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Study of the tidal dynamics of the Korea Strait using the extended Taylor method

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Tidal wave propagation in one-dimensional channels with abrupt depth/width changes

S1. Basic Equations

We study tidal wave propagation in channels with abrupt depth/width changes. To be specific, we consider a one-dimensional problem corresponding to the model shown in Fig. 3 of our paper. For simplicity, Area3 is combined into Area2, and the Coriolis force and friction are neglected, then Eqs. (10) and (11) in the Sect. 2.2 of our paper can be simplified as follows:

$$u_{1,-}(x) = -a_1 \exp[ik_1(x - l_1)] \tag{S1}$$

$$\zeta_{1,-}(x) = p_1 a_1 \exp[ik_1(x - l_1)] \tag{S2}$$

$$u_{1,+}(x) = b_1 \exp[-ik_1(x - l_1)] \tag{S3}$$

$$\zeta_{1,+}(x) = p_1 b_1 \exp[-ik_1(x - l_1)] \tag{S4}$$

$$u_{2,-}(x) = -a_2 \exp[ik_2(x - l_2)] \tag{S5}$$

$$\zeta_{2,-}(x) = p_2 a_2 \exp[ik_2(x - l_2)] \tag{S6}$$

$$u_{2,+}(x) = b_2 \exp[-ik_2(x - l_2)] \tag{S7}$$

$$\zeta_{2,+}(x) = p_2 b_2 \exp[-ik_2(x - l_2)] \tag{S8}$$

where $k_j = \sigma/c_j$ is the wave number, with $c_j = \sqrt{gh_j}$ representing the wave speed in Areaj, j=1, 2; $p_j = \sqrt{h_j/g}$; l_1 is the x coordinate at the opening of Area1; and $l_2 = l_1 + L_1$ is the x coordinate of the connecting point of Area1 and Area2, where an abrupt change in depth and/or width occurs. In Eqs. (S1) to (S8), we have changed the notations $u_{j,1}$, $\zeta_{j,1}$, $u_{j,2}$ and $\zeta_{j,2}$ in Eqs. (10) and (11) of Wu et al. (2021) to $u_{j,-}$, $\zeta_{j,-}$, $u_{j,+}$, and $\zeta_{j,+}$ (j=1, 2), respectively, to indicate the directions of wave propagation. That is, $\zeta_{j,+}(x)$ and $u_{j,+}(x)$ represent the complex amplitudes of tidal level and tidal current of the tidal waves that travel in the positive x direction in Areaj, respectively; and $\zeta_{j,-}(x)$ and $u_{j,-}(x)$ represent those travelling in the negative x direction in Areaj, respectively.

The open boundary condition at $x = l_1$ can be specified as follows:

$$\zeta_{1,+}(l_1) = H_I \exp(-i\theta_1), \tag{S9}$$

where H_I and θ_1 represent the amplitude and phase lag of the incident wave at the opening of Area1, respectively. From Eqs. (S9) and (S4) we obtain

$$p_1 b_1 = H_I \exp(-i\theta_1), \tag{S10}$$

and

$$\zeta_{1,+}(x) = H_I \exp\{-i[k_1(x - l_1) + \theta_1]\}. \tag{S11}$$

Therefore,

$$\zeta_{1,+}(l_2) = H_I \exp\left[-i(\chi_1 + \theta_1)\right], \quad \text{(S12)}$$

where

$$\chi_1 = k_1 L_1. \quad \text{(S13)}$$

The matching conditions at $x = l_2 = l_1 + L_1$ are as follows:

$$\zeta_{1,+}(l_2) + \zeta_{1,-}(l_2) = \zeta_{2,+}(l_2) + \zeta_{2,-}(l_2), \quad \text{(S14)}$$

and

$$\left[u_{1,+}(l_2) + u_{1,-}(l_2)\right] h_1 W_1 = \left[u_{2,+}(l_2) + u_{2,-}(l_2)\right] h_2 W_2. \quad \text{(S15)}$$

To use the relationship among tidal elevations instead of tidal currents, we multiply Eq. (S15) by $p_1/h_1 W_1$ and obtain

$$\zeta_{1,+}(l_2) - \zeta_{1,-}(l_2) = \rho\left[\zeta_{2,+}(l_2) - \zeta_{2,-}(l_2)\right], \quad \text{(S16)}$$

where

$$\rho = \frac{p_1 h_2 W_2}{p_2 h_1 W_1} = \frac{\sqrt{h_2} W_2}{\sqrt{h_1} W_1} \ . \quad \text{(S17)}$$

