Ocean Sci. Discuss., 12, C455–C467, 2015
www.ocean-sci-discuss.net/12/C455/2015/

Ocean Science
Discussions
Open Access

Interactive comment on "The open boundary equation" *by* D. Diederen et al.

D. Diederen et al.

d.diederen@student.tudelft.nl

Received and published: 26 July 2015

We'd like to thank Anonymous Referee 2 for the review. First we give a summary of the reply, than we answer point-by-point.

Summary

We have tried to formulate our 'claim' for the existence of an additional equation (22) as careful as possible. In the article we indicate that the numerical analysis of convergent and frictional estuaries by definition has (numerical) errors, so that it cannot be used as conclusive evidence (page 934, line 10). The only solution that can be used as a full reference test is an exact, analytical solution of the 'Saint-Venant' equations, which does not (yet) exist and may not exist for some time to come or may never exist. However, we will further discuss the counterexample presented and we will show that it does not agree with the numerical solution of Eqs. (1) and (2), including scenarios with small amplitude-to-depth ratios. We will further argue that the Lagrangean phase lag analysis is an addition to existing methods. Finally, we will argue that the application by Gosh can not be compared to the applications in Sect. 6.

4 Major objections

4.1 The analysis regarding Eqs. (26) and (27)

‘I am completely at a loss as to why to use/present/discuss Eqs. (26) and (27). The authors want to check the validity of Eq. (22) and this can be done easily (at least in principle) directly (as they attempt with Fig. 4). Why they instead want to consider two very nonlinear equations completely escapes me.’

In order to understand why checking the validity of Eqs. (26) is equally direct as checking Eq. (22), we rewrite the (forced) governing equations (1) and (2) as:

$h_t u_x + u h_x u_x + h u_x^2 - \beta u h u_x = 0,$

$u_t \zeta_x + u u_x \zeta_x + g \zeta_x^2 + W \zeta_x = 0.$

These two equations can be combined as:

$(\zeta_t u_x) + (h u_x^2 - \beta u h u_x) = (u_t \zeta_x) + (u u_x Z_x + g \zeta_x^2 + W \zeta_x),$

since $h_t = \zeta_t$ and $\zeta_x = h_x + Z_x$.

Now it can be observed that, with the forcing of Eqs. (1) and (2), if Eq. (22) applies so does Eq. (26) or vice versa, if Eq. (26) applies so does Eq. (22). The reason to show a validity check of Eq. (26) in Fig. 2 is that it holds the (relatively small) terms containing friction and convergence of width and depth. From Fig. 2 it can be observed that none of these terms can be neglected (see also the reply to Referee 1), so that it appears that Eq. (26) equates exactly (at this location in this scenario), and therefore also Eq. (22) equates exactly.

The same principle applies to Eq. (27). Time series of the ‘Tidal Rating Curve’ (TRC), which follows from Eq. (27) (as it is an ordinary differential equation in time), are

available in the supplementary material in the reply to Referee 1.

‘The agreement shown in Fig. 2 does not impress me in view of the large scatter in Fig. 4 (see below), that is: Fig. 2 may simply refer to a case for which (22) happens to hold to some extend.’

To remove the impression that Fig. 2 only shows good agreement for a single case, in the supplementary material we show time series of evaluations of Eq. (26) for 100 scenarios, within the parameter range indicated in the article.

‘The discussion of the Pearson correlation coefficient is not relevant either: one wants to check an exact equality, say "A = B", not whether A and B correlate. Obviously A = sin(t) and B = 0.001 sin(t) correlate perfectly but they are clearly unequal.’

It is a fair point that the linear correlation may correlate a left hand side and a right hand side which differ by a constant factor as $y = ax$. However, this problem is avoided by using the correlation analysis (Fig. 3) in combination with the parity analysis (Fig. 4), where the left hand side is directly plotted against the right and where both sides are scaled by the same scaling factor F_{sc}. From Fig. 4 it can be observed that the points (closely) line up on $y = x$, so that $a = 1$.

4.2 The validity of Eq. (22)

4.2.1 Problems with the presented evidence for Eq. (22)

'Suggesting that the huge scatter is due to numerical errors does not mean that Eq. (22) holds for those cases and does not release the authors from the responsibility to demonstrate the validity of Eq. (22) explicitly.'

