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Multivariate extreme value analysis of storm surges in SCS on peak over threshold method

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Abstract

We use a novel statistical approach-MGPD to analyze the joint probability distribution of storm surge events at two sites and present a warning method for storm surges at two adjacent positions in Beibu Gulf, using the sufficiently long field data on surge levels at two sites. The methodology also develops the procedure of application of MGPD, which includes joint threshold and Monte Carlo simulation, to handle multivariate extreme values analysis. By comparing the simulation result with analytic solution, it is shown that the relative error of the Monte Carlo simulation is less than 8.6 %. By running MGPD model based on long data at Beihai and Dongfang, the simulated potential surge results can be employed in storm surge warnings of Beihai and joint extreme water level predictions of two sites.

1 Introduction

There have been significant advances in the modelling procedures available for multivariate extreme values. In particular, the research about application of MGPD (Multivariate Generalized Pareto Distribution) draws more and more attention. MGPD, as the natural distribution of MPOT (Multivariate Peak Over Threshold) sampling method, has the extreme value theory background and can retain more extreme information from the raw data than the annual maxima of a series.

In multivariate extreme values analysis, two sampling approaches have been advocated, which are called Annual Maxima Series (AMS) method and MPOT method respectively. The MGEVD (Multivariate Generalized Extreme Value Distribution) is the natural distribution of AMS, in which the sample is consist of the annual maxima of all components (Morton and Bowers, 1996; Sheng, 2001; Yang and Zhang, 2013). However, Zaijin You and Baoshu Yin (2006) propose, taking the extreme waves estimation as an example, AMS method often ignores multiple severe storm waves that occur in the same year, which may be much larger than the annual largest waves in

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many other years. Consequently, this method may result in underestimation of extreme variables. MGPD is the natural distribution of MPOT method, in which the sample is consist of independent exceedances of a suitably high threshold for all components (Falk et al., 2004). Obviously, MPOT can retain more independent extreme values from the raw data than AMS, and the additional data would likely lead to greater estimation precision (Luo and Zhu, 2014). Besides, the fluctuation of the estimation of the extreme waves by POT is smaller than by AMS under different sample lengths (Luo et al., 2012).

MGPD and MPOT method are widely used recently. Rootzén and Tajvidi (2006) suggest, based on the idea of Tajvidi (1996), that MGPD should be characterized by the following couple properties: (i) exceedances (of suitably coordinated levels) asymptotically have a MGPD if and only if componentwise maxima asymptotically are EVD, (ii) the MGPD is the only one which is preserved under (a suitably coordinated) change of exceedance levels. Morton and Bowers (1996) are based on the response function with wave and wind speed of anchoring semi-submersible platforms enabling to analyze extreme anchorage force and corresponding wave height and wind speed by using logical extreme value distribution. But the study did not use the natural distribution MGPD of the MPOT method but MGEVD for fitting the MPOT samples. Coles and Tawn (1994) and Bhunya et al. (2011) used the same mind too. More details about MGPD can be found in Rootzén and Tajvidi (2005), Tajvidi (1996), Beirlant et al. (2005) and Falk et al. (2004).

In the paper, our aim is to develop procedures of MGPD and to demonstrate how the methodology can be exploited as part of the analysis of extreme surges at two adjacent sites. The theory and associated statistical methodology is presented in Sect. 2. Fundamental to the application of MGPD is the choice of the joint threshold and the estimation of the joint density. These aspects included in an example are discussed in Sects. 3 and 4. Finally, the advantage of MPOT and new possibilities based on Monte Carlo simulation are outlined.

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2 Methodology

2.1 MGPD theory

It is well known that MGEVD (Coles and Tawn, 1991, 1994; Beirlant et al., 2005) arise, like in the univariate case, as the limiting distributions of suitably scaled componentwise maxima of independent and identically distributed random vectors. If for independent $X_1, \ldots, X_n$ following F, there exist vectors, $a_n, b_n \in \boldsymbol{R}^d, a_n > 0$, such that

$$P\left(\frac{\max_{i=1,\ldots,n}(X_i - b_n)}{a_n}\right) = F^n(a_n x + b_n) \overset{n\to\infty}{\longrightarrow} G(x) \tag{1}$$

where $G(x)$ is a MGEVD, and F is in the domain of attraction of G. We note this by $F \in D(G)$.

