Referee #2

Dear Reviewer,

We found your criticism really interesting, in particular your first critique. This gave us the opportunity to better understand the dynamics of the EMDW along the Sicily Channel and to clarify some crucial aspects related to its relation with the LIW at the sill region. Please find below our comments to your critiques and suggestion.

Critique (C) #1

It is not clear to me why the EMDW layer should even be treated with a streamtube model when it dynamically seems to be better described as the lower portion of the weakly-stratified Levantine Intermediate Water (LIW) layer. Although the EMDW can be identified by its density and T-S anomaly, as described in the previous paper by Astraldi et al (JPO 2001), the thermal wind shear appears to be quite small compared to the shear at the top of the LIW [we guess the Referee here means the EMDW]*, so the layer may not really be dynamically distinct.*

We believe that the Referee is here wondering about the Ekman transport effect induced by the LIW to the EMDW and how strong this Ekman transport is with respect to the geostrophic flow (i.e., thermal wind). We explored carefully this intriguing observation since no similar dynamics have ever been taken into account in the previous literature. Within the Sicily Channel, MAW, LIW and EMDW have been always seen as three distinct water masses, with their own origins, histories, fluxes, hydrologic characteristics and thus potential vorticities (Malanotte-Rizzoli et al., 1997; Sparnocchia et al., 1999; Astraldi et al., 1996, 1999, 2001; Gačić et al., 2013; and many others).

The relation among EMDW and LIW velocities, and the related Ekman effects (i.e., spiral effects), were examined by Astraldi et al. (1996), A01, and Sparnocchia et al. (1999) by means of current meter measurements at the western sill of the Channel and in the Tyrrhenian Sea. These current meter data show that cross-shore velocities of the two deepest veins (i.e., LIW and EMDW) are much smaller than the along-channel component. This suggest the absence of spiral effects and, in turn, supports the geostrophic description for these two distinct flows, as in agreement with the historical literature. These findings receive support by the *Johnson et al.* (1976) tank experiment, regarding the thickness of each Ekman layer h_{EK} between adjacent flows. For our case *Johnson et al.*'s analysis gives a Ekman layer thickness $h_{EK} \approx$ O(10^{-1}) m between LIW and EMDW. It also results that the strongest Ekman effect for the EMDW is the one due to the bottom drag , which gives $h_{EK} \approx$ O(1) m (*Salon et al.*, 2008).

All this enlightens two important conclusions for this case study:

i) the Ekman transport effect induced by the LIW to the EMDW is negligible;

ii) ii) bottom friction effects on the EMDW dynamics cannot be disregarded, especially in the sill region.

We included all these findings and discussions in the revised version of the manuscript and we thanks the reviewer for giving us the opportunity to explore this point. See line 99-128.

C #2

The theoretical development of potential vorticity conservation in a shallow-water layer is not exactly a new concept, and the model does not, in the end, even yield a prediction that can be tested with the observations. The end result is a set of friction parameters estimated by requiring the model to fit the data (i.e., the friction is used to explain the residual in the budget). As a result, these friction parameters are a combination of all un-resolved aspects of the dynamics and can't really be said to yield any insight about topographic effects in dense-water flows.

Our theoretical development of PV conservation in shallow water layer is indeed not a new concept. We totally agree with this observation and, in fact, our analysis concerns a different aspect. The deep flow dynamics through marine channels under the effect of friction are rarely considered in the literature, although friction is the most important component for such dynamics, mainly at the channel sill. In addition, studies on the potential vorticity evolution in presence of friction and entrainment are even rarer. So our effort aims to provide an original approach that should fill an ancient scientific gap.

Indeed, we do not feel that the main aim of this paper is to provide a prognostic model to be tested with observations. Rather, we plan to introduce the potential effect of bottom friction and entrainment effects in integral forms of vorticity and PV.

We are also aware of the fact that the friction parameter we estimate is obtained by fitting the data, as frequently done in Physics. In this regard, contrary to many previous studies, our goal is to demonstrate that a constant-PV approach, as well as a constant frictional parameter, are not particularly suitable approximations for the investigation of channel flows dynamics where important sills are present.

That said, the perception by this referee that the thrust of this paper was different from our intended goal is our fault. The wording of the original manuscript did not frame the nature of our work in a clear way. Therefore we thank the Referee and rephrase some crucial paragraphs… See lines 14-27; 34-40; 47-49; 57-59; 62-66; 343-344; 395-396; 416-418; 446-448;

C #3

I did not find the paper easy to follow–particularly with regards to understanding the assumptions being made in the theoretical model and evaluating the results and their implications.

We pursued a careful revision of the manuscript in order to make the main framework of this study as clearer and easier as possible. We indeed revised, tentatively, the paragraphs related to the explanation of the vorticity and PV models and the assumptions that have been made. Results and implications should also be clearer now. See lines 34-40; 47-49; 57-59; 62-66; 99-128; 134-136; 142-150; 223-225; 343-344; 395-396; 453-461.

Suggestion (S) #1

*The abstract should clearly state the *purpose* of the research, the essential elements of the *method*, and the *results* obtained. At present, it is mostly background and doesn’t really describe the methods used or the results.*

Thanks for this suggestion. The abstract now follows this scheme: Topic of the paper; purpose; methods; results; implications. See lines 14-27.

S #2

*Give a clear explanation of how the LIW and EMDW layer thicknesses and velocities were estimated. The A01 paper may include all of the details, but the essential method and assumptions should be stated here since these are the core "measurements" used. How is EMDW defined if the density interface changes? What level of no motion was used for geostrophic calculations? Were *any* velocity measurements made or only density? Was other information used to constrain transports? Are the velocities in Table 2 and Fig. 5 averages over a few stations or total transport divided by cross-sectional area?*

We now provide a clearer description of A01 methodologies used in this 15 years old paper. The reader can now easily figure out how all hydrodynamic quantities were estimated, in particular those concerning the EMDW. In the previous version we missed to mention the current meter measurements that actually constrained the velocity estimates for the EMDW. This was totally our fault! See lines 291-298; 312-317 and caption of Table 1

S #3

Since the theory for the multi-layer streamtube model (Eq.1+2) is not, in the end, used in the data analysis, it ends up mostly being a distraction. I'd suggest reducing the treatment to a single layer.

We removed the "three layer" formalism and we reduced the Eqs. (1a)-(1c) to a single equation. Eqs. (2) is now written for the bottom layer only. However, we did not lose the generality of the model (see line 94-115). Thanks for this suggestion.

S #4

Similarly, the parameterization of friction with arbitrary exponents n and m on velocity and thickness (Eq.4) seems unnecessary, since it is never used.

It is indeed not used here but in the classical study of Baringer and Price (1997a, b) of the Gibraltar outflow of EMDW a very complex friction is considered. We can however simplify this part as well. We defined just one general formulation and then we specify in the text which particular friction we will consider (see line 116-122).

S #5

Entrainment is included in the thickness budget (2c) but not in the momentum budget (2a and 2b). Why not? Since friction with the overlying layer (F) is included, the momentum impact of entrainment (entrainment drag) should probably be as well. (Although it just ends up being another term that is lumped into the residual.)

This is exactly why we did not include the entrainment drag in the two momentum equations. This entrainment is, in fact, proportional to the two velocity difference and this in our case is a small quantity. We now provide some more details about this, in order to justify our choice. See line 99-115; 123-128.

S #6

The Sannino et al (2009) numerical model results really could and should be used in the evaluation of the vorticity and PV results (Fig.6). This is a simple comparison, much like the velocity. Friction and momentum budget terms in the model could also be compared with the streamtube results (Figs.7+8), though this will be more difficult.