For detailed analysis, we have investigated time series (like in Fig. 2, see supplementary material), since this gives a better view on the behavior of the equations. Fig. 4 is intended as a summary, since the results of 100 scenarios can be plotted in one figure. However, to show where the scatter in Fig. 4 comes from, we have added parity plots of Eq. (22) for 100 scenarios, at different locations in the estuaries. Spatial correlation overviews (like Fig. 3) can be found in the 'TRC-figures' in the supplementary material of the reply to Referee 1, where the same scenarios were analyzed.

4.2.2 Examples discussed in Appendix A

'Regarding the examples discussed in the Appendix A I point out that (A1,2) actually refer to solutions which are constant on characteristics $\phi(x,t) = const$. In that case Eq. (22) always holds and in fact, I think this is the only case in which it is true. I think that Eq. (22) is merely a mathematical reformulation of the existence of invariants for a specific case rather than a physics based law with general validity. I don't think it holds for propagating tidal waves that have a spatial variation of amplitude. If the authors think otherwise, they should come up with a clear example for which Eq. (22) holds and discuss it in detail. I don't think that Eq. (22) holds for the case presented in A2 as Eqs. (A5) and (A6) actually constitute a counterexample (see below).'

As correctly rephrased by Referee 1, 'the Appendix shows that some approximate analytical progressive wave solutions also satisfy eq(22)'. What the examples have in common is that the dynamics of the argument $\phi(x,t)$ are the same for the water level ζ and the velocity u. This has nothing to do with characteristics. In fact, example A2 includes varying amplitudes, which implies that the characteristics are not constant. The assumption of proportionality is used here, which has limitations since it is a (rough) approximation, especially if the phase lag is large. Therefore this analytical approximation is in the appendix.

4.2.3 Counterexample

The counterexample derived by Referee 2 yields an additional term to Eq. (22) in the right hand side:

$\zeta_t u_x - u_t \zeta_x = -\frac{4\pi^2\mu}{k}\frac{\eta^2}{\overline{h}_0 T^2}\exp(-2\mu x)\sin(\phi).$

The additional term contains $\mu, k, \eta, \overline{h}_0, T, \phi$, of which none varies in time. Therefore we have added time series of $\zeta_t u_x - u_t \zeta_x$ in the numerical approximations to the supplementary material. It can be observed that the additional term in the numerical approximations (which may be the numerical error) shows a time varying behavior. Therefore the numerical approximations do not agree with the analytical approximation which is used as a counterexample. Overview images of the wavelike behavior of the 'additional term' $\zeta_t u_x - u_t \zeta_x$ in the numerical approximations can be found in the 'ErrorImages' in the supplementary material of the reply to Referee 1.

Finally, it is claimed that the counterexample applies for small amplitude-to-depth ratios. Scenarios (21, 24, 65, 74 and 98) have an amplitude-to-depth ratio that is smaller than 2.5 percent.

4.3 Phase difference between horizontal and vertical tide (Sects. 1 and 5)

'I don't think that Lagriangian phase lags translate into Eulerian ones in a straightforward way, certainly not for non-linear tides. .. Here I would point out the fact that there is a clear example where Eq. (22) will certainly not hold for an infinitely long channel either, namely the case of a linear tidal wave in an exponentially converging channel without friction.'

The Eulerian and the Lagrangian perspective are fundamentally different. They complement each other and cannot be seen as competitors. Therefore, the Lagrangean phase lags do not necessarily have to be translated to Eulerian phase lags, or vice versa.

However, when we consider tidal waves (which all are non-linear), the tidal excursion, which is the distance water particles travel up and down the estuary in a tidal period, is in the order of 5-20 km, whereas the tidal wave length is in the order of 100-1000 km. Therefore, although the shape of Eulerian and the Lagrangean tidal curves will show some (important) differences, the timing between water level and velocity will show similar behavior. Analysis of the Lagrangean phase lag can be seen as an addition to the analysis of the Eulerian phase lag, and may be compared.

Assignment of a 'wave character' may be seen as semantics. However, the timing between water level and velocity may be used in the discussion of the wave character, and subsequently the wave character may be used to draw conclusions about the occurrence of reflection. Since Eq. (22) will not apply in the case of (classical) reflection, it confirms the distinction between 'apparent reflection' and 'classical reflection', which is already made in the literature.