In the paper, we stick to the MGPD definition of Falk et al. (2004). Similarly to the relationship of GPD and GEVD in one dimension: $H(x) = 1 + \log(G(x)), \log(G(x)) > -1$, the distribution function of MGPD can be deduced:

$$\begin{aligned} W(X) &= 1 + \log(G(x_1, \ldots, x_d)) \\ &= 1 + \left(\sum_{i=1}^{d} x_i\right) D\left(\frac{x_1}{\sum_{i=1}^{d} x_i}, \ldots, \frac{x_{d-1}}{\sum_{i=1}^{d} x_i}\right), \qquad \log(G(x_1, \ldots, x_d)) > -1 \end{aligned} \tag{2}$$

where $(x_1, \ldots, x_d) = x \in U$, U is a neighborhood of zero in the negative quadrant $(-\infty, 0)^d$, D is the Pickands dependence function in the unit simplex $\overline{R_{d-1}}$ on the domain of definition, $\overline{R_d} = \{x \in [0, \infty)^d | \sum_{i=1}^{d} x_i = 1\}$; $G(x_1, \ldots, x_n)$ is a MGEVD function which marginal distribution is negative exponential distribution (detail in René, 2007). The MGPD can use existing multivariate extreme dependence function directly because it is deduced from MGEV, and this enrichs greatly the expression of MGPD for various dependence relationships. MGPD of logistic type is ease to use and has the favorable statistical properties from Pickands dependence function Eq. (3), and widely

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used to hydrology, financial and other fields.

$$D_r(t_1,\ldots,t_{d-1}) = \left(\sum_{i=1}^{d-1} t_i^r + \left(1-\sum_{i=1}^{d-1} t_i\right)^r\right)^{1/r} \tag{3}$$

$$W_r(x) = 1-\left(\sum_{i=1}^{d}(-x_i)^r\right)^{1/r} = 1-\|x\|_r, \tag{4}$$

where r is the correlation parameter of dependence function and $r > 1$. x_i in the interval $(-1, 0)$ are variables of standardization. The Bivariate Logistic GPD density function is

$$w(x,y) = \frac{\partial W}{\partial x \partial y} = (r-1)(xy)^{r-1}[(-x)^r + (-y)^r]^{1/r-2} \quad x<0, y<0 \tag{5}$$

The correlation parameter r can be evaluated by step by step method: evaluate by using two marginal distributions firstly, and then introduce to MGPD; By the other method, the correlation parameter r can be evaluated by global method. r is estimated directly by using the maximum likelihood for the density function w. The global method evaluated results more reliable due to the final function form are to be concerned, but the processes of evaluate are more complex. The maximum-likelihood function is

$$L(r) = \sum_{i=1}^{n} \ln(w_r(x_i,y_i)). \tag{6}$$

2.2 Simulation method

The Monte Carlo Simulation method of multivariate distribution is relatively complex, because of generating multivariate random and relevant vectors involved. By a transformation method, the variables become independence. And then, every variable is generated a random vector. Finally by the inverse transformation, the random vectors

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of the multivariate distribution are obtained. The simulation method was suggested by René (2007).

Using polar coordinate to demonstrate the simulated method of MGPD better:

$$T_p(x_1,\ldots,x_d) = \left(\frac{x_1}{x_1+\ldots+x_d},\ldots,\frac{x_{d-1}}{x_1+\ldots+x_d},x_1+\ldots+x_d\right) = (z_1,\ldots,z_{d-1},c), \qquad (7)$$

where T_p change vector $(x_1,\ldots,x_d)$ into polar coordinate. $\boldsymbol{C} = x_1 + \ldots x_d$ and $\boldsymbol{Z} = (x_1/\boldsymbol{C},\ldots,x_{d-1}/\boldsymbol{C})$ are radial component and angular component, respectively. They called Pickands polar coordinate.