We agree with the reviewer and, in the past, we tried to figure out how to make use of numerical outputs in order to validate (or just compare) our analytic/ experimental results with numerical ones. From several discussions we had with our colleagues (numerical ocean modelers) it came out that such a calculation may result to be meaningless, due to grid problems over the sea bottom (Dr. G. Sannino, personal communication, 2015; Dr. L. Palatella, personal communication, 2015).
In such a context the calculations of vorticity field (i.e., the vertical vorticity ζ) from the horizontal velocity components provided by Mediterranean Sea numerical models is not

trivial all. Indeed such a calculation may finally result to be meaningless, due to grid problems over a highly irregular, tormented sea bottom: i) numerical outputs are provided on staggered-grids and it would be quite hard to retrieve the original non-staggered grid information; ii) spatial resolutions are too coarse and, in particular, bottom velocities are available on a very few cross-stream grid points (one or two at the western sill; see figure below).

We are aware of some other ocean models that provide, directly, the vorticity field (and then they obtain the velocity field from those), but these products are not available on the main web portal for operational oceanography (e.g., MyOcean). In synthesis we believe that to include in this work numerical runs that provide the bottom vorticity field for the seasons we investigated (i.e., January 1997 and May 1998) is not in short time feasible and it would constitute a fascinating suggestion for another work.

In the revised manuscript we included this discussion. See line 355-356; 378-382.

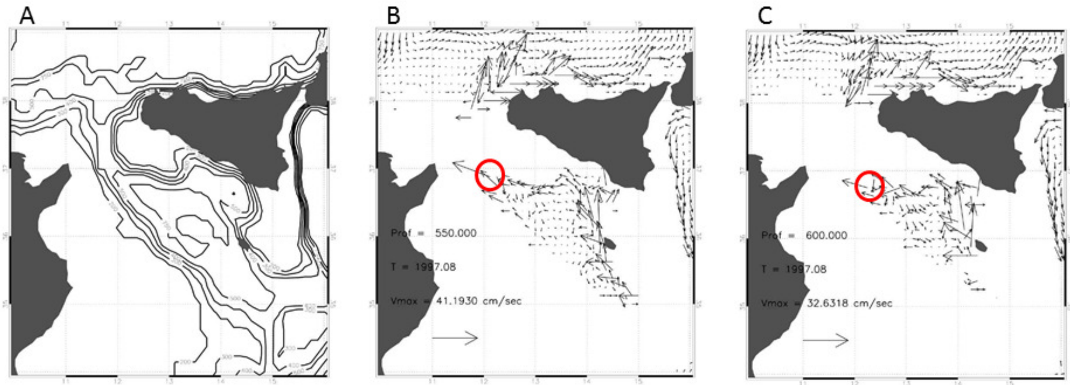

Figure. Monthly mean velocities from PROTHEUS numerical data (January 1997), a relatively coarse resolution Mediterranean model (1/8° ×1/8°) based on the MIT general circulation model (MITgcm; Sannino et al., 2009; Sannino, personal communication). Panel A: Bathymetry used in the model; Panel B: velocity filed at 550 m depth; Panel C: : velocity filed at 600 m depth. The figure show that bottom velocities within the channel (red circles) do not allow to evaluate the vorticity filed due to the coarse horizontal resolution of the model.

Additional references (net present in the main text)

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Friction and mixing effects on potential vorticity for bottom current crossing a marine strait: an application to the Sicily Channel (central Mediterranean Sea)

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Abstract

We discuss here the evolution of vorticity and potential vorticity (PV) for a bottom current crossing a marine channel in shallow-water approximation, focusing on the effect of friction and mixing. The purpose of this research is indeed to investigate the role of friction and vertical entrainment on vorticity and PV spatial evolution in channels or straits when along-channel morphology variations are significant. To pursue this investigation, we set the vorticity and PV equations for a homogeneous bottom water vein and we calculate these two quantities as an integral form. Our theoretical findings are discussed by means of in situ hydrographic data related to the Eastern Mediterranean Deep Water, i.e., a dense, bottom water vein that flows northwestward, along the Sicily Channel (Mediterranean Sea). Indeed, the narrow sill of this channel implies that friction and entrainment need to be considered. Small tidal effects in the Sicily Channel allow for a steady theoretical approach.

We argue that bottom current vorticity is prone to significant sign changes and oscillations due to topographic effects when, in particular, the current flows over the sill of a channel. These vorticity variations are, however, modulated by frictional effects due to seafloor roughness and morphology. Such behavior is also reflected in the PV spatial evolution, which shows an abrupt peak around the sill region. Our diagnoses on vorticity and PV allow us to obtain general insights about the effect of mixing and friction on the pathway and internal structure of bottom-trapped currents flowing through channels and straits, and to discuss spatial variability of the frictional coefficient. Our approach significantly differs from other PV-constant approaches previously used in studying the dynamics of bottom currents flowing through rotating channels.

1 Introduction

An ongoing debate in diagnostic models for currents that flow over a sill in a rotating channel with varying cross sections concerns the effect of friction and mixing, which clearly play an important role in the presence of morphological constraints (Pratt et al., 2008; Pratt and Whitehead, 2008). Despite such a role, these two key effects are often not considered in the literature. Idealized models for marine currents flowing through rotating channels (e.g., Whitehead et al., 1974; Gill, 1977; Borenas and Lundberg, 1986, 1988; Killworth, 1992) usually assume a steady state and are often simplified, out of necessity, for a feasible analytic investigation (Pratt and Whitehead, 2008). This, for instance, leads to friction being neglected, assuming a uniform potential vorticity (PV) and considering channels with rectangular or smooth, idealized cross sections in order to avoid dynamic pathologies at the current lateral edges (Lacombe and Richez, 1982; Hogg,1983; Pratt et al., 2008).

In particular, the most often cited models for these currents assume a zero-potential vorticity flow (Whitehead et al., 1974; Borenas and Lundberg, 1988). Such an assumption is mostly applied for fluid columns coming from a quasi-quiescent upstream state and then severely squashed as they cross the sill of a channel. Along-channel profiles of depth and velocity of these approximated currents are particularly simple to predict and, for the case of a rectangular cross section, it has been demonstrated that such flows are also stable (Paldor, 1983). In fact, realistic bottom marine currents that are confined to channels or straits show a thickness that goes to zero at the lateral edges, which can lead to pathological features in terms of flow stability (Pratt et al., 2008).

A second, often adopted approximation is given by disregarding friction and vertical entrainment of bottom currents flowing in rotating channels (Armi and Farmer, 1985; Bryden and Kinder, 1991; Whitehead et al., 1974; Gill, 1977; Borenas and Lundberg, 1986). Friction and entrainment in fact play an important role for currents crossing channels or straits (Johnson and Ohlsen, 1994), in particular when along-channel morphology variations are present (Borenas and Lundberg, 1986, 1988; Killworth, 1992, among others). Experimental data on this regard have shown complicated dynamics that suggest a strong effect of both interfacial and bottom friction that may induce a secondary circulation (Johnson et al., 1976).

These considerations are at the base of our interest for a more realistic analysis of bottom currents that cross a narrow marine channel, in the presence of an irregular morphology, and flow underneath upper layers that have different dynamics. We do not aim to provide a prognostic model to be tested with observations, but rather, to introduce the potential effect of bottom friction and entrainment effects in integral forms of vorticity and PV. To pursue such an investigation, we derive vorticity and PV equations from the classic stream-tube model (Smith, 1975; Killworth, 1977), which describes the steady properties of a homogeneous, viscous bottom water vein, also considering entrainment in the mass conservation equation (Turner, 1986). We then discuss these equations in order to figure out the role of seafloor morphology, friction, and mixing in marine channel dynamics. We finally introduce the hydrographic settings of the Sicily Channel (Fig. 1) (Astraldi et al., 2001; A01 hereafter) and employ interpolated, cross-averaged flow velocity ($\bar{u}$) and thickness ($\bar{h}$) data related to the Eastern Mediterranean Deep Water (EMDW, a bottom vein flowing northwestward through the

Sicily Channel) in order to diagnose our vorticity and PV equations. The EMDW flows underneath the Levantine Intermediate Water (LIW) and the Modified Atlantic Water (MAW). Those currents constitute a three layer system (Fig. 2), whose hydrodynamics are strongly affected by baroclinic, mixing, and topographic effects (A01).