4.4 Applications (Sect. 6)

‘Explicitly, if water levels and the estuary‘s geometry (i.e. B(x) and Z(x)) are known there is a far more powerful method to obtain velocities, namely the cubature method (e.g. ()Gosh 1998). .. From this discharge and width averaged velocity are readily derived provided the discharge is known somewhere in the estuary (e.g. upstream river discharge).’

- The ‘cubature method’ may be used if the river discharge is known at some location. This is not necessary for the applications in Sect. 6. Therefore it is not a comparable application.
- By only using the mass balance, the ‘cubature method’ will show larger errors farther away from the location where the discharge is measured, since it will use numerical approximation of the ‘exact’ mass balance.
- The measurement of the discharge will contain measurement errors.
- Many estuaries in the world are ungauged.
- Finally, the applications in Sect. 6 also include the mass balance Eq. (1). Additionally, they include the momentum balance Eq. (2) and the additional open boundary Eq. (22).

5 Minor issues

- ‘Sect. 2.3: what are the boundary conditions that are used here, in particular at $x = \infty$? If one assumes only a landward propagating wave, doesn‘t this already imply that R2 = 0 as one does not want information to travel seaward? Please clarify this.’

  For this analysis no landward boundary is required, since the domain is infinitely long in landward direction, which implies that there is no landward boundary. For a numerical simulation (which will not work because numerical simulations require some friction) a perfect open boundary would be that $R_2(L, t)$ is constant of which $R_2(L, t) = 0$ is a single example.

- ‘Sect. 3.2, lines 19-20 on pg 935: "The seaward boundary ... wave to adjust". This sounds strange. Isn‘t the boundary condition at the seaward side something that can be accurately imposed numerically? The solution near x = 0 may have to adjust in time (depending on the initial condition) but not in space. I don‘t expect any effect from the reflected wave here as I expect it to have decayed. For the landward boundary this may indeed be different due to reflection.’

  By imposing a boundary condition seaward, a part of the solution is imposed at this location. As long as we do not have an exact solution, we can only speculate about the right harmonic equation to use. We do know, however, that a pure sinus is not a correct solution of the (non-linear) ‘Saint-Venant equations’. Not only is the friction term determined by a quadratic velocity, there is also the hydraulic radius in the denominator. So even if we linearize the quadratic velocity, then the friction term is still non-linear for a finite amplitude. A wrongly imposed downstream boundary will therefore result in a compensation in other variables (resulting in erroneous velocity and phase lag). It appears that the error introduced by the (wrong) seaward boundary condition requires some length to adjust in the upstream direction. What are precisely the mechanics of adjustment, and the adjustment length required is not easily answered. Fact is that a pure sinus imposed at the downstream boundary will become gradually more deformed until a stable pattern between velocity, water level and phase lag is achieved. We hypothesize that the new equation describes the stable relationship between velocity and water level.

- 'lines 3-7 on pg. 937, Fig. 5. I think cases B, C and D are only relevant for transient behaviour, not for the long term (purely periodic) time behaviour. I think the authors interest is with this latter case.'

  If the domain is infinitely long, there will always be cases like B, C and D in the domain. Therefore they are relevant.

- 'Sect. 5.2, Fig. 7. How is an "ideal estuary" defined for the present case with nonlinear bottom friction? Is this in a time averaged sense? If so, this is effectively the same as adopting Lorentz linearization of the friction. Please clarify.'

  In our definition, the 'ideal estuary' is a theoretical case in which the phase lag and amplitudes remain constant in space (see page 947 line 2-4). It was shown

by Cai and Savenije (2013) that this is the asymptotic solution of tidal waves in infinite estuaries. It has nothing to do with linearized friction.

Cai, H. and H.H.G. Savenije. Asymptotic behavior of tidal damping in alluvial estuaries, Journal of Geophysical Research: Oceans, 118, 1–16, doi:10.1002/2013JC008772, 2013.

Please also note the supplement to this comment:
http://www.ocean-sci-discuss.net/12/C455/2015/osd-12-C455-2015-supplement.zip

---

Interactive comment on Ocean Sci. Discuss., 12, 925, 2015.