In the Pickands polar coordinate, $W(X)$ presents different properties. Presume that $(X_1,\ldots,X_d)$ follow multivariate generalized Pareto distribution $W(X)$ and its Pickands dependence function D exists d order differential, define the Pickands density of $H(X)$

$$\phi(z,c) = |c|^{d-1}\left(\frac{\partial^d}{\partial x_1,\ldots,\partial x_d}H\right)T_p^{-1}(z,c) \qquad (8)$$

Presume $\mu = \int_{R_{d-1}}\phi(z)\mathrm{d}z > 0$ and constant $c_0 < 0$ existing in a neighborhood of zero, then the simulation method of MGPD is: (1) generate uniform random numbers on unit simplex $\overline{R_{d-1}}$, (2) generate random vector $(z_1,\ldots z_d)$ based on the density function $f(z) = \frac{\varnothing(z)}{\mu}$ of $Z = (z_1,\ldots,z_{d-1})$ in the Pickands polar coordinate combined with Acceptance–Rejection Method, (3) generate uniform random numbers on $(c_0,0)$, and (4) calculate vector $(cz_1,\ldots,cz_{d-1},c-c\sum_{i=1}^{d-1}z_i)$ which is random vector of satisfy the multivariate over threshold distribution.

The c_0 above is the joint threshold in MGPD method. This paper determines the threshold by using the principle on Coles and Tawn (1994).

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3 The data and declustering

3.1 The data

The data used in this study are provided by The Joint Archive for Sea Level (JASL) of UHSLC (http://uhslc.soest.hawaii.edu/home), which consist of simultaneous hourly sea-level observations at Beihai and Dongfang, which are on the Beibu Gulf coast in SCS (Fig. 1). The data set used extends from June 1975 to December 1997. The data can be used for the analysis of the extreme surge in this study, since hourly sampling sufficiently captures the high water level. Only 0.023 % of the data set was lost in Beihai and 0.173 % of the data in Dongfang was not being used was due to gauge failure or others; And the 201 578 and 201 275 hourly values are yet to be processed at Beihai and Dongfang respectively. The amount of data can ensure enough extreme value information of surge.

3.2 Data analysis and declustering

Beihai city is located on the coast of the Beibu Gulf, which is a semi-closed and shallow bay. Due to special geomorphology, the typhoon surge in Beibu Gulf is violent and might cause floods to the city. The surge levels at a site are defined as the residuals after removal of the astronomically induced tidal component from the sea-level observations. The tidal component is cyclical and does not satisfy the basic hypothesis of random variables. Tidal analysis was undertaken using the method of Godin (1972).

The first stage in an extreme value analysis is declustering: identify a set of independence events. This is done to make adjacent elements of the sample, which consists of the maxima of all events, to be independent of each other. Declustering techniques have been used by Morton and Bowers (1996) and Coles and Tawn (1991), in which the cluster interval are 30 and 40 h respectively. But the feature of storms and surges caused storms every place is different. The declustering method is illustrated in an application to a sequence of surges in Fig. 2. The duration of a typhoon surge in the Beibu

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Gulf is approximately 100 h. The components of each vector are taken as the maximum surge at each site over a 100 h event. The peak events for both vectors in a cluster may be happen at the different time. Dongfang was 3–5 h ahead of Beihai for the arrival of the peak of surges from Fig. 2. Since the purpose of the case is to analysis the relationship between the extreme surges at both sites, the declustering method is proper because all peaks were included in these clusters basically. The 100 h cluster interval has enough spare to make two surge peaks at two sites in the same typhoon surge be in a cluster. According to the above principles, the total number of independent events N is 2016.