Our approach differs from a similar investigation proposed by Hogg (1983) and Whitehead (1998), among many others, who analyzed the hydraulic control and frictionless flow separation in the Vema Channel. The Sicily Channel has relatively unimportant tides; its sill is 300 m deep and shows an irregular and narrow morphology, all features that make this channel particularly suitable for our goals and theoretical approaches. In particular, the usual inviscid quasi-geostrophic approach does not seem particularly adequate in the Sicily Channel.

2 Momentum and mass conservation of dense flows for realistic channels

Here we consider the dynamics of a shallow, homogeneous, bottom layer of fluid flowing in a deep channel, underneath upper moving layers of water that have a slightly lower density. The channel is thought to be aligned along the *x* direction and has a realistic, quasi-rounded cross section (Fig. 2a). The stream-wise evolution of such a bottom flow is governed by the shallow-water equations. The use of the full equations, rather than "balance" equations or other approximations, is required in order for hydraulic effects to be accurately captured (Pratt et al., 2008).

To take into account the role of upper layers, we consider a shallow-water model for multiple homogeneous layers with thicknesses h_j , densities ρ_j , and velocities $\vec{u}_j \equiv (u_j, v_j)$, where j = 1, 2, 3 indicates the different layers; z is the vertical coordinate (positive upward); t is the time; $b(x,y)$ is the sea bottom, with $\frac{\partial b}{\partial x} << \frac{\partial b}{\partial y}$ and $W_j(x)$ being the cross-channel layer widths (Fig. 2a).

The hydrostatic pressure related to the third layer (j = 3) can be written as (Hogg, 1983; A01)

$$p_3 = p_0'' + g\,\rho_3\,(h_3 - z) + g\,\rho_2\,(h_2 - h_3) + g\,\rho_1\,(h_1 - h_2) \qquad (1)$$

where p_0'' *is a* constant and g is the gravitational acceleration (m s−1) (Fig. 2a).

The full shallow-water equation for a streamline in the third layer is as follows (Gill, 1982, p. 231–232; Pratt et al., 2008):

$$\delta \frac{\partial}{\partial t} u_3 + \delta u_3 \frac{\partial}{\partial x} u_3 + \delta v_j \frac{\partial}{\partial y} u_3 - f\ v_3 = -\frac{1}{\rho_3} \frac{\partial}{\partial x} p_3 + \delta * \frac{\vec{F}_3}{\rho_3} \qquad (2a)$$

$$\delta \frac{\partial}{\partial t} v_3 + u_3 \frac{\partial}{\partial x} v_3 + v_3 \frac{\partial}{\partial y} v_3 + f\, u_3 = -\frac{1}{\rho_3} \frac{\partial}{\partial y} p_3 + \delta^* \frac{\vec{F}_3}{\rho_3} \qquad \text{(2b)}$$

$$\delta \frac{\partial}{\partial t} h_3 + h_3 \frac{\partial}{\partial x} u_3 + h_3 \frac{\partial}{\partial y} v_3 = \delta^* E |u_3 - u_2|, \qquad \text{(2c)}$$

where f is the Coriolis parameter; $\vec{F}_3$ and $E|u_3 - u_2|$ represent, respectively, friction and entrainment between adjacent layers; and E is a suitable entrainment parameter. In Eq. (2) $\delta = 0$ gives the steady, quasi-geostrophic approximation, while $\delta^* = 0$ leads to the inviscid case. $\vec{F}_3$ contains both inter-layer friction and bottom stress, schematizing the upper and lower friction, which mainly occurs at the boundaries of the bottom layer. These stresses induce both upper and lower Ekman spirals, in addition to some entrainment effects (Johnson and Ohlsen, 1994). We point out that entrainment should be also included in the momentum budget (2a and 2b). Since friction with the overlying layer is included, the momentum impact of entrainment (entrainment drag) has indeed a potential role. However, this results in being another term that is lumped into a residual and we therefore omit such a term.

A general formulation for bottom friction can be defined as (Baringer and Price, 1997a, b; A01)

$$\vec{F}_3 = -\rho_3 X(\vec{u}_3, h_3)\vec{u}_3\,, \qquad \text{(3)}$$

where X (s^{-1}) is, in general, an empirical, nonlinear relation. In the following we will use the formulation $X = K^* \frac{\rho \bar{u}}{\bar{h}}$ - with K^* = *constant* - that takes account for the averaged flow thickness and velocity (A01).

Ekman transport effects induced by the intermediate layer to the bottom layer, and how strong this transport is with respect to the geostrophic flow (i.e., thermal wind), can be explored by means of Ekman layer thickness $h_{EK} \approx (2\nu/f)^{1/2}$. For a laminar case (Johnson et al., 1976) such a thickness is $\approx O(10^{-1})$ m, where ν is the fluid viscosity. All this enlightens that for our case study the Ekman transport effect induced by the LIW to the EMDW is negligible. On the other hand we stress that the effect of friction in the bottom layer is more complex, mostly in the sill region. Real seafloors are indeed irregular, with bathymetric heterogeneities of many space scales. This gives a much thicker benthic layer, i.e., $(2K/f)^{1/2} \approx O(10)$m for a turbulent viscosity $K >> \nu$ (Salon et al., 2008). Moreover, Johnson et al. (1976) noted the occurrence of a secondary, frictional-induced cross-channel circulation, which forces spun-down fluid into the interior, further limiting the sill flow (see Fig. 5 of Johnson and Ohlsen, 1994).

Vorticity is therefore strongly affected by these frictional effects. Moreover, because the bottom frictional coefficient K^* may reasonably vary along-stream due to the spatial pattern of bottom irregularities, the effect of friction on flow vorticity and PV further increases.

3 The vorticity equation

By focusing on the narrow bottom layer (j = 3, where the index "3" will be disregarded hereafter), we make use of a stream-tube model (Fig. 2b) in a stream-wise coordinate system (ξ, ψ). In this frame, ξ is the along-flow coordinate, centered along the midline of the vein, and ψ is the cross-flow coordinate (Smith, 1975; Killworth, 1977). Such a model is that of a steady flow where the bottom water is assumed to be well mixed. The flow has strong axial velocity nearly uniform over a cross-section of the stream (i.e., $v << u$ is anti-symmetric and vanishes at the vein lateral boundaries $\psi = \pm W/2$; Baringer and Price, 1997b). Consequently, the cross-stream scale is assumed to be much smaller than the local radius of curvature of the stream axis. All this implies that the velocity of a stream line is a function of ξ only. The angle between the stream-tube axes (ξ, ψ) and the fixed axes (x, y) is β (Fig. 2b). Consequently, in this new frame, the horizontal gradient operator can be written as (Smith, 1975)

$$\nabla_h = \left(\frac{1}{1-\psi\dfrac{\partial\beta}{\partial\xi}} \frac{\partial}{\partial\xi}, \left(\frac{\partial}{\partial\psi} - \frac{\dfrac{\partial\beta}{\partial\xi}}{1-\psi\dfrac{\partial\beta}{\partial\xi}} \right) \right) \approx \left(\frac{\partial}{\partial\xi}, \frac{\partial}{\partial\psi} \right) \tag{4}$$

where the approximation on the right-hand side of Eq. (4) is justified by a small $\psi\dfrac{\partial\beta}{\partial\xi}$, as for the Sicily Channel case (Fig. 1), where β is close to zero because of the straight E–W path of the bottom vein (see Sect. 7).

