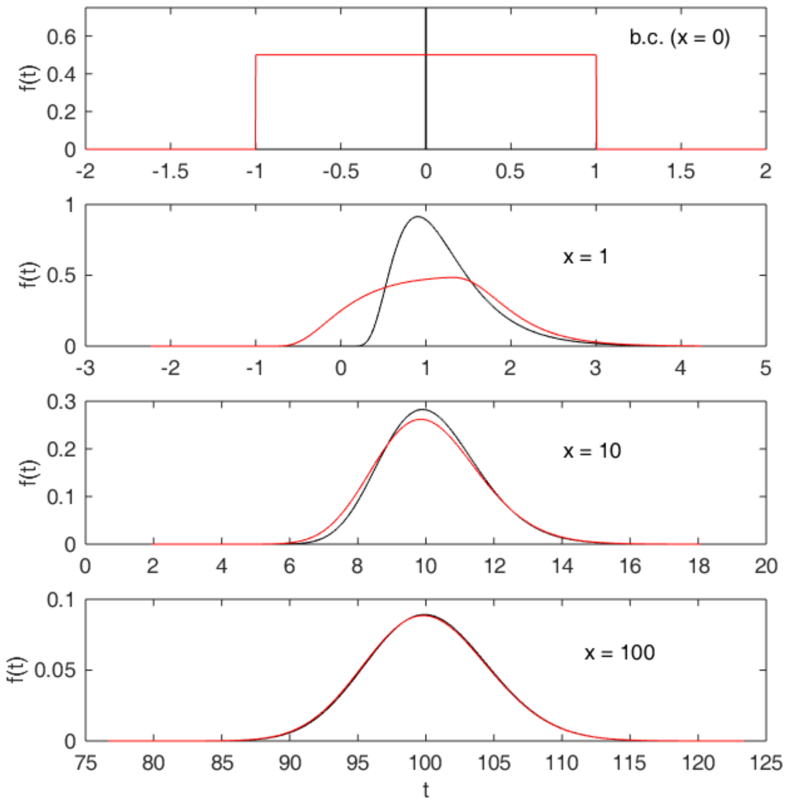

Fig. 2. Temporal variation in different positions.