3.3 Constructing conditional probability functions

The extreme surge in Beibu Gulf is cause mainly by typhoons from lower-latitude areas in SCS. Typhoons move usually through Beibu Gulf from south to north with a small number of them moving from east to west. The extreme surge at Dongfang, which is to the southeast of Beihai, should be as an early warning signal for Beihai. Multivariate extreme value analysis can be used for the warning.

In order to analyze the joint probability of the extreme surge of Beihai and Dongfang, CP (conditional probability) distributions can be used (Eq. 10). CP can represent the probability of encounter between extreme surges. The joint distribution of bivariate Pareto distribution function $W(x,y)$ is

$$W(x,y) = Pr(X < x, Y < y), \tag{9}$$

where x and y represent the surge (m) in Beihai and Dongfang respectively (the same in the paper). $W_x(x)$ and $W_y(y)$ are marginal distribution of x and y respectively. Con-

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ditional probability distributions are:

$$\begin{aligned}
\text{CP1:}\quad & Pr(X \geq x | Y \geq y) = \frac{Pr(X\geq x, Y\geq y)}{Pr(Y\geq y)} = \frac{1-W_X(x)-W_Y(y)+W(x,y)}{1-W_Y(y)} \\
\text{CP2:}\quad & Pr(X \leq x | Y \geq y) = \frac{Pr(X\leq x, Y\geq y)}{Pr(Y\geq y)} = \frac{W_X(x)-W(x,y)}{1-W_Y(y)} \\
\text{CP3:}\quad & Pr(X \geq x | Y \leq y) = \frac{Pr(X\geq x, Y\leq y)}{Pr(Y\leq y)} = \frac{W_Y(y)-W(x,y)}{W_Y(y)} \\
\text{CP4:}\quad & Pr(X \leq x | Y \leq y) = \frac{Pr(X\leq x, Y\leq y)}{Pr(Y\leq y)} = \frac{W(x,y)}{W_Y(y)}
\end{aligned} \tag{10}$$

Other four CP distributions can be deduced by swapping two variables.

4 Extreme-value analysis

In this section, the focus is on problems in extreme surges whose solution would require advances in the methodology of the statistics of extremes. These problems include analysis of joint threshold, stochastic simulation, and statistics of multivariate extreme surges. Finally, the issue of how to analyze the statistic results of extreme surges at two locations is briefly discussed.

4.1 Marginal transformation and joint threshold

After many experiments, it is found that marginal distributions of 2016 independent events can be described by GEVD:

$$F(x) = P(X < x) = \exp\left\{-\left[1-\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{1/\xi}\right\}, \quad \xi \neq 0, \tag{11}$$

where ξ, σ and μ are three variables of GEVD. They are estimated by using maximum likelihood estimate. This is an approach of estimation suggested in Sect. 5.1 of Beirlant et al. (2005) and Sect. 3.3 of Coles (2001). Figure 3 shows the probability plot and

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probability plots (including the 95 % confidence intervals) of marginal distribution before fitting MGPD.

In Sect. 2.1, the variables of MGPD must be in a neighborhood of zero in the negative quadrant. By a suitable marginal transformation, we can transfer a margin into a uniform margin in a neighborhood of zero by the idea of Rene Michel (2007). To standardize the margins, the marginal distribution of MGPD must be a negative exponential distribution. According to Taylor expansion, we get

$$y = \log F(x_i) = \log(1 + F(x_i) - 1) \approx F(x_i) - 1, \tag{12}$$

where $F(x_i)$ is GEVD of variable i ($i = 1,2$),which represent Beihai and Dongfang respectively. In Eq. (15), $F(x_i)$ closes to 1 since we care about extreme observations.

Many dependence models between extreme variables have been suggested: Logistic, Bilogistic, Dirichlet, etc. However, it appears that the choice of dependence model is not usually critical to the accuracy of the final model (Morton and Bowers, 1996). So the simple bivariate Logistic GPD was selected. The MGPD model of the paper is based on multivariate extreme value distribution, the joint threshold can be calculated by the method in Sect. 2.2. The joint threshold is $c_0 = -0.28$, and there are 218 groups of combination of Beihai and Dongfang over c_0. Figure 4a shows that the samples of over threshold value. In the left subfigure, $c_0 = -0.28$ is a curve, and c in right side of the curve are greater than c_0. The converted data is shown in the right subfigure and $c_0 = -0.28$ is a line.