By cross-differentiating the horizontal components of Eq. (2), for a dense water streamline one obtains the classical vorticity equation (Gill, 1982; p. 231)

$$\frac{d}{dt}\zeta + (\zeta + f)(div\,\vec{u}) = \frac{1}{\rho}(curl\vec{F})_z\,, \tag{5}$$

which, in steady state, is

$$u\frac{\partial}{\partial\xi}\zeta+(\zeta+f)(div\,\vec{u})=\frac{1}{\rho}(curl\,\vec{F})_z \qquad (6)$$

It is useful to recall that ζ , in Eqs. (5) and (6), is the sum of a “shear vorticity”, related to the lateral shear of the current, and a “curvature vorticity” due to the bending streamline of the current (Holton, 1972; Chen et al., 1992). The frictional term in Eqs. (5) and (6) can be explicated as $\frac{1}{\rho}\left(curl\,\vec{F}\right)_z=-X\zeta$. We finally emphasize that our Eq. (6) looks rather different from the steady, quasi-geostrophic, and inviscid version proposed by Hogg (1983):

$$(\frac{\partial v}{\partial\xi}+f)\,u-\frac{\partial}{\partial\psi}B=0 \qquad (7)$$

where $B=\frac{p}{\rho}+\frac{v^2}{2}$ is the Bernoulli function.

Equation (6), once integrated, gives an exact diagnostic relation for the spatial evolution of ζ by assuming the knowledge of $h(\xi,\psi)$ and $u(\xi,\psi)$:

$$\frac{\zeta}{f}=e^{-\int_0^{\xi}\frac{1}{u}(X+div\,\vec{u})\,dx}\left\{\frac{\zeta_0}{f}-\int_0^{\xi}e^{\int_0^{x}\frac{1}{u}(X+div\,\vec{u})\,dx'}\;\frac{1}{u}(div\;\vec{u})\,dx\right\} \qquad (8)$$

Let us also note that an approximated solution of Eq. (6) for $\zeta<<f$ is

$$\frac{\zeta}{f}=e^{-\int_0^{\xi}\frac{X}{u}dx}\left\{\frac{\zeta_0}{f}-\int_0^{\xi}e^{\int_0^{x}\frac{X}{u}dx'}\;\frac{1}{u}(div\,\vec{u})\,dx\right\} \qquad (9)$$

Intuitively, the two solutions Eqs. (8) and (9) are rather similar, although Eq. (9), analytically speaking, is relatively more subject to eventual irregularities in the flow velocity *u*, such as sharp and large peaks around the sill region. Moreover, we note that the approximation that leads to Eq. (9) cannot be applied near the sill of a channel if the flow there is subject to hydraulic control.

In real field cases, the knowledge of $h(\xi,\psi)$ and $u(\xi,\psi)$ is often difficult to infer form in situ hydrographic data. By seeking for a more applicable relation we therefore consider cross-sectional averages of the various terms of Eq. (6). This leads to the following solution for $\bar{\zeta} << f$ (Appendix A):

$$\frac{\bar{\zeta}}{f} = e^{-\int_0^{\xi} \frac{X}{\bar{u}} dx} \left\{ \frac{\bar{\zeta}_0}{f} - \int_0^{\xi} e^{\int_0^{x} \frac{X}{\bar{u}} dx'} \frac{1}{\bar{u}} (\overline{div\vec{u}})\, dx \right\} \tag{10}$$

where the overbars indicate the cross-channel average.

Such a cross-averaging approach is further justified by the fact that the bottom vein is assumed to flow along a narrow and long channel, where the longitudinal length scale is greater than the transversal one. In this way, one can diagnose the cross-channel average of flow vorticity ($\bar{\zeta}$) from the experimental knowledge of the cross-channel averaged $\bar{h}$ and $\bar{u}$, which are bulk quantities easily inferable from in situ measurements. Moreover, the cross-channel averaging allows for further perturbations to be avoided that can be given by waves occurring along the lateral edges of the current, which are known, however, to have a small local effect (Lacombe and Richez, 1982; Pratt et al., 2008). Similar discussions can be had regarding the presence of upper and bottom Ekman boundary layers, which can perturb the non-averaged vorticity field, as was found in the study of Johnson and Ohlsen (1994).

4 Continuity equation and vertical entrainment

To include dynamical effects due to entrainment between the two lowest, cross-sectionally homogeneous layers, we consider here the mass continuity equation (Appendix A)

$$\frac{d}{dt}\bar{h} + \bar{h}\,\overline{div\vec{u}} = E\left|\,\bar{u} - \bar{u}_2\right| \tag{11}$$

or, in steady state,

$$\bar{u}\frac{\partial}{\partial \xi}\bar{h} + \bar{h}\;\overline{div\vec{u}} = E\left|\,\bar{u} - \bar{u}_2\right| \tag{12}$$

where $E\left|\bar{u}-\bar{u}_2\right|$ describes the vertical displacement of the interface between the two lowest layers due to mixing. Layer 2 (i.e., the middle layer; Fig. 2a) has velocity ($\bar{u}_2$, 0), and the entrainment dimensionless parameter E is assumed to be ~10^{-4} (Ellison and Turner, 1959; Turner, 1986). Entrainment also implies an exchange of momentum between layers, and thus an additional resistive force (Baringer and Price, 1997b; Gerdes et al., 2002) that should be considered in the momentum balance. However, if $\bar{u}$ is ~ $\bar{u}_2$ (Tables 1 and 2), momentum variations due to entrainment can be reasonably neglected as previously discussed.

We finally point that the continuity Equations (11) and (12) can be formulated if one assumed that both velocity and density profiles within the bottom layer exhibit similarity forms, so that the rate of entrainment may be related solely to the mean velocity and the layer thickness (Smith, 1975).

5 Vorticity equation with entrainment

By substituting the $\overline{div\vec{u}}$ in Eq. (10) with that from Eq. (12), one obtains

$$\frac{\bar{\zeta}}{f}=\frac{\bar{u}_0}{\bar{u}}e^{-\int_0^{\xi}\frac{X}{\bar{u}}dx}\left(\frac{\bar{\zeta}_0}{f}+\frac{1}{\bar{u}_0}\int_0^{\xi}e^{\int_0^{x}\frac{X}{\bar{u}}dx'}\left[\frac{\bar{u}}{\bar{h}}\frac{d\bar{h}}{dx}-\frac{1}{\bar{h}}E(\bar{u}-\bar{u}_2)\right]dx\right), \qquad (13a)$$

while, disregarding the entrainment, equations (10) and (12) simply give

$$\frac{\bar{\zeta}}{f}=\frac{\bar{u}_0}{\bar{u}}e^{-\int_0^{\xi}\frac{X}{\bar{u}}dx}\left(\frac{\bar{\zeta}_0}{f}+\frac{1}{\bar{u}_0}\int_0^{\xi}e^{\int_0^{x}\frac{X}{\bar{u}}dx'}\frac{\bar{u}}{\bar{h}}\frac{d\bar{h}}{dx}dx\right) \qquad (13b)$$

Note that, for the sake of simplicity, we hereafter omit overbars on all the cross-channel averaged variables.