S2. Solution for the case with semi-infinite Area2

Here, we first investigate a simpler case that has been previously studied by Dean and Dalrymple (1984). In this case, Area2 is assumed to be semi-infinitely long so that the wave can propagate freely in the positive *x* direction without reflection, meaning that $a_2 = 0$. Thus, the terms $\zeta_{2,-}$ in Eqs. (S6), (S14) and (S16) are all equal to zero. From Eqs. (S14) and (S16) with $\zeta_{2,-}(l_2) = 0$ we obtain

$$\zeta_{1,-}(l_2) = \kappa_R \zeta_{1,+}(l_2), \quad \text{(S18)}$$

and

$$\zeta_{2,+}(l_2) = \kappa_T \zeta_{1,+}(l_2), \quad \text{(S19)}$$

where κ_R and κ_T are called reflection and transmission coefficient respectively. These coefficients are equal to the following:

$$\kappa_R = \frac{1-\rho}{1+\rho} \ , \quad \text{(S20)}$$

and

$$\kappa_T = \frac{2}{1+\rho} \ , \quad \text{(S21)}$$

If $\rho > 1$, namely, if $\sqrt{h_2} W_2 > \sqrt{h_1} W_1$, then $\kappa_R < 0$. It is more desired to write Eq. (S20) in the following form:

$$\kappa_R = \frac{\rho-1}{\rho+1} \exp(-i\pi), \quad \text{(S22)}$$

which is Eq. (33) in the text.

From Eqs. (S2), (S12), (S18) and (S22) we obtain

$$\zeta_{1,-}(x) = \frac{\rho-1}{\rho+1} H_I \exp\{-i[-k_1(x - l_1) + 2\chi_1 + \theta_1 + \pi]\}, \quad \text{(S23)}$$

and from Eqs. (S8), (S12), (S19) and (S21) we obtain

$$\zeta_{2,+}(x) = \frac{2}{1+\rho} H_I \exp\{-i[k_2(x - l_2) + \chi_1 + \theta_1]\}. \quad \text{(S24)}$$

Finally, we obtain the following solution:

$$\begin{cases} \zeta(x) = H_I\left(\exp\{-i[k_1(x - l_1) + \theta_1]\} + \frac{\rho-1}{\rho+1}\exp\{-i[-k_1(x - l_1) + 2\chi_1 + \theta_1 + \pi]\}\right), & l_1 \ll x \ll l_2, \\ \zeta(x) = \frac{2}{1+\rho} H_I \exp\{-i[k_2(x - l_2) + \chi_1 + \theta_1]\}, & l_2 \ll x, \end{cases} \quad \text{(S25)}$$

which is Eq. (34) in the text of our paper.

S3. Solution for the case with finite Area2

In the following, we investigate a more complicated case that is more suitable to the KS-JS basin. In this case, Area2 is closed at its right end, and a boundary condition is thus involved:

$$u_{2,+}(l_2 + L_2) + u_{2,-}(l_2 + L_2) = 0. \quad \text{(S26)}$$

From Eqs. (S5) to (S8), we see that Eq. (S26) is equivalent to the following:

$$\zeta_{2,+}(l_2 + L_2) - \zeta_{2,-}(l_2 + L_2) = 0. \quad \text{(S27)}$$

This further gives us

$$b_2 = a_2 \exp(i2\chi_2), \quad \text{(S28)}$$

where

$$\chi_2 = k_2 L_2. \quad \text{(S29)}$$

Hence we have

$$\zeta_{2,-}(l_2) = \zeta_{2,+}(l_2)\exp(-i2\chi_2). \quad \text{(S30)}$$

Therefore, Eqs. (S14) and (S16) can be rewritten in the following respective forms:

$$\zeta_{1,+}(l_2) + \zeta_{1,-}(l_2) = [1 + \exp(-i2\chi_2)]\zeta_{2,+}(l_2), \quad \text{(S31)}$$

and

$$\zeta_{1,+}(l_2) - \zeta_{1,-}(l_2) = \rho[1 - \exp(-i2\chi_2)]\zeta_{2,+}(l_2). \quad \text{(S32)}$$

Eliminating $\zeta_{2,+}(l_2)$ in above two equations results in

$$\zeta_{1,-}(l_2) = \frac{[1+\exp(-i2\chi_2)] - \rho[1-\exp(-i2\chi_2)]}{[1+\exp(-i2\chi_2)] + \rho[1-\exp(-i2\chi_2)]}\zeta_{1,+}(l_2). \quad \text{(S33)}$$