The correlation parameter r of dependence function is estimated by the maximum-likelihood method and $r = 2.14992$. Having obtained estimates for all of the parameters, the joint extreme pdf was constructed, as illustrated in Fig. 4b.

4.2 Comparison of stochastic simulation results

According to the simulation method in Sect. 2.2, we generate the enormous simulation data. The section will compare the CP results from the two approaches: simulation and directly solve. Figure 5 shows the data of stochastic simulation by $N = 10\,000$ and

$N = 100\,000$. The simulation results are in basic conformity with the observations, and this represents that the MGPD simulation method is worked. The scatter diagrams show the result of simulation directly, however they need further quantitative analysis to show the differences of them objectively.

A couple of CP are to be used in the paper: CP1 $P(X > x|Y > y)$ and CP4 $P(X < x|Y < y)$ which means CP1 is the probability of the surge in Beihai over x under the surge in Dongfang over y and CP4 is the probability of the surge in Beihai less than x under the surge in Dongfang less than y, respectively. As it's showed in Fig. 6, the relative difference value of the simulation and the analytic solution are related to the simulation times N. it is obvious from all subplots in Fig. 6 that relative difference value is reduced with the increase of simulation times. When the simulation times up to 1.5×10^6, the maximum relative error value of simulation results and calculation results is 5.258 %, which appears satisfactory.

We conducted also runtime experiments. 1×10^4, 5×10^4, 10×10^4, 1×10^6, 1.5×10^6 random vectors were generated on a desktop with an Intel Core i7 Processor with 3.4 GHz. Their runtimes are 3, 24, 82, 9649 and 22 106 s respectively.

For estimating M-year surge of Beihai and Dongfang, the Poission–Gumbel distribution is used. The Poission–Gumbel distribution is

$$F(x) = P(X < x) = e^{-\lambda[1-G(x)]}, \tag{13}$$

where $G(x)$ is Gumbel distribution.

Based on the results by simulation times 1.5×10^6, Tables 2 and 3 show the value of CP1 and CP4. The tables represent 5 groups' calculation and stochastic simulation results of CP1 and CP4 on different combination of M-year surges at Dongfang and Beihai. It is found that two results are closely. For instance, the analytic solution of $P(X > x_{50}|Y > y_{10})$ is 12.94 % and its simulation result is 14.06 %. Their relative error is 8.6 %, which is the maximum of all relative errors for CP1; The analytic solution of $P(X < x_5|Y < y_{50})$ is 99.35 % and its simulation result is 94.13 %. Their relative error is 5.25 %, which is the maximum of all relative errors for CP4.

4.3 Warning of extreme surges

The relations between extreme surges at Beihai and Dongfang can be analyzed by CP. Because the peak surge at Dongfang occurred earlier than one at Beihai, we use CP1: $P(X > x|Y > y)$ as a standard for warning of extreme surge at Beihai. From Table 2, we can know that when 50 year surge is appeared at Dongfang, the probability of greater 50 year surge is 94.55 % at Beihai.

According to long-term records of surges, we can build the relationship between extreme surges at Beihai and Dongfang. Additionally, because of the special geographical relation of two places, the peak surge at Dongfang is a precursory signal for the prediction of the probability of the largest surge's occurrence at Beihai. So we can predict the probability of different surges at Beihai ahead, and then take preventive measures in order to prevent society and people from suffering some pains.

5 Discussion and conclusions

5.1 Conclusions

The primary theme of this paper concerns how recent developments about MGPD can be applied to marine disaster forecasting. The paper not only develops the process of determining joint threshold and simulation, but makes some analyses contributed to warning of extreme surges by MGPD. The MGPD is the nature distribution of MPOT method, which can dig up more extreme information from the raw data. The model based on multivariate extreme value theory which is well-founded. The intrinsic properties of all extreme variables are also into consideration. The method of determining the joint threshold was introduced to MGPD in the paper. The Monte Carlo simulation of MGPD was used for the conditional probability of two extreme surges, and the accuracy was verified to be acceptable.