Equations (13a) and (13b) show that the main forcing on ζ is given by (i) a vorticity stretching term $\frac{u}{h}\frac{\partial h}{\partial \xi}$ (Gill, 1977), (ii) the entrainment effect, and (iii) friction. In particular, we note that:

1. ζ is the sum of an initial condition (ζ_0) plus the integral of both stretching and entrainment terms $\left[\frac{u}{h}\frac{dh}{dx}-\frac{1}{h}E(u-u_2)\right]$ due to bathymetric forcing and vertical mixing, respectively;

2. the entrainment term $\frac{1}{h}E(u-u_2)$ is, however, small for $u \approx u_2$, a condition that occurs when the two adjacent bottom and intermediate layers flow in the same direction;

3. both initial condition and stretching terms are multiplied by $\frac{u}{u_0} e^{-\int_0^{\xi} \frac{X}{u} dx}$, which is related to friction, and it vanishes progressively over a distance ~ 3*u/X*. One can therefore argue that the role of frictional effects largely depend on the friction function *X* and thus on the local sea-bottom roughness. All these features are particularly valid where topographic changes are significant and therefore represent general effects for deep, steady, baroclinic currents in marine channels, straits, and ridges.

Our considerations imply that the evolution of *ζ/f* is not strictly related to the initial or downstream conditions but rather that it is mainly ruled by $\frac{u}{h}\frac{\partial h}{\partial x}$. Indeed, upstream of the sill of a marine channel $\frac{u}{h}\frac{\partial h}{\partial \xi} \leq 0$, while $\frac{u}{h}\frac{\partial h}{\partial \xi}$ becomes positive downstream, which means that *ζ* must decrease as the sill is approached, eventually becoming negative. Once downstream of the sill, *ζ* will increase again, reaching pre-existing upstream values. This is an important point since it differs from classical stream-tube models that require, for hydraulically supercritical flows, the integral from the upstream location to be taken in order to obtain solutions for *ζ*. Moreover, "if the ordinary differential equation can be solved analytically in closed form, the constant of integration in the analytic solution can be determined from the boundary condition; consequently the location of the control section, where the boundary condition is prescribed, is of no concern" (Jain, 2001).

6 PV equation

By combining Eqs. (5) and (11), for cross-section averaged quantities, one obtains the shallow-water vertical PV equation

$$\left(\frac{d}{dt}+\Gamma\right)\Pi = \frac{(curl\vec{F})_z}{\rho h} = -\frac{X\zeta}{h}, \text{ with } \Pi = \frac{\zeta+f}{h} \text{ and } \Gamma = \frac{E|u-u_2|}{h}. \tag{14}$$

In a steady case, Eq. (15) gives

$$\Pi = e^{-\int_0^{\xi}\frac{\Gamma}{u}dx}\left\{\Pi_0 - \int_0^{\xi} e^{\int_0^{x}\frac{\Gamma}{u}dx'}\,\frac{X\zeta}{hu}\,dx\right\}, \tag{15a}$$

which can be significantly simplified if the exponential length scale u/Γ in Eq. (15a) is much larger than the channel length:

$$\Pi \approx \Pi_0 - \int_0^{\xi} \frac{X\zeta}{hu} dx. \tag{15b}$$

Equations (16a) and (16b) confirm that variations in ζ and h, along with frictional effects represented by the presence of X, play a direct role in Π variations. Moreover, because u, h, and X are, in general, rather regular and positive quantities, while ζ is much more variable, Eqs. (14) and (15) suggest that for positive ζ and weak friction – as occurs upstream of a sill – Π must decrease; for a negative ζ and strong friction at the sill region, Π increases.

7 Diagnostic analysis in the Sicily Channel

We now analyze Eqs. (14) and (16), namely $\zeta(\xi)$ and $\Pi(\xi)$, for the realistic case of the EMDW flowing through the Sicily Channel (Fig. 1).

7.1 Sicily Channel hydrographic settings

Cross-channel vertical sections of potential temperature (θ) and salinity (S) along the whole Sicily Channel were performed by A01 during MATER II (10–31 January 1997) and MATER IV (21 April–14 May 1998) cruises (Fig. 1a) in order to investigate the three-layer flow properties, in particular, around the west sill (Figs. 3 and 4). CTD casts were collected over a regular grid (CTD stations ~ 9 km apart from each other; near the sill the distance was reduced to ~ 5 km).

The analysis of potential density (σ), θ, and S, combined with the assumption that the LIW flux is conserved, allowed A01 to estimate the thickness and cross-sectional areas of EMDW, LIW, and MAW layers (Table 1). Current meter measurements were also collected over the western sill (i.e., section IV) and in correspondence with section V (Figs. 1, 3, and 4). These measurements allowed to estimate EMDW and LIW velocities for all sections by the use of continuity. The upper part of LIW was defined by σ ~ 28.80 and the interface between LIW and EMDW by σ ~ 29.11–29.16 (Figs. 3 and 4). The EMDW was defined by using θ-S diagrams, recognizing the bottom density observed along each transect.

A01 analysis showed that the EMDW enters the channel from the east at a depth of ~ 400–550 m, banked against the Sicilian shelf break (Figs. 1b, 3, and 4). There, the width (W) of the current is about 20 km, σ is ~ 29.17, the cross-channel averaged velocity $\bar{u}$ is 12 – 13 cm s^{-1}, and the cross-channel averaged thickness $\bar{h}$ is ~ 75 – 120m (Tables 1 and 2). Further west, the EMDW was observed to sink to depths greater than 700m (transect III in Figs. 3 and 4), rising again at 300–350m depth at the western sill but, rather surprisingly, banked against the Tunisian shelf break (transects IV–V). There, *W* is ~ 8–15 km, *σ* is ~ 29.15, $\bar{h}$ is ~ 25–50 m, and $\bar{u}$ reaches ~ 27–46 cm s^{-1}. At the western mouth of the channel the EMDW sinks again along the Sicilian coast at ~ 1100–1200m (transect VII). Then, it attains a buoyancy equilibrium in the southern Tyrrhenian Sea, where *W* is ~ 20 km, *σ* is ~ 29.12, $\bar{u}$ is ~ 8–17 cms–1, and $\bar{h}$ is ~ 130–200m (Sparnocchia et al., 1999; Figs. 3 and 4). This final sinking is allowed by the small density of the Tyrrhenian LIW (*σ* ~ 29.05).

The initial *θ*–*S* characteristics of the EMDW at the eastern entrance are progressively modified along the vein route (Figs. 3 and 4). These changes are rather weak east of the sill and within the channel, while they become larger in the region west of the sill. The most substantial changes in the hydrographic characteristics are observed between sections V and VII: a gradual increase of both temperature and salinity, indicates a progressive lightening of EMDW from section I (eastern sill) to section IV. This stresses the important role of friction and mixing around the sill region in modifying the hydrographic characteristics of the bottom water and, in turn, enlightens a rather weak mixing processes that is fully discussed in A01.

From these data, A01 also estimated Rossby (*Ro* ~ 0.1) and Froude numbers. Far from the sill the EMDW was characterized by a Froude number of *Fr* ~ 0.1, a small value that would inhibit a strong mixing between LIW and EMDW. Over the sill *Fr* is ~ 0.6–0.8 (Tables 1 and 2). These values, however, are obtained from time averaging and thus depict a steady condition (A01). We believe that *Fr* may reach higher values during strong transient phenomena.

Finally, by assuming quadratic friction, $\vec{F} = -K^* \frac{\rho\,\bar{u}}{\bar{h}}\,\vec{u}$, A01 estimated a dimensionless frictional coefficient, $K^* = 2.6\times10^{-2}$, from the vein momentum balance. This value is rather large with respect to those proposed in the literature – which lie within the range of 2–12 ($\times10^{-3}$) (Baringer and Price, 1997b) – and is likely justified by the very irregular topography of the Sicily Channel around the sill region.