A few steps of algebra give us

$$\frac{1-\exp(-i2\chi_2)}{1+\exp(-i2\chi_2)} = \frac{i\sin 2\chi_2}{1+\cos 2\chi_2}. \quad \text{(S34)}$$

Substitution of Eq. (S34) in Eq. (S33) yields

$$\zeta_{1,-}(l_2) = \frac{1+\cos 2\chi_2 - i\rho\sin 2\chi_2}{1+\cos 2\chi_2 + i\rho\sin 2\chi_2}\zeta_{1,+}(l_2). \quad \text{(S35)}$$

Let

$$\begin{cases}\cos\delta = \frac{1+\cos 2\chi_2}{[(1+\cos 2\chi_2)^2+(\rho\sin 2\chi_2)^2]^{1/2}}, \\ \sin\delta = \frac{\rho\sin 2\chi_2}{[(1+\cos 2\chi_2)^2+(\rho\sin 2\chi_2)^2]^{1/2}},\end{cases} \tag{S36}$$

then (S35) reduces to

$$\zeta_{1,-}(l_2) = \zeta_{1,+}(l_2)\exp(-i2\delta), \tag{S37}$$

which is an equivalent form of Eq. (34) in the text of our paper.

From Eqs. (S4) and (S10) we have

$$\zeta_{1,+}(x) = H_I \exp\{-i[k_1(x-l_1)+\theta_1]\}, \tag{S38}$$

and from Eqs. (S12) and (S37) we have

$$\zeta_{1,-}(l_2) = H_I \exp[-i(\chi_1+\theta_1+2\delta)]. \tag{S39}$$

Meanwhile, Eq. (S2) gives us

$$\zeta_{1,-}(l_2) = p_1 a_1 \exp(i\chi_1). \tag{S40}$$

Comparison of Eq. (S40) with Eq. (S39) gives

$$p_1 a_1 = H_I \exp[-i(2\chi_1+\theta_1+2\delta)]. \tag{S41}$$

On substituting Eq. (S41) into Eq. (S2) we have

$$\zeta_{1,-}(x) = H_I \exp\{-i[-k_1(x-l_1)+2\chi_1+\theta_1+2\delta]\}. \tag{S42}$$

From Eqs. (S31) and (S32) we obtain

$$\zeta_{2,+}(l_2) = 2[(\rho+1)-(\rho-1)\cos 2\chi_2 + i(\rho-1)\sin 2\chi_2]^{-1}\zeta_{1,+}(l_2). \tag{S43}$$

Let

$$\begin{cases}E\cos\phi = (\rho+1)-(\rho-1)\cos 2\chi_2, \\ E\sin\phi = (\rho-1)\sin 2\chi_2,\end{cases} \tag{S44}$$

and

$$\varepsilon = 2E^{-1}, \tag{S45}$$

then (S43) reduces to

$$\zeta_{2,+}(l_2) = \varepsilon\exp(-i\phi)\zeta_{1,+}(l_2). \tag{S46}$$

Inserting Eq. (S12) into Eq. (S46) yields

$$\zeta_{2,+}(l_2) = \varepsilon H_I \exp[-i(\chi_1+\phi+\theta_1)]. \tag{S47}$$

From Eq. (S8), we know that $p_2 b_2 = \zeta_{2,+}(l_2)$, thus we further have

$$\zeta_{2,+}(x) = \varepsilon H_I \exp\{-i[k_2(x-l_2)+(\chi_1+\phi+\theta_1)]\}. \tag{S48}$$

Likewise, we can obtain the following solution for $\zeta_{2,-}(x)$ from Eqs. (S6) and (S47):

$$\zeta_{2,-}(x) = \varepsilon H_I \exp\{-i[-k_2(x-l_2)+(2\chi_2+\chi_1+\phi+\theta_1)]\}. \tag{S49}$$

Finally from Eqs. (S38), (S42), (S48) and (S49), we obtain the solution for $\zeta(x)$:

$$\begin{cases} \zeta(x) = H_I(\exp\{-i[k_1(x-l_1)+\theta_1]\} + \exp\{-i[-k_1(x-l_1)+2\chi_1+\theta_1+2\delta]\}), & l_1 \ll x \ll l_2, \\ \zeta(x) = \epsilon H_I(\exp\{-i[k_2(x-l_2)+(\chi_1+\phi+\theta_1)]\} + \exp\{-i[-k_2(x-l_2)+(2\chi_2+\chi_1+\phi+\theta_1)]\}), & l_2 \ll x \ll l_3, \end{cases} \tag{S50}$$

which is Eq. (37) in the text of our paper.