5.2 New possibilities based on Monte Carlo simulation

Warning of extreme surges will be more reliable if we can build the relationship among extreme surges at three or more sites. In the paper, the theory about MGPD and its simulation is derived for multidimensional variables. The methodology could be extrapolated to higher dimensional space. So difficulties of solving procedure for MGPD can not restrict its application under the condition of high dimensionality. A potentially better warning approach is possible based on Monte Carlo simulation. Once the long-term (such as thousands of years) sea state data has been simulated, several ocean environment factors can be assessed quickly by the law of large numbers.

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Table 1. Parameters of marginal distribution.

	ξ	σ	μ
Beihai	0.3376	0.1187	−0.0465
Dongfang	0.1933	0.0890	−0.0720

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Table 2. Comparision of the results of CP1.

RP (year)		5		10		20		50		100	
RP	D(m)	0.57		0.77		0.84		0.93		0.99	
	B(m)	a	s	a	s	a	s	a	s	a	s
5	0.87	62.52	62.59	99.33	99.54	99.89	99.79	99.99	100.00	100.00	100.00
10	1.13	5.20	5.34	81.00	82.52	96.67	97.44	99.76	100.00	99.97	100.00
20	1.22	1.70	1.74	49.38	50.71	88.05	88.70	99.13	100.00	99.89	100.00
50	1.34	0.34	0.37	12.94	14.06	47.95	49.68	94.55	95.12	99.29	100.00
100	1.42	0.11	0.11	4.29	4.47	19.09	19.40	80.75	82.93	97.37	100.00

RP: return periods, a: analytic solution, s: simulation results, D: Dongfang, B: Beihai (the same below).

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Table 3. Comparision of the results of CP4.

RP (year)		5		10		20		50		100	
RP	D(m)	0.57		0.77		0.84		0.93		0.99	
	B(m)	a	s	a	s	a	s	a	s	a	s
5	0.87	99.75	97.61	99.36	94.26	99.35	94.15	99.35	94.13	99.35	94.12
10	1.13	100.0	99.99	99.98	99.81	99.97	99.72	99.97	99.69	99.97	99.69
20	1.22	100.0	100.0	100.0	99.97	99.99	99.93	99.99	99.90	99.99	99.90
50	1.34	100.0	100.0	100.0	100.0	100.0	99.99	100.0	99.98	100.0	99.98
100	1.42	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.99

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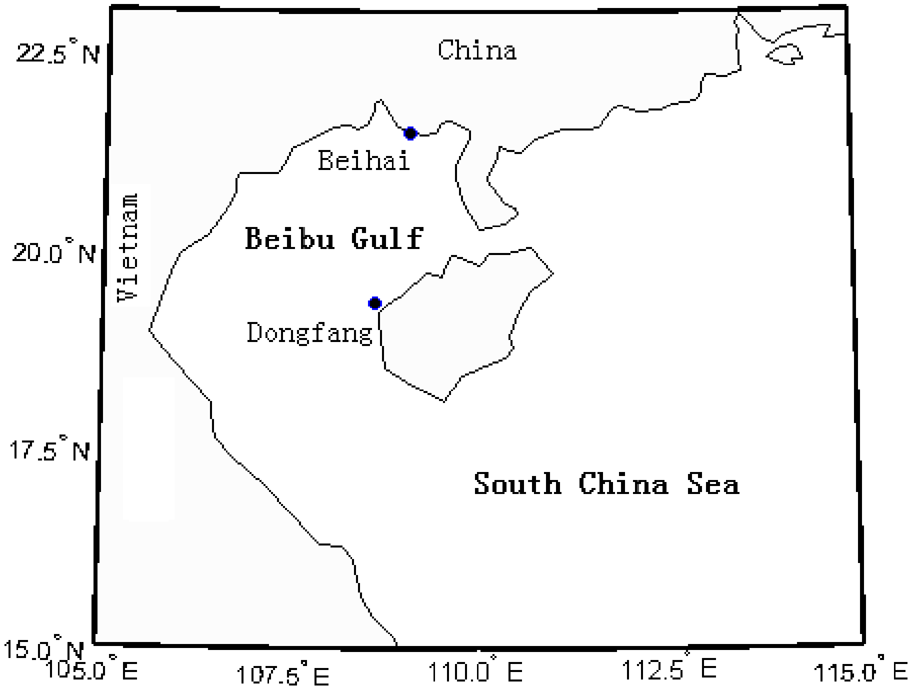