7.2 Diagnostic analysis for vorticity and PV

From hydrographic and current-meter data for the EMDW above described we perform a scale analysis of Eq. (5): considering $L \sim 10^5$ m and $W \sim 10^4$ m as the along-channel and cross-channel space scale, respectively, and $U \sim 10^{-1}$ ms^{-1} as the along-channel velocity, we obtain

$$
\begin{array}{lll}
u\dfrac{\partial}{\partial x}\zeta & +\quad (\zeta+f)\,(div\vec{u}) & =-X\zeta \\
\dfrac{U}{L}\left(\dfrac{U}{R}+\dfrac{U}{W}\right) & +\left(\dfrac{U}{R}+\dfrac{U}{W}+f\right)\left(\dfrac{U}{L}\right) & =X\left(\dfrac{U}{R}+\dfrac{U}{W}\right) \\
\dfrac{1}{T}\left(10^{-6}+10^{-5}\right) & +\dfrac{1}{T}\left(10^{-6}+10^{-5}+10^{-4}\right) & =10^{-5}\times10^{-4}
\end{array}
\qquad (16)
$$

where T ~ $10^{4–5}$ s is the EMDW timescale, f is ~ 10^{-4} s^{-1}, and R ~ 10^5 m is an estimated curvature radius for the EMDW pathway around the sill region; the friction coefficient X ~ 10^{-5} s^{-1} is estimated by considering the value proposed by A01 (i.e., K^*), multiplied by U^2/H ~ 10^{-4} s^{-1} (where H ~ 100m scales for the EMDW thickness). We remark that ζ in Eq. (16) is the sum of a "shear vorticity" (U/W ~ 10^{-5} s^{-1} in the Sicily Channel) and a "curvature vorticity" (U/R ~ 10^{-6} s^{-1} in the Sicily Channel) due to the bending pathway of the EMDW.

The scale analysis in Eq. (16) shows that each term of Eq. (5), and thus of Eq. (14), plays a role in the EMDW dynamics. Friction, in particular, results in being a crucial term in the along-channel evolution of ζ and it brings to a non-conservative PV. Moreover, since (i) $\zeta << f$ in Eq. (16), (ii) $u \approx u_2$ in Eq. (13a), and (iii) the length scale u/Γ ~ 10^6 m in Eq. (15) results in being larger than the entire channel length, one can reasonably use the approximated solutions for vorticity and PV in Eqs. (13b) and (15b). From these considerations we therefore expect a negative trend for ζ when approaching the sill region, followed by a positive trend and a rather large peak of Π immediately after the sill, as confirmed by the detailed results we describe below. The following analysis of Eqs. (13) and (15) in their closed form is performed by using continuous functions for $u(\xi)$ and $h(\xi)$, which are computed from modified spline interpolations of $\overline{u}_i$ and $\overline{h}_i$ as obtained from the in situ data (Appendix B). Velocity interpolations are also compared with PROTHEUS numerical data (Fig. 5), a relatively coarse resolution Mediterranean model (1/8° ×1/8°) based on the MIT general circulation model (MITgcm; Sannino et al., 2009; Sannino, personal communication), and show fair agreement with the splines. Due to the coarse vertical resolution of this model, such a comparison cannot be provided for the bottom water thickness $\overline{h}_i$.

MATER II cruise (January 1997)

For this data set (Figs. 3 and 5, Table 3) we see that both ζ and Π are rather small upstream of the sill, namely ζ ~ 5×10^{-6} s^{-1} or less and Π ~ 8×10^{-7} s^{-1} m^{-1} (Fig. 6). Approaching the sill, vorticity changes sign (Fig. 6) due to the stretching term $\dfrac{u}{h}\dfrac{\partial h}{\partial \xi}$ in Eq. (13). A negative value ζ ~ -6×10^{-5} s^{-1} is then reached at transect IV, and consequently Π reaches a very large peak ~ 6×10^{-6} s^{-1} m^{-1} at transect V.

Downstream of the sill, in the southern Tyrrhenian Sea, ζ again has a positive value, $\zeta \sim 6 \times 10^{-5}$ s^{-1}, and Π strongly decreases to $8{\times}10^{-7}$ s^{-1} m^{-1} (Fig. 6).

MATER IV cruise (April–May 1998)

This springtime data set (Figs. 4 and 5, Table 4), although similar to the one described above, shows lower velocities and fluxes than those of the winter case (Fig. 6; Stansfield et al., 2001). Upstream of the sill ζ is ~ $5{\times}10^{-6}$ s^{-1} and Π is ~ $1{\times}10^{-6}$ s^{-1} m^{-1}, while, for a region about 120 km long before the sill, ζ goes from ~ 10^{-6} to $-7{\times}10^{-5}$ s^{-1}. In the same way, Π goes from ~ 10^{-6} s^{-1} m^{-1} to ~ $6{\times}10^{-6}$ s^{-1} m^{-1} immediately after the sill (Fig. 6). In the southern Tyrrhenian Sea, downstream of the sill, ζ ~ $4{\times}10^{-5}$ s^{-1}. This shows a sudden change in vorticity, as for the MATER II cruise data (Fig. 6). Accordingly, Π decreases strongly from the largest value at the sill to ~ $1.5{\times}10^{-6}$ s^{-1} m^{-1} in the Tyrrhenian Sea, and then strongly decreases.

8 Discussions

The lack of specific current-meter measurements does not allow for a realistic determination of vorticity and, in particular, for a validation of our model. Moreover, the use of available numerical outputs in order to validate and/or compare our analytic results is not an easy task due to grid problems (Dr. G. Sannino, personal communication, 2015; Dr. L. Palatella, personal communication, 2015): spatial (vertical and horizontal) resolutions are often too coarse and, in particular, bottom velocities are available on a very few cross-stream grid points (i.e., one or two at the western sill).

A rough, although reasonable, way to infer the EMDW vorticity independently from our model is given by the following considerations: since the EMDW path is rather straight upstream of the sill (Fig. 1), the curvature vorticity of this flow along the upstream region of the channel is very small (Holton, 1972). Therefore, initial values of vorticity for our analysis are taken from the shear vorticity only, which is approximately $\zeta_0 \sim U/W$ (Fig. 6). Although this approximation – taken as an initial condition for our vorticity analysis – can be affected by a large error, Eq. (13) shows that the “memory” of the initial vorticity ζ_0 vanishes within a few kilometers.

A different option for determining ζ is suggested through use of the classical Π conservation: $\zeta = \frac{f\,h}{h_\infty} - f$ (Gill, 1982), where h_∞ is the bottom depth far upstream, in the Ionian Sea. This suggests that a vorticity stream-wise profile should look approximately like the EMDW thickness profile. However, such an estimate of ζ only holds far from the sill, where friction and mixing certainly do not affect the deep current.

Our diagnosis, through use of the A01 experimental data set, confirms the “memory-loss” effect of upstream vorticity conditions due to the role of friction. We found that the region around the sill (~ 70 km length) has an unexpected negative peak of ζ that, moreover, seems to be also in agreement with the EMDW–LIW interface tilting that occurs at the sill (Figs. 3 and 4) in terms of change in flow curvature.

Abrupt changes in vorticity are also reflected in the downstream evolution of PV, which is definitely not constant around the sill region. This interesting result points out that an increase in Π violates the all those assumptions for flow stability theorems (see, for instance, Wood and McIntyre, 2010).

An interesting aside, we check the reliability of the idealized friction coefficient by investigating the balance of the PV Eq. (14) for the EMDW along-channel evolution. The nonlinear friction $\vec{F} = -K^* \frac{\rho\,\overline{u}}{\overline{h}}\,\vec{u}$ described above, with the constant friction coefficient $K^* = 2.6\times10^{-2}$ (A01), gives the following PV balance:

$$\left(\frac{d}{dt} + \Gamma\right)\Pi = \frac{(curl\vec{F})_z}{\rho h} \approx -\frac{2K^* u\zeta}{h^2} \sim 10^{-11}\ \mathrm{m}^{-1}\mathrm{s}^{-2}. \tag{17}$$

Equation (17) is nicely satisfied in the upstream part of the Sicily Channel, while this agreement fails over the sill (Fig. 7). Therefore, to investigate such a discrepancy, we analyze

$$\begin{aligned}\varepsilon = \left(u\frac{\partial}{\partial\xi} + \Gamma\right)\frac{\zeta+f}{h} - \frac{(curl\vec{F})_z}{\rho h} &= \left(u\frac{\partial}{\partial\xi} + \Gamma\right)\frac{\zeta+f}{h} + \frac{2K^*}{h^2}u\zeta \\ &= \underbrace{u\frac{d}{d\xi}\Pi}_{\varepsilon_1} + \underbrace{\Gamma\Pi}_{\varepsilon_2} + \underbrace{\frac{2K^*}{h^2}u\,\zeta}_{\varepsilon_3} \approx 10^{-10} - 10^{-11} s^{-2} m^{-1}\end{aligned} \tag{18}$$

We point out that the analysis of each term of Eq. (18) does not use the explicit solution for Π in Eqs. (15a) and (15b) but only the vorticity ζ(ξ) in Eq. (13b) since $\Pi = \frac{\zeta + f}{h}$. All this represents therefore a sort of independent validation of the PV balance (17).

For both MATER cruises, the along-channel profiles of the three terms ε_1, ε_2, and ε_3 (Fig. 7) are rather small but never exactly balanced, in particular around the sill region. For the MATER IV cruise, which was characterized by lower velocities, this unbalance seems to be due to the variability of the entrainment term ε_2, when approaching the sill, and to the advection term ε_1, which results in being too small for balancing the friction term ε_3.

This suggests that some tuning of the quadratic friction coefficient is needed. Consequently, we propose the use of a varying friction coefficient, namely $K^* \rightarrow K^* + X^*(\xi)$. Indeed, large values of ε around the sill (Fig. 7) suggest that both local roughness due to the sea-bottom morphology over the sill and an additional frictional effect due to the strong mixing occurring at the sill could affect the local schematization for friction. To optimize the balance of Eq. (18), we set

$$0 = u\frac{d}{dx}\Pi + \Gamma\Pi + \frac{K^* + \chi^*(\xi)}{h^2}u\,, \qquad (19)$$

which leads to local solutions for $X^*(\xi)$ (Fig. 8).

In the region where the bottom of the channel is rather flat, i.e., until 350 km from the beginning of the channel (around transect IIIa, Fig. 1), one obtains $X^*(\xi) << K^*$, in good agreement with the A01 coefficient. Then, approaching the sill, a ~ 50% greater friction coefficient is required to satisfy Eq. (19) (Fig. 8). We note that a similar approach for seeking a more realistic frictional coefficient along particular morphological settings (such as straits and channels) was also pursued by Baringer and Price (1997a, b). Their results showed that (i) “large bottom friction coupled with the relatively small thickness of outflows may lead to a turbulent bottom boundary layer that extends over much of the total thickness of the outflow” and (ii) “the bottom stress appears to follow a quadratic drag law, though the appropriate c_D [i.e., dimensionless friction coefficient] will vary considerably with the type of average velocity available for the parameterization”. Both conclusions are in agreement with our results.

9 Conclusions

We investigated vorticity (ζ) and PV (Π) evolution of the EMDW flowing along the Sicily Channel by making use of a shallow-water, stream-tube approach. The model allowed us to explore bottom current properties under the effect of sea-bottom changes, bottom friction, and vertical entrainment. Our analysis reveals sharp negative vorticity peaks over the sill region, while ζ again becomes positive downstream of the sill, as they were in the eastern basin. All this reflects on the PV behavior of the bottom currents, which experience large variations in Π, and reveals how PV-constant models are not suitable for exploring bottom currents dynamics along rotating channels. We argue that the along-channel evolution of both vorticity and PV is due to bathymetric effects occurring approaching the sill, which are also modulated by frictional effects that significantly change the structure of vorticity and PV equations for describing such dynamics.

Knowledge of the downstream evolution of ζ allowed us (i) to infer the deep vein dynamics, in particular, around the sill region, where frictional, entrainment, and stretching effects all play a crucial role; (ii) to diagnose the PV balance; and thus (iii) to tune the parameterization for bottom friction. In this regard, our analysis is a general implication of the steady, deep, and baroclinic current theory in marine straits (Smith, 1975; Killworth, 1977; Hogg, 1983) and it can provide an analytic support to numerical and tank experiments aimed to the investigation of rotating hydraulic dynamics.

Appendix A: The cross-sectional averages

We evaluate here the cross-sectional averages of various terms of the vorticity Eq. (7). Let us first assume that, for a narrow and long strait or channel, the derivative $\frac{\partial}{\partial \psi}a >> \frac{\partial}{\partial \xi}a$ cross-strait, where *a* is a general flow property. We then define the cross-channel average as $\bar{a} = \int adzd\psi$.

For a nonlinear friction $\vec{F} = -\rho_3 X(\vec{u}_3, h_3)\vec{u}_3$, the cross-channel averaging would therefore give

$$\overline{\frac{1}{\rho}(curl\vec{F})_z} = K\frac{\bar{u}}{\bar{h}}\overline{\frac{\partial u}{\partial \psi}} = -K\frac{\bar{u}}{\bar{h}}\bar{\zeta} \equiv -\mathrm{X}\bar{\zeta} \qquad \text{(A1)}$$

since $\bar{u}$ and $\bar{h}$ by definition are functions of ξ only.

Accordingly, the second term on the left-hand side of the vorticity Eq. (6) can be averaged by considering that $\int d\psi \partial_\psi v = 0$ since v is symmetric and vanishes at the vein lateral borders. Therefore, one obtains

$$\overline{div\,\vec{u}} \equiv \int dz\,d\psi(\partial_\xi u + \partial_\psi v) = \int dz\,d\psi\,\partial_\xi u = \overline{\partial_\xi u} \qquad \text{(A2)}$$

This, moreover, results that

$$\overline{\zeta\frac{\partial u}{\partial \xi} + u\frac{\partial \zeta}{\partial \xi}} = \iint \frac{\partial}{\partial \xi}(u\zeta)\,dz\,d\psi \approx \frac{\partial}{\partial \xi}\iint (u\zeta)\,dz\,d\psi \approx \frac{\partial}{\partial \xi}\bar{u}\iint \zeta\,dz\,d\psi = \bar{u}\frac{\partial \bar{\zeta}}{\partial \xi} + \bar{\zeta}\frac{\partial \bar{u}}{\partial \xi}$$

(A3)

since *u* is less variable than *ζ*. All of this leads to the cross-averaged vorticity equation in the steady case

$$\overline{u}\frac{\partial}{\partial \xi}\overline{\zeta} + (f + \overline{\zeta})\,\overline{div\vec{u}} \approx \overline{\frac{1}{\rho}\left(curl\vec{F}\right)_z} = -\mathrm{X}\overline{\zeta} \tag{A4}$$

and thus to the corresponding solution, Eq. (10), in the main text for $\overline{\zeta} << f$.

Similarly, the mass conservation equation

$$u\frac{\partial}{\partial \xi}h + h\ div\ \vec{u} \approx E\left|u - u_2\right| \tag{A5}$$

becomes Eq. (11) in the main text.