Figure 1. Location of two stations in Beibu Gulf, SCS.

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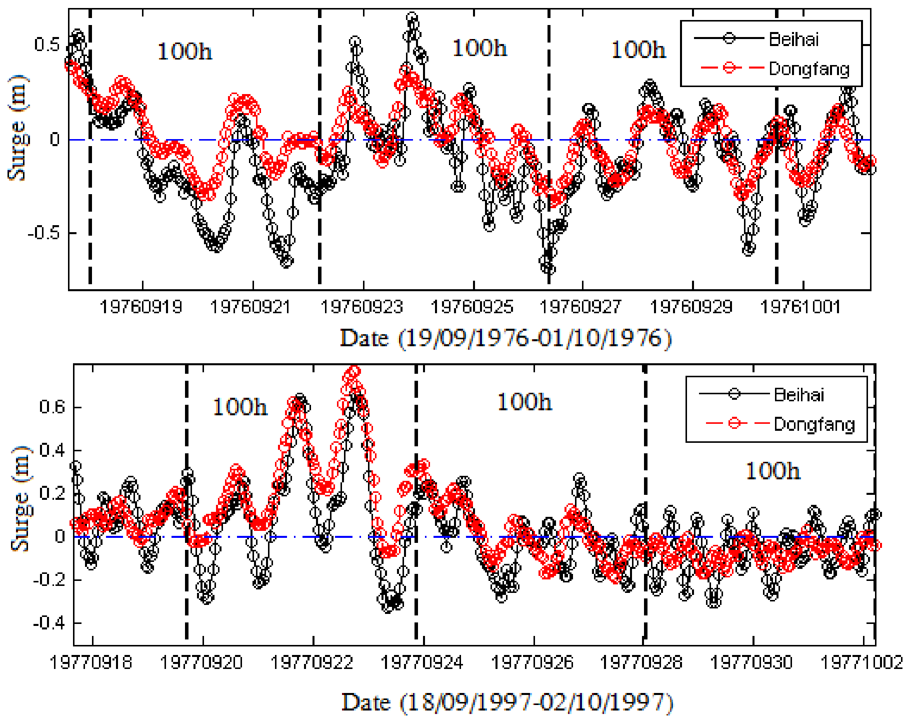

Figure 2. Declustered surge.

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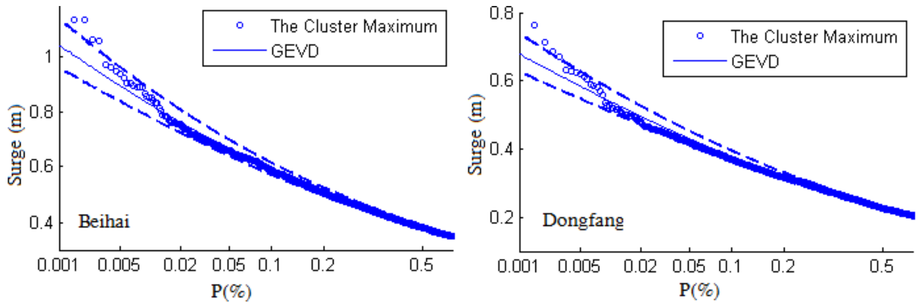

Figure 3. Fitting testing of marginal distribution.