Appendix B: The spline interpolation of the *ui* and *h*i

To use the cross-averaged EMDW data of A01 (Tables 1 and 2), we computed a continuous $u(\xi)$ and $h(\xi)$ spline interpolation of the u_i and h_i from transect i = 1, 2, . . . (Fig. 6). This problem may be solved exactly by fitting a polynomial of degree n–1. Unfortunately, for such a "polynomial" solution, it is not easy to control for the influence of any particular observation. Moreover, it can behave very strangely at the boundaries.

Spline interpolation achieves a better result. In order to enhance the spline flexibility around the sill, following Durbin and Koopman (2001) we introduce a scale parameter σ that varies as $\sigma^2 = v^2 + \mu\Lambda(\xi)$ with $\mu >> v^2$. For such a "modified" spline interpolation, one has

$$\Lambda(\xi) = \frac{35}{32}\left[1 - \left(\frac{2(\xi - \Delta\xi_m)}{\xi_m - \xi_{m-1}}\right)^2\right]^3, \text{ where } \Delta\,\xi_m = (\xi_m - \xi_{m-1})/2, \tag{B1}$$

for all points in an interval $\xi_{m-1} < \xi < \xi_{m+1}$ and zero elsewhere. We moreover impose that our transect V is a local minimum for $h(\xi)$ and a maximum of $u(\xi)$ as shown in Fig. 6. Note that, although these last plots might look rather discontinuous over the sill, in reality this apparent effect is due to the vigorous evolution of the current over the sill. We moreover compare such a modified spline interpolation of u_i with monthly averaged data of PROTHEUS (see text). Along-channel velocities for January 1997 and April 1998 cruises are shown in Fig. 5, superimposed on modified spline interpolations of both MATER II and MATER IV.

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Transect	**I**	**II**	**III**	**IV**	**V**	**VIc**	**VII**	Units
σ_{bottom}	29.168	29.165	29.163	29.157	29.150	29.124	29.117	kg m^{-3}
h	120	150	140	100	50	150	200	m
g'	3	3	4	6	6	3	2	10^{-4} m s^{-2}
bottom depth	550	600	800	530	350	600	1200	m
distance between transects	0	80	135	170	25	65	25	km
W	20	80	40	15	15	30	20	km
u_2 (LIW)	12	5	3.2*	18	53	11	7	cm s^{-1}
u_3 (EMDW)	13	8	5*	14	46	15	17	cm s^{-1}
ϕ (EMDW)	0.23	0.26	0.20	0.23	0.32	0.35	0.34	Sv
$F_r = \left\lvert \frac{u - u_2}{\sqrt{g'h}} \right\rvert$	0.1	0.2	0.1	0.2	0.8	0.2	0.5	
E^*	/	~ 0	10^{-5}	10^{-4}	2×10^{-4}	9×10^{-4}	3×10^{-4}	

Table 1. Main experimental quantities measured by A01 in the Sicily Channel and in the Southern Tyrrhenian Sea during the MATER II cruise (Figs. 1 and 3). Here σ_{bottom} is the maximum σ_θ observed in the hydrographic casts; h is the bottom layer thickness; g' the reduced gravity; ϕ (EMDW) is the EMDW volume transport; the Froude number is Fr; the entrainment parameter E^* is the one computed by Baringer and Price (1997b). *LIW and EMDW velocities for section III were obtained from current meter measurements. Velocities for the sections were obtained by using continuity, considering the total transport and dividing by the cross-sectional area

Transect	**I**	**III**	**IIIa**	**IV**	**V**	**VII**	*Units*
σ_{bottom}	29.167	29.165	29.163	29.156	29.148	29.119	kg m^{-3}
h	75	125	100	50	25	130	m
g'	8.3	8.2	8	7.8	7.4	6	10^{-4} m s^{-2}
bottom depth	550	700	650	500	360	1150	m
distance between transects		155	170	60	25	90	Km
W	15	5	22	15	8	18	Km
u_2 (LIW)	10	2	12	13	35	5	cm s^{-1}
u_3 (EMDW)	12	6	3	8	27	8	cm s^{-1}
ϕ (EMDW)	5	5.4	7.2	8	10	12	10^{-2} Sv
$F_r = \left\lvert \frac{u - u_2}{\sqrt{g'h}} \right\rvert$	0.1	0.05	0.3	0.2	0.6	0.03	
E^*	2x10^{-5}	10^{-5}	1.3x10^{-4}	4x10^{-4}	4.5x10^{-4}	3x10^{-4}	

Table 2. As Table I, but for the MATER IV cruise (Fig. 1 and 4).

Figure Captions

Figure 1a. General map of the Sicily Channel: the channel length is ~ 500 km, with two sills at its eastern and western entrances (~ 550 and ~ 350m deep, respectively). Dots indicate the hydrographic stations of all cross-section vertical transects; triangles indicate the position of current-meter chains. The Ionian Sea is on the southeastern side of the map. From Astraldi et al. (2001).

Figure 1b. Main routes of the principal water masses flowing through the region: LIW (Levantine Intermediate Water, dashed line), EMDW (Eastern Mediterranean Deep Water, solid line), and MAW (Modified Atlantic Water, bold line). The trajectory of the EMDW corresponds to the centerline of the vein in the different hydrographic sections. After Astraldi et al. (2001).

Figure 2. (a) Schematic representation of the three layers in a cross-flow vertical transect. The interface are at $z = h_1$ for the air–sea surface, at $z = H_1+h_2$, and at $z = H_1+H_2+h_3$ for the lower interfaces, with H_1 = const and bottom depth = $H_1 + H_2 + H_3$. (b) Diagram of a bottom current also showing the (x, y) and (ξ, ψ) coordinate systems. Modified from Astraldi et al. (2001).

Figure 3. MATER II cruise (January 1997): (a) characteristic isopycnal cross sections between MAW, LIW, and EMDW. In these sections, Tunisia is on the left side. Note that, in section IV, the EMDW flows only in the western passage of the cross section; interfacial slope modification is also visible in section V. (b) Evolution of $\theta - S$ values of EMDW close to the bottom. From Astraldi et al. (2001).

Figure 4. MATER IV cruise (April–May 1998): (a) characteristic isopycnal cross sections between surface Atlantic water, LIW, and EMDW. In these sections, Tunisia is on the left side. Note that, in section IIIa, the EMDW flows only in the western passage of the cross section; interfacial slope modification is also visible in sections IIIa, IV, and V. (b) Evolution of $\theta - S$ values of EMDW close to the bottom. From Astraldi et al. (2001).

Figure 5. Modified spline interpolation of h_i (m) and u_i in (cm s^{-1}) along ξ (km); Roman numerals indicate hydrograph transects shown in Figs. 3 and 4, for MATER II cruise and MATER IV, respectively. The black arrows at the top show the position of the sill. Diamonds represent the cross-sectional maximum velocities as obtained by the Sannino et al. (2009) numerical model (see text).

Figure 6. Analytic profiles for ζ (s–1) and Π (m^{-1} s^{-1}) along ξ (km) as obtained from Eqs. (13) and (15), respectively. The dashed lines indicate approximate solutions for $\zeta << f$ and $\frac{1}{h}E(u-u_2)\approx 0$, i.e., Eq. (13b). Position of the transects is also shown (see Figs. 3 and 4) for MATER II and MATER IV cruises. The arrows show the position of the sill.

Figure 7. Analytic profile for the various terms in Eq. (18) along ξ (m), namely the friction term (bold line), the PV-advection term (dots line), and the entrainment term (thin line). Position of the transects is shown in Figs. 3 and 4, for MATER II cruise and MATER IV, respectively. The black arrow shows the position of the sill.

Figure 8. Variations in χ^*, defined as $K^* \rightarrow K^* + \chi^*(\xi)$, along ξ (m), obtained through optimizing the balance of Eq. (19).