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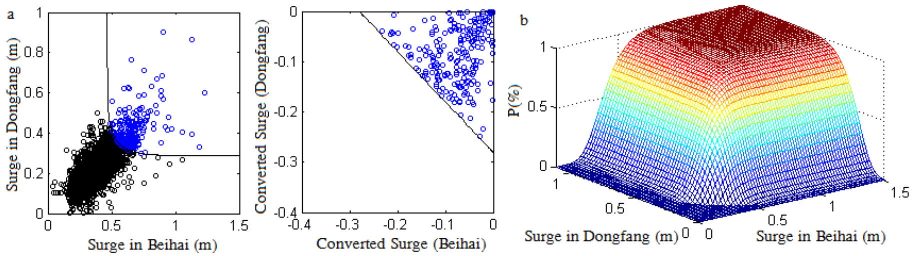

Figure 4. **(a)** Over threshold value of Dongfang and Beihai, **(b)** joint distribution of extreme value of Dongfang and Beihai.

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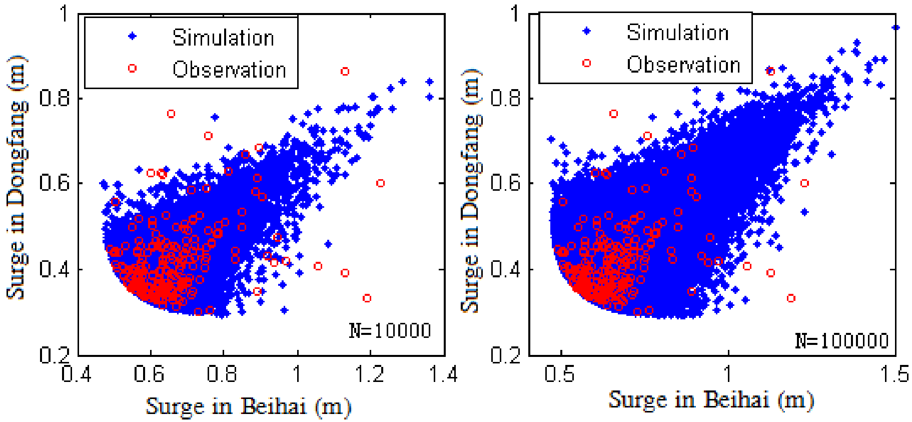

Figure 5. Value over threshold and data of stochastic simulation.

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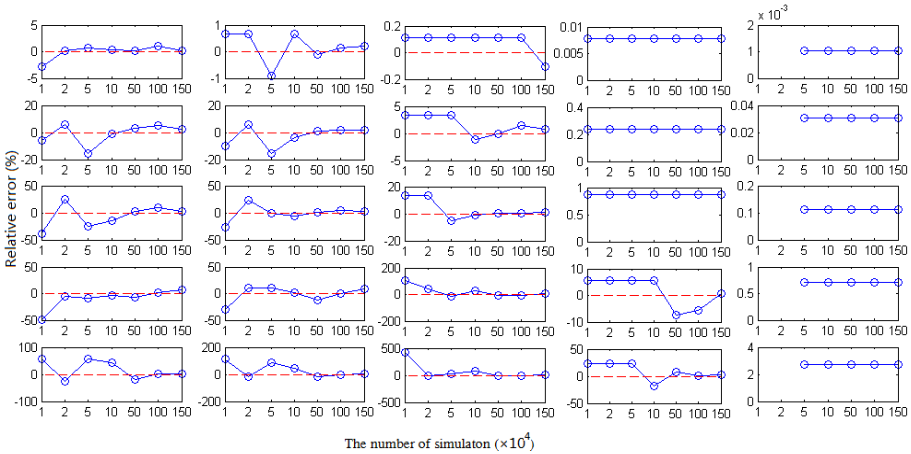

Figure 6. The variance of the relative error under the different number of simulation N and the distribution of case is the same to Tables 2 and 